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PREFACE.

In this book the subject has been treated in the manner recommended in the Report, issued by the Mathematical Association, on the Teaching of Arithmetic in Preparatory Schools.

A full account of the plan of the book is unnecessary in this place, as it will be understood more clearly from the table of contents. The matter is there given fully and in detail, and it will be seen that a very complete and comprehensive course is provided. The steps are very gradual; abstract and concrete questions are dealt with side by side as far as possible, and problems such as easy questions involving the unitary method, averages, etc., are not treated separately, but are worked into the general scheme. No questions are marked out specifically for oral work, as it will be found that the earlier sections can be used for this purpose when a pupil is working a later section on paper.

I should express my acknowledgments of the great help which I received from Mr. W. S. Beard in the preparation of this book, and I must thank my brother, Mr. W. M. Pendlebury, for his kindness in verifying many of the answers.

C. P.

1*st July*, 1912.

NOTE TO THE FIFTH AND ENLARGED EDITION.

Much new matter has been added to cover the syllabus issued by the Joint Committee of the Headmasters' Conference and the Association of Preparatory Schools—the basis of the papers of the Common Examination for Entrance to Public Schools. The

new sections are numbered A^1—P^1; the detailed table of contents shows the systematic and progressive arrangement.

At the end of the book is a collection of well graduated revision papers, which can be used at convenient specified stages. The 101 short revision tests are followed by longer examination papers to be worked in a stated time.

Throughout the book such guides and directions as are given for the solution of questions are deduced as the outcome of simple explanation and simple reasoning. In the preparation of the sections on elementary Mensuration, I must thank Mr. W. J. Stainer for illustrations, suited to young students, adapted from his *Junior Practical Mathematics*, published by Messrs. G. Bell and Sons.

C. P

April, 1924.

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MULTIPLICATION TABLE.

Twice			3 times			4 times			5 times			6 times		
1	are	2	1	are	3	1	are	4	1	are	5	1	are	6
2	...	4	2	...	6	2	...	8	2	...	10	2	...	12
3	...	6	3	...	9	3	...	12	3	...	15	3	...	18
4	...	8	4	...	12	4	...	16	4	...	20	4	...	24
5	...	10	5	...	15	5	...	20	5	...	25	5	...	30
6	...	12	6	...	18	6	...	24	6	...	30	6	...	36
7	...	14	7	...	21	7	...	28	7	...	35	7	...	42
8	...	16	8	...	24	8	...	32	8	...	40	8	...	48
9	...	18	9	...	27	9	...	36	9	...	45	9	...	54
10	...	20	10	...	30	10	...	40	10	...	50	10	...	60
11	...	22	11	...	33	11	...	44	11	...	55	11	...	66
12	...	24	12	...	36	12	...	48	12	...	60	12	...	72

7 times			8 times			9 times			10 times			11 times			12 times		
1	are	7	1	are	8	1	are	9	1	are	10	1	are	11	1	are	12
2	...	14	2	...	16	2	...	18	2	...	20	2	...	22	2	...	24
3	...	21	3	...	24	3	...	27	3	...	30	3	...	33	3	...	36
4	...	28	4	...	32	4	...	36	4	...	40	4	...	44	4	...	48
5	...	35	5	...	40	5	...	45	5	...	50	5	...	55	5	...	60
6	...	42	6	...	48	6	...	54	6	...	60	6	...	66	6	...	72
7	...	49	7	...	56	7	...	63	7	...	70	7	...	77	7	...	84
8	...	56	8	...	64	8	...	72	8	...	80	8	...	88	8	...	96
9	...	63	9	...	72	9	...	81	9	...	90	9	...	99	9	...	108
10	...	70	10	...	80	10	...	90	10	...	100	10	...	110	10	...	120
11	...	77	11	...	88	11	...	99	11	...	110	11	...	121	11	...	132
12	...	84	12	...	96	12	...	108	12	...	120	12	...	132	12	...	144

BRITISH MONEY TABLE.

Farthings.		Pence.			Shillings.		
	s. *d.*		*s.*	*d.*		£	*s.*
4 make	1	12 make	1	0	20 make	1	0
6	1½	20	1	8	30	1	10
8	2	24	2	0	40	2	0
10	2½	30	2	6	50	2	10
12	3	36	3	0	60	3	0
14	3½	40	3	4	70	3	10
16	4	48	4	0	80	4	0
18	4½	50	4	2	90	4	10
20	5	60	5	0	100	5	0
22	5½	70	5	10	110	5	10
24	6	72	6	0	120	6	0
26	6½	80	6	8	130	6	10
28	7	84	7	0	140	7	0
30	7½	90	7	6	150	7	10
32	8	96	8	0	160	8	0
34	8½	100	8	4	170	8	10
36	9	108	9	0	180	9	0
38	9½	110	9	2	190	9	10
40	10	120	10	0	200	10	0
42	10½	130	10	10	250	12	10
44	11	132	11	0	300	15	0
46	11½	140	11	8	400	20	0
48	1 0	144	12	0	500	25	0

BRITISH MEASURES.

AVOIRDUPOIS WEIGHT.

16 ounces make 1 **pound** (1 lb.).
14 pounds ..... 1 **stone** (1 st.).
2 stones ..... 1 **quarter** (1 qr.).
4 quarters, or 112 pounds ..... 1 **hundredweight** (1 cwt.).
20 hundredweights ..... 1 **ton**.

7000 grains make 1 **pound**.

LINEAR MEASURE.

12 inches make 1 **foot** (1 ft.).
3 feet ..... 1 **yard** (1 yd.).
22 yards ..... 1 **chain** (1 ch.).
10 chains ..... 1 **furlong** (1 fur.).
8 furlongs, or 1760 yards ..... 1 **mile**.

Also
100 links make 1 **chain**.
$5\frac{1}{2}$ yards ..... 1 **rod** or **pole**.
4 poles ..... 1 **chain**.

SQUARE MEASURE.

12^2 or 144 sq. inches make 1 sq. **foot** (1 sq. ft.).
3^2 or 9 sq. feet ..... 1 sq. **yard** (1 sq. yd.).
22^2 or 484 sq. yards ..... 1 sq. **chain** (1 sq. ch.).
10 sq. chains ..... 1 **acre** (1 ac.).
640 acres ..... 1 sq. **mile**.

Also
4840 sq. yards make 1 **acre**.
$30\frac{1}{4}$ sq. yards ..... 1 sq. **pole** or **perch**.
40 perches ..... 1 **rood**.
4 roods ..... 1 **acre**.

CUBIC MEASURE

12^3 or 1728 cubic inches make 1 cubic **foot** (1 c. ft.).
3^3 or 27 cubic feet ..... 1 cubic **yard** (1 c. yd.).

LIQUID MEASURE.

4 gills make 1 **pint** (1 pt.).
2 pints ..... 1 **quart** (1 qt.).
4 quarts ..... 1 **gallon** (1 gall.).

METRIC MEASURES.

The more important measures are in heavier type.

MEASURES OF WEIGHT.

10 **milligrams** (mg.)	= 1 **centigram** (cg.).
10 **centigrams** (cg.)	= 1 *decigram* (dg.).
10 *decigrams* (dg.)	= 1 **GRAM** (g.).
10 **grams** (g.)	= 1 *decagram* (Dg.).
10 *decagrams* (Dg.)	= 1 *hectogram* (Hg.).
10 *hectograms* (Hg.)	= 1 **kilogram** (Kg.).
100 **kilograms** or 10^5 grams	= 1 **quintal** (q.).
1000 **kilograms**	= 1 **tonne** (t.).

MEASURES OF LENGTH.

10 **millimetres** (mm.)	= 1 **centimetre** (cm.).
10 **centimetres**	= 1 *decimetre* (dm.).
10 *decimetres*	= 1 **METRE** (m.).
10 **metres**	= 1 *decametre* (Dm.).
10 *decametres*	= 1 *hectometre* (Hm.).
10 *hectometres*	= 1 **kilometre** (Km.).

MEASURES OF AREA.

100 sq. **millimetres** (sq. mm.)	= 1 sq. **centimetre** (sq. cm.).
100 sq. **centimetres**	= 1 sq. **decimetre** (sq. dm.).
100 sq. **decimetres**	= 1 sq. **metre** (sq. m.) = 1 **centiare.**
100 sq. *metres*	= 1 sq. **decametre** (sq. Dm.) = 1 **are.**
100 sq. *decametres*	= 1 sq. *hectometre* (sq. Hm.) = 1 **hectare.**
100 sq. *hectometres*	= 1 sq. *kilometre* (sq. Km.).

LAND MEASURE.

100 **centiares**	= 1 **ARE** (a.).
100 **ares**	= 1 **hectare** (Ha.).
100 **hectares**	= 1 **sq. kilometre.**

MEASURES OF VOLUME.

1000 **cubic millimetres** (c.mm.)	= 1 **cubic centimetre** (c.cm.).
1000 **cubic centimetres**	= 1 **cubic decimetre** (c.dm.).
1000 **cubic decimetres**	= 1 **cubic metre** (c.m.) = 1 **stere.**

MEASURES OF CAPACITY.

10 **centilitres** (cl.)	= 1 **decilitre** (dl.).
10 **decilitres**	= 1 **LITRE** (l.).
10 **litres**	= 1 **decalitre** (Dl.).
10 **decalitres**	= 1 **hectolitre** (Hl.).

MEASURES OF NUMBER AND TIME.

MEASURES OF NUMBER.

12 units	make	1 dozen.
20 units	.....	1 score.
12 dozen, or 144 units	.....	1 gross.
24 sheets of paper	.....	1 quire.
20 quires	.....	1 ream.
10 reams	.....	1 bale.

MEASURES OF TIME.

60 seconds	make	1 minute.
60 minutes	.....	1 hour.
24 hours	.....	1 day.
7 days	.....	1 week.
28, 29, 30, or 31 days	.....	1 calendar month.
12 calendar months	.....	1 year.
365 days	.....	1 common year.
366 days	.....	1 leap year.
100 years	.....	1 century.

PREPARATORY ARITHMETIC.

A. INTRODUCTION: NOTATION AND NUMERATION.

1. A **unit** is a single thing; as *one boy*, *one penny*, *one book*.

2. Quantities *of the same kind*, or **like** quantities, are such as have the same unit; for example, *five books* and *eight books* are like quantities, the unit, one book, being the same for both.

3. A **number** is a unit, such as *one* shilling, or a collection of units of the same kind, as *six* pennies. In the latter case the *unit* is *one* penny.

4. All numbers are written by means of the symbols

0, 1, 2, 3, 4, 5, 6, 7, 8, 9,

which are called **digits**, and are named respectively, *nought*, *one* or *unity*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine*.

Each of these digits denotes one unit more than the digit which precedes it; 0 denotes that there are *none* of the units or things referred to.

To express numbers greater than *nine*, two or more of these digits are written side by side.

5. The art of expressing numbers by symbols is called **Notation.**

The art of expressing them by words is called **Numeration.**

6. The number which denotes *one* more than nine is called **ten**, and plays a most important part in Arithmetic.

Numbers are told off in groups of ten.

The number in *one* group of ten is written 10; the number in *two* groups of ten is called *twenty*, and is written 20;
„ *three* „ „ *thirty*, „ 30;
„ *four* „ „ *forty*, „ 40;
„ *five* „ „ *fifty*, „ 50;
„ *six* „ „ *sixty*, „ 60;
„ *seven* „ „ *seventy*, „ 70;
„ *eight* „ „ *eighty*, „ 80;
„ *nine* „ „ *ninety*, „ 90;
„ *ten* „ „ *one hundred*, „ 100.

7. All numbers between 10 and 100 are represented by *two* digits. The highest, *ninety-nine*, is represented by 99, and means

9 *tens* + 9 *units*.

If to this we add 1, we get

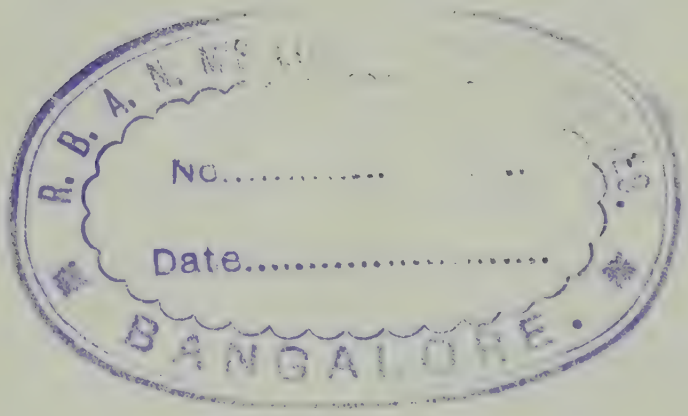
9 *tens* + 10 *units*,
or 10 *tens* + 0 *units*,
or 1 *hundred* + 0 *tens* + 0 *units*.

This is represented by 100, and therefore requires *three* digits.

8. In the number 99 the digit 9 appears twice; in one place it means 9 units, and in the other 9 tens. Hence, although a digit always has the same *absolute* value, yet in consequence of the difference of position the one 9 here denotes ten times as much as the other.

9. The principle on which the **value** of a digit depends upon its **position** among the others is a fundamental one in our method of notation, and is called the PRINCIPLE OF LOCAL VALUE.

This system of notation is called the **Arabic** system.



B. NUMBERS LESS THAN 100.

Pupils should be led to discover the processes, or rules, by the use of objects such as coloured sticks, cubes, and discs, which can be arranged in bundles or in piles to show the 'place-value' of a digit: white objects can represent, and be referred to as, 'ones'; blue objects can represent, and be referred to as, 'tens,' and so on.

B (1). Notation and Numeration.

EXERCISE 1.

(1) How many groups of **ten** each can be made with the following numbers? And how many 'ones' will be left over?

(2) Write in figures, placing 'tens' under 'tens,' and 'ones' (units) under 'ones' (units):

1. Ten.	2. Thirteen.	3. Thirty-one.
4. Twenty.	5. Twenty-four.	6. Forty-two.
7. Fifty.	8. Fifty-six.	9. Sixty-five.
10. Eighty.	11. Eighty-seven.	12. Seventy-eight.

EXERCISE 2.

(1) Analyse each of the following numbers into tens and units, and (2) write each number in words:

1. 14.	2. 41.	3. 25.	4. 52.	5. 68.
6. 86.	7. 37.	8. 73.	9. 94.	10. 49.

B (2). Addition.

10. Addition is the process of finding a *single* number to represent the same as two or more given numbers together, the numbers being of the same kind.

11. The result of addition is called the **sum**, or **total**, or **amount** of the given numbers.

12. The mark + placed between two numbers denotes that they are to be added together, and is called the sign of **addition**, or **plus**.

Thus $3 + 4$ denotes that 4 is to be added to 3.

13. For the word *equals*, or the words *is equal to*, the sign = is used.

Thus $3 + 4 = 7$ is read "3 *plus* 4 *is equal to* 7."

14. For the word *therefore* the sign ∴ is used.
For the word *because* the sign ∵ is used.

EXERCISE 3.

(1) *Add from left to right, and* (2) *verify each result by adding from right to left.*

Write down the sum of the following numbers without altering the horizontal arrangement:

1. $4+3+2+2+3+4+5+4+6=$
2. $3+2+4+5+6+3+2+5+6=$
3. $4+5+6+2+2+3+4+6+4=$
4. $3+4+5+6+3+4+5+3+6=$
5. $5+3+2+4+6+3+6+5+4=$
6. $4+6+3+3+2+3+6+3+5=$
7. $7+8+4+5+8+6+8+8+7=$
8. $6+3+7+8+7+8+8+7+6=$
9. $5+4+8+7+6+7+8+5+8=$
10. $7+8+8+8+4+5+8+8+4=$

EXAMPLE. *There are fifteen sticks in one heap and twenty-seven in another; how many are there in both heaps together?*

Arrange the sum as in the margin.

Tens.	Units.
1	5
2	7
4	2

Notice that 15 is one bundle of *ten* and five *ones*; that 27 is two bundles of *ten* and seven *ones*.

1st. Add together the sticks not tied into bundles. Put together they make 12. We have now to change the single sticks into bundles of *ten*. Notice that they make 1 bundle of *ten*, and 2 sticks are left.

Put the 2 in the *units'* column; and keep the 1 *ten* to be put to the *tens*.

NOTE.—The removal from one column to another is called "carrying."

2nd. Add together the bundles of sticks. There are 1 ten and 2 tens, and also the 1 ten which we got out of the *units*, *i.e.*, 4 tens altogether.

Put down 4 in the *tens* column.

The number of sticks is therefore forty-two, written 42.

EXERCISE 4.

Verify each result.

1. There are twenty-one pencils in one box and thirty-seven in another; how many pencils are there in both together?

2. One book has fifty-three pictures and another has thirty-five; how many pictures are there in both together?

3. Tom makes twelve runs in a game of cricket and Dick forty-five; what is the total?

4. A woman spends ten pence on bread, sixteen pence on tea, and five pence on sugar; what is the total?

5. One book has thirty-two pages, another one forty-eight pages, and another sixteen pages; how many pages are there in the three books?

6. In a certain school there are 17 boys in the first class, 28 in the second, 25 in the third, and 24 in the fourth; how many boys are in the school?

7. In an orchard there are 39 apple trees, 27 pear trees, 10 cherry trees, and 17 plum trees; what is the total number of trees?

8. A boy has four bags of marbles; one of the bags contains 23, another 32, another 19, and another 17; what is the total number of marbles?

9. A man has four fields; in the first there are 13 cows, in the second 24, in the third 25, and in the fourth 17; how many cows are there in the four fields?

10. In a wood there are 26 elm trees, 29 oak trees, 17 beech trees, and 19 pine trees; what is the total number of trees?

EXERCISE 5.

Check each answer by adding (1) *upwards*, (2) *downwards.*

Add together

1. 58 39	2. 64 29	3. 27 13	4. 38 24	5. 49 35	6. 18 39
7. 19 38 8	8. 17 26 32	9. 9 37 42	10. 15 9 47	11. 16 34 9	12. 63 10 8
13. 9 63 8 15	14. 15 26 33 9	15. 4 59 7 28	16. 14 34 16 10	17. 70 9 8 12	18. 41 27 9 8
19. 32 4 15 7 16	20. 41 9 19 8 13	21. 46 9 8 17 4	22. 29 18 6 6 16	23. 40 9 30 6 13	24. 8 62 4 19 6

EXERCISE 6.

Check each answer by adding (1) *from left to right,* (2) *from right to left.*

Write down the sum of the following numbers arranged as they are :

1. $61+13+19=$
2. $42+21+18=$
3. $23+32+15=$
4. $14+39+42=$
5. $35+27+19=$
6. $46+16+21=$
7. $37+28+15=$
8. $28+32+14=$
9. $19+17+45=$
10. $28+33+19=$

B (3). Subtraction.

15. **Subtraction** is the process of finding what remains when one number is taken away from another of the same kind.

16. The result of subtraction is called the **difference** or **remainder.**

17. The mark – placed between two numbers denotes that the number which follows it is to be taken from the number which precedes it. It is called the **sign of subtraction** or **minus.**

Thus 7 – 3 denotes that 3 is to be taken from 7.

And 7 – 3 = 4 is read "7 *minus* 3 *is equal to* 4."

18. Subtraction is the converse of Addition.

Thus 7 – 3 also means—'What number added to 3 units makes 7 units?'

EXERCISE 7.

What number must be added to

1. 4 to make 5?
2. 5 to make 6?
3. 6 to make 7?
4. 7 to make 9?
5. 6 to make 9?
6. 4 to make 9?
7. 3 to make 14?
8. 5 to make 17?
9. 4 to make 16?
10. 2 to make 15?
11. 6 to make 18?
12. 7 to make 19?

Find the missing numbers in

13. $18-5=$
14. $19-4=$
15. $17-3=$
16. $16-2=$
17. $15-3=$
18. $14-2=$
19. $19-6=$
20. $13-1=$

EXAMPLE 1. *A heap contains* 57 *stones. I take away* 33 *: how many will be left?*

Arrange the sum as in the margin.

The mental steps are

1st. 3 and **4** make 7 ; write down **4.**

2nd. 3 and **2** make 5 ; write down **2.**

The number of stones left is twenty-four, written **24.**

Verification. Adding 33 to 24 the result is 57.

Tens.	Units.
5	7
3	3
2	4

EXERCISE 8.

Verify each result.

1. There are twelve books on the table; how many more must I add to make up eighteen?

2. There are eighteen books on a shelf; if three are taken away, how many will be left?

3. Tom has thirteen pennies and George has eighteen; how many has Tom less than George?

4. A boy took sixteen apples to school and gave away twelve; how many had he left?

5. A farmer sold seventeen sheep out of a flock of thirty-eight; how many had he left?

6. One window has thirty-six panes of glass in it; another has twenty-four; how many more has the one than the other?

7. Tom is eighteen years old; how old was he six years ago?

8. Bob is thirteen years old; in how many years' time will he be twenty-five?

9. Tom is fifteen years old and his brother Fred is twenty-seven years old; what is the difference between their ages?

10. Willie's father is forty-eight years old; how many years ago was he fourteen years of age?

EXERCISE 9.

Verify each result.

1. How many must be added to 62 to make 78?
2. How many must be added to 26 to make 87?
3. How many must be taken from 39 to leave 17?
4. How many must be taken from 68 to leave 23?
5. By how many is 75 greater than 32?
6. By how many is 87 greater than 44?
7. 78 is how many more than 25?
8. 25 is how many more than 13?
9. 32 is how many less than 89?
10. 10 is how many less than 32?

EXAMPLE 2. *A heap contains* 57 *stones. I take away* 19 *; how many will be left ?*

Arrange the sum as in the margin.
The mental steps are

1st. 9 and 8 make 17 ; write down 8.
2nd. 1 and 1 and 3 make 5 ; write down 3.

The number of stones left is thirty-eight, written 38.

Verification. By adding 19 to 38 we get 57.

Tens.	Units.
5	7
1	9
3	8

EXERCISE 10.

Verify each result.

1. How many must be added to 33 to make 72 ?
2. How many must be taken from 91 to leave 72 ?
3. What number added to 17 makes 23 ?
4. What number taken from 30 leaves 23 ?
5. By how many is 80 greater than 18 ?
6. By how many is 72 greater than 27 ?
7. By how many is 35 less than 53 ?
8. By how many is 58 less than 71 ?
9. What is the difference between 47 and 62 ?
10. What is the difference between 93 and 39 ?

EXERCISE 11.

Check each result.

Write down the answers to the following :

1. $86-17=$	2. $93-39=$	3. $60-11=$	4. $64-38=$
5. $87-48=$	6. $70-36=$	7. $72-37=$	8. $38-19=$
9. $90-52=$	10. $93-24=$	11. $86-29=$	12. $30-18=$

Find the number denoted by x in each of the following :

13. $90-x=18$.	14. $96-x=78$.	15. $87-x=28$.
16. $73-x=35$.	17. $x-12=78$.	18. $x-46=36$.

EXERCISE 12.

Verify each result.

1. Tom has fifty-eight marbles ; George has sixty-five ; how many more has George than Tom ?

2. There were eighty-two boys at school this morning, and twenty-eight less this afternoon ; how many boys were present this afternoon ?

3. A book had forty-eight pages, but nine were torn out; how many pages are now in the book?

4. A book has ninety-six pages, and all but eight have pictures on them; how many pages have pictures?

5. A farmer had sixty sheep, and sold thirty-six; how many had he left?

6. A woman takes ninety-six eggs to market and brings back thirty-eight; if she sells the rest, how many does she sell?

7. A box of oranges contains ninety-one, but fourteen are bad; how many good ones are there?

8. One tree is twenty-five feet high and another is eighteen feet; how many feet is one higher than the other?

9. A woman has eighty-four apples for sale; how many will she have left when she has sold fifty-nine?

10. Tom has sixty-one marbles; how many more does he want to make ninety?

EXAMPLE 3. *In one operation take the sum of* 27 *and* 39 *from* 84.

Arrange the sum as in the margin.
The mental steps are
1st. 9 and 7 make 16, and **8** make 24; write down **8** in the *units'* column, and carry forward 2 *tens.*
2nd. 2, 3, and 2 make 7, and **1** makes 8; write down **1** in the tens' column.
The result is eighteen, written **18.**
Verification. Add the last three lines.

Tens.	Units.
8	4
2	7
3	9
1	8

EXERCISE 13.

Verify each result.

In one operation take the sum of the second and following lines from the first line:

1.	2.	3.	4.	5.	6.
80	70	90	90	80	90
13	4	15	17	9	42
8	15	8	7	27	7
48	39	59	49	39	14

7.	8.	9.	10.	11.	12.
85	62	93	84	95	82
17	6	9	18	16	9
5	14	18	6	24	33
29	27	37	36	9	8
6	8	4	9	28	15

EXERCISE 14.

Verify each result.

Find the missing number denoted by x in each of the following:

1.	2.	3.	4.	5.	6.
x	x	8	27	21	41
16	14	x	14	8	9
6	47	12	x	10	19
15	7	16	8	x	x
80	94	65	81	66	82

7. $8+x+6+4+5=31$.
8. $4+5+x+2+7=24$.
9. $9+9+8+x+8=40$.
10. $21+12+19+13+x=83$.

B (4). Brackets.

19. Brackets ( ), [ ], { } are used to bind together two or more quantities into one.

Thus $12-(3+4)$ means—'From 12 take the sum of 3 and 4'; therefore $12-(3+4)=12-7=5$.

But $12-3+4$ means—'From 12 take 3 and then add 4'; therefore $12-3+4=9+4=13$.

20. When a + sign precedes a bracket, the bracket may be removed without altering the value of the expression.

Thus $12+(5-3)=12+5-3$.

21. When a − sign precedes a bracket, the bracket may be removed without altering the value of the expression, provided the signs + and − within the bracket are changed respectively to − and +.

Thus $12-(5-3)=12-5+3$.

EXERCISE 15.

(1) Explain the meaning of each of the following, and (2) write down the answers:

1. $(16+7)-5=$
2. $16+(7-5)=$
3. $(16-7)+5=$
4. $16-(7+5)=$
5. $16-(7-5)=$
6. $16-7-5=$
7. $(25-9)+(8+5)=$
8. $(25+9)+(8-5)=$
9. $(25-9)+(8-5)=$
10. $25+9+(8-5)=$
11. $25-9-8-5=$
12. $25+(9+8-5)=$
13. $(25-9)-(8-5)=$
14. $(25+9)-(8+5)=$

15. $25 - 9 + (8 - 5) =$ 16. $(25 + 9) - (8 - 5) =$
17. $25 - 9 + (8 + 5) =$ 18. $(25 + 9) + 8 - 5 =$
19. $(10 + 4) - (8 - 2) + (8 + 6) - (5 - 2) =$
20. $(10 + 8 - 4 - 2) - (10 - 6 - 4) - 5 =$

EXERCISE 16.

Verify each result.

1. A boy gets 4 pence from his father and 7 pence from his mother; how much will he have left after spending 5 pence?

2. A boy put into the bank 10 shillings and then 6 shillings; he afterwards drew out 7 shillings; how much had he then in the bank?

3. A boy won 10 marbles in the first game and lost 3 in the second game; but in the third game he won 5; how many more marbles has he now?

4. A man had 18 shillings in his purse; he buys a hat for 7 shillings and a pair of gloves for 3 shillings; how many shillings has he left?

5. Tom has 12 marbles and Bob has 13 marbles. If Tom gives 4 of his marbles to Bob, how many will each then have?

6. George has 20 apples and Harry has 11 apples. If George gives Harry 6 of his apples, how many will each then have?

7. Tom made 36 runs in a cricket match, and Harry made 19 less than Tom; how many runs did they make together?

8. Tom has 62 marbles and Bob has 35 marbles less than Tom. If Tom gives Bob 27, how many will Bob have more than Tom?

9. A boy has two copy-books; one has in it 30 pages and the other 24 pages. If he has filled up 16 pages in the first book already, how many more pages has he to write to fill both the books?

10. In one field a farmer had 40 sheep and in another field he had 35 sheep. He took 6 sheep from the first field and put them into the other field; how many more are there now in the second field than in the first?

B (5). Multiplication.

22. Multiplication is a short way of finding the sum of any number of given *equal numbers.*

Thus $3 + 3 + 3 + 3$ is equal to 4 times 3, that is, 12.

Multiplication is another form of Addition.

23. The number of given equal numbers is called the **multiplier.**

24. The number to be multiplied, *i.e.*, any one of the given equal numbers, is called the **multiplicand.**

25. The result of the multiplication is called the **product** of the multiplier and the multiplicand; and the multiplier and the multiplicand are called the **factors** of the product.

26. Multiplication is denoted by an *inclined* cross $\times$, which is called the **sign of multiplication**, and is placed between the numbers.

Thus 3 multiplied by 4 is written 3×4.

And $3 \times 4 = 12$ is read—"3 *multiplied by* 4 *is equal to* 12," or "4 *times* 3 *equal* 12."

27. The multiplier and the multiplicand can be interchanged without altering the product.

For example, consider the dots in the margin. The number of dots altogether is the product of the number of dots in a row, 3, multiplied by the number of rows, 4. It is also the product of the number of dots in a column, 4, multiplied by the number of columns, 3.

* * *
* * *
* * *
* * *

Hence $3 \times 4 = 4 \times 3$.

28. The result of multiplying three or more numbers together is called the **continued** product of the numbers.

Thus the continued product of 2, 3, and $5 = 2 \times 3 \times 5 = 6 \times 5 = 30$.

It is assumed that pupils have committed the **Multiplication Table** *to memory before proceeding to the Exercises which follow.*

EXERCISE 17.

Find the missing number in each of the following:

1. $1+1+1=$	2. $4+4+4+4=$	3. $3+3+3+3+3=$
4. $5+5+5=$	5. $6+6+6+6=$	6. $2+2+2+2+2=$
7. $5 \times 4=$	8. $4 \times 5=$	9. $3 \times 6=$
10. $12 \times 6=$	11. $6 \times 4=$	12. $8 \times 3=$
13. $12 \times 2=$	14. $4 \times 6=$	15. $2 \times 3 \times 4=$
16. $2 \times 4 \times 5=$	17. $3 \times 5 \times 4=$	18. $6 \times 3 \times 2=$
19. $3 \times 3 \times 3=$	20. $4 \times 4 \times 4=$	21. $2 \times 2 \times 2 \times 2=$

EXAMPLE. *How many sticks will 3 children have altogether, if each has 24?*

[PROCESS: *Multiply 24 by 3.*]

Arrange the sum as in the margin: write the multiplier under the multiplicand, and draw a line below.

We begin to multiply at the right-hand figure.

1st. 3 times 4 units are 12 units, which equal 1 ten and 2 units; write down **2** and carry forward 1 ten.

2nd. 3 times 2 tens are 6 tens; 6 tens and 1 ten are 7 tens; write down **7.**

The product is seventy-two, written **72.**

24
3
72

Verification. By addition: $24+24+24=72$.

EXERCISE 18.

1. A penny is equal in value to 2 halfpennies; how many halfpennies are equal to 18 pennies?

2. A penny is equal in value to 4 farthings; how many farthings are equal to 17 pennies?

3. A shilling is equal in value to 12 pennies; how many pennies are worth 8 shillings?

4. Twelve inches make a foot; how many inches long is a 2-foot rule?

5. Three feet make a yard; how many feet long is a piece of string 18 yards long?

6. Two pints make a quart; how many pint jugs can be filled from a cask holding 18 quarts?

7. Four quarts make a gallon; how many quart jugs can be filled from a tank holding 18 gallons?

8. Sixteen ounces make a pound; how many ounces make 6 pounds?

9. Twenty-four hours make a day; how many hours are there in 4 days?

10. Twelve things are called a dozen; how many things make 4 dozen?

EXERCISE 19.

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	
1. Multiply	15;	17;	26;	35;	37;	46	by 2.
2. Multiply	14;	16;	15;	24;	26;	25	by 3.
3. Multiply	13;	15;	14;	17;	16;	18	by 4.
4. Multiply	12;	14;	13;	16;	15;	17	by 5.
5. Multiply	10;	12;	14;	13;	16;	15	by 6.

29. The expression $4+3\times5$ means that 4 is added to 5 times 3.

Thus $4+3\times5=4+15=19.$

The sign $\times$ links the numbers 3 and 5 into one term and draws the 3 away from the 4.

But $(4+3)\times5$, or $5\times(4+3)$, means that the sum of 4 and 3 is to be multiplied by 5.

Thus $(4+3)\times5=7\times5=35.$

Multiplication must be performed before Addition or Subtraction, unless a bracket indicates otherwise.

30. Multiplication may be indicated by writing the quantities in brackets side by side without any sign between them.

Thus $(4+2)(5-3)=6\times2=12.$

EXERCISE 20.

(1) Explain the meaning of each of the following, and (2) write down the answers:

1. $6\times4+2=$
2. $6\times(4+2)=$
3. $8\times5-2=$
4. $8\times(5-2)=$
5. $12+3\times5=$
6. $(12+3)\times5=$
7. $5\times12-3=$
8. $5\times(12-3)=$
9. $3\times3+2\times5=$
10. $3\times(3+2)\times5=$
11. $7\times6-3\times4=$
12. $7\times(6-3)\times4=$
13. $40-12+6\times2=$
14. $40-(12+6)\times2=$
15. $9\times(4-2)+3=$
16. $9\times(4-2+3)=$
17. $9\times4-2+3=$
18. $9\times4-(2+3)=$
19. $6+3\times4-2-2\times4=$
20. $(6+3)\times(4-2)-2\times4=$
21. $(6+3)\times4-(5+2)\times4=$
22. $(6-3)\times4+(5-2)\times4=$
23. $(7+8)(9-4)=$
24. $(8+5)(8-5)=$

EXERCISE 21.

(1) Write each of the following expressions with signs and brackets, and (2) write down the answers:

1. Five times the difference between 13 and 30.
2. The product of the sum and the difference of 5 and 9.
3. The sum of 3 times 3 and 5 times 8.
4. The sum of 6 times 9 and 4 times 10.
5. The difference between 6 times 5 and 5 times 3.
6. The difference between 6 times 7 and 3 times 9.

7. The sum of 5 times 7 and 6 times 3.
8. 19 diminished by 6 times the difference between 6 and 3.
9. 18 added to 6 times the sum of 6 and 3.
10. 13 added to 5 times the sum of 4 and 5.

EXERCISE 22.

Verify each result.

1. There are 16 sheep in one field, and three times as many in the next; how many are there altogether?

2. John has 16 marbles, and Fred has five times as many; how many have they between them?

3. Sam has 18 marbles, and Tom has four times as many; how many more has Tom than Sam?

4. A boy earned tenpence every day for 6 days, and then spent elevenpence; how many pence did he save?

5. There are 20 boys in a class. Nine of these did 2 sums each, and the rest did 3 sums each; how many sums were worked by this class?

6. A man paid 3 shillings each to 19 boys, and 6 shillings each to 7 girls; how many shillings did he pay away?

7. A woman took to market 3 baskets, each containing 29 eggs. She could sell only 39 eggs; how many did she take back with her from market?

8. 81 parcels were packed in 6 vans. Five of the vans held 13 parcels each; how many were there in the other van?

9. If a boy takes 18 steps in walking the length of a room, how many steps will he take in walking up and down the room twice?

10. Harry bought 28 marbles and lost 9; how many must he now buy so as to place 30 marbles in each of 3 bags?

B (6). Division.

31. **Division** is the method of finding how many times one number contains another, *i.e.* how many times in succession a smaller number can be taken from a larger number.

Thus 2 is contained 3 times in 6, *i.e.* 2 can be taken 3 times from 6. Hence when 6 is *divided by* 2 the result is 3.

Division is thus a shortened form of Subtraction.

32. The number to be divided is called the **dividend.**

33. The number by which it is divided is called the **divisor.**

34. The number which tells how many times the divisor is contained in the dividend is called the **quotient.**

The number which is left after dividing one number by another is called the **remainder**; thus 7 contains 2 three times, leaving a *remainder* 1.

When there is no remainder, the division is said to be **exact.**

35. Division is denoted by the mark ÷, which is called the **sign of division.**

Thus $8 \div 2 = 4$ is read "8 *divided by* 2 *equals* 4."

Division may be also indicated thus:

$$\frac{8}{2} = 4,$$

which is read—"8 *over* 2 *equals* 4."

Division is the converse of Multiplication.

Thus $8 \div 2$ also means—'What number multiplied by 2 makes 8?'

36. In **exact** division

$$\textbf{Dividend} \div \textbf{Divisor} = \textbf{Quotient},$$

and $$\textbf{Quotient} \times \textbf{Divisor} = \textbf{Dividend}.$$

In **inexact** division

$$\textbf{Quotient} \times \textbf{Divisor} + \textbf{Remainder} = \textbf{Dividend}.$$

EXERCISE 23.

How many times can

1. 2 be taken from 12?
2. 3 be taken from 12?
3. 4 be taken from 12?
4. 6 be taken from 12?
5. 3 be taken from 15?
6. 5 be taken from 15?

How many are there in each share if

7. 24 marbles are divided into 2 shares?
8. 24 marbles are divided into 3 shares?
9. 24 marbles are divided into 4 shares?
10. 24 marbles are divided into 6 shares?

How many shares of

11. 2 marbles each can be made with 48 marbles?
12. 3 marbles each can be made with 48 marbles?
13. 4 marbles each can be made with 48 marbles?
14. 6 marbles each can be made with 48 marbles?

Find the number denoted by x in each of the following:

15. $18 \div 2 = x$.
16. $18 \div 3 = x$.
17. $18 \div 6 = x$.
18. $30 \div 5 = x$.
19. $\frac{48}{6} = x$.
20. $\frac{36}{4} = x$.
21. $\frac{27}{3} = x$.
22. $\frac{30}{6} = x$.
23. $5 \times x = 25$.
24. $x \times 6 = 48$.
25. $x \times 4 = 32$.
26. $4 \times x = 48$.

EXAMPLE 1. *If* 69 *sticks are arranged in* 3 *equal groups, how many are there in each group?*

[PROCESS: *Divide* 69 *by* 3.]

Arrange the sum as in the margin.
We begin to divide at the left-hand figure.
Notice that 69 = six bundles of *ten* and nine *ones*.
1st. Divide 6 bundles into 3 equal groups; there will be 2 bundles in each group. Write **2** in the tens place.

	Tens.	Units.
3	6	9
	2	3

2nd. Divide 9 sticks into 3 equal groups; there will be 3 sticks in each group.
Write **3** in the units' place.
The number of sticks in each of the required groups is **23.**

Verification. $69-23-23-23=0$; and $23\times3=69$.

EXAMPLE 2. *If* 96 *sticks are arranged in groups of* 4 *sticks, how many groups are there?*

[PROCESS: $96\div4$.]

Notice that 96 = 9 tens and 6 units.
1st. Divide 9 tens by 4; that is, arrange 9 tens in equal groups of 4 tens.
There will be 2 groups with 1 ten over.
Write **2** in the tens' place.

$$\begin{array}{r|rr} 4 & 9 & 6 \\ \hline & \underline{2} & \underline{4} \end{array}$$

2nd. Change the 1 ten which remained over into 10 units; these 10 units and the 6 units make 16 units.
Divide the 16 units by 4; that is, arrange the 16 units in equal groups of 4 units. There will be 4 such groups.
Write **4** in the units' place.
The number of groups is **24.**

Verification. $24+24+24+24=96$; and $24\times4=96$.

EXERCISE 24.

Verify each result.

1. How many marbles will each boy receive, if 68 are equally divided between 2 boys?

2. How many apples will you receive, if I give you the fourth part of 84 apples?

3. If you have 96 oranges and give me one-third of them, how many shall I receive?

4. If 59 plums are equally divided among 5 boys, how many will each boy get, and how many plums will be left over?

5. There are 64 pears in 4 bags and an equal number in each; how many pears are there in each bag?

6. If 84 chestnuts are arranged in 5 equal groups, how many are there in each group, and how many chestnuts will be left over?

7. How many inches are there in each length, when 85 inches of string are measured off into 5 equal lengths?

8. How many pence are there in a group, if 68 pennies are arranged in 4 equal groups?

9. How many times in succession can you take 5 walnuts at a time out of a bag which contains 75 walnuts?

10. How often can you take 3 filberts out of a bag which contains 75 filberts?

11. How many groups, each containing 4 pennies, can you make with 56 pennies?

12. If I buy 65 apples at 5 for a penny, how many pence do I spend?

13. If 6 apples weigh a pound, how many pounds will 78 apples weigh?

14. How many bags, each to contain half-a-dozen chocolates, can be filled from a box containing 84 chocolates?

15. How many times can 4 shillings be subtracted from 68 shillings?

16. How often can 6 inches of string be measured off from 94 inches of string, and what is the length of the piece left over?

EXERCISE 25.

Verify each result.

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	
1. Divide	46;	48;	84;	64;	86	by 2.
2. Divide	39;	63;	96;	69;	93	by 3.
3. Divide	13;	29;	79;	95;	47	by 2.
4. Divide	20;	35;	59;	88;	94	by 3.
5. Divide	77;	19;	27;	51;	83	by 4.
6. Divide	93;	90;	66;	99;	49	by 4.
7. Divide	87;	43;	61;	73;	89	by 5.
8. Divide	96;	59;	72;	98;	91	by 5.
9. Divide	69;	79;	71;	89;	17	by 6.
10. Divide	29;	98;	47;	77;	95	by 6.

SUPPLEMENTARY EXAMPLES.

In each of the examples 3–10, find how many must be added to the dividend to make the quotient greater by one.

37. When a number is divided into two equal parts, each part is called *one half* of the number. When the divisor is 3, 4, 5, ... each part is called *one third*, *one fourth* (or *a quarter*), *one fifth*, ... of the number. The term *five-ninths* denotes what we have when we divide a number into *nine* equal parts and take five of them.

EXERCISE 26.

1. How many articles are there in

(*a*) half-a-dozen? (*b*) one-quarter of a dozen?

(*c*) three-quarters of a dozen? (*d*) one-third of a dozen?

(*e*) two-thirds of a dozen? (*f*) one-sixth of a dozen?

(*g*) five-sixths of a dozen? [1 dozen = 12 units.]

2. How many articles are there in

(*a*) half-a-score? (*b*) one-fourth of a score?

(*c*) three-fourths of a score? (*d*) one-fifth of a score?

(*e*) two-fifths of a score? (*f*) three-fifths of a score?

(*g*) four-fifths of a score? [1 score = 20 units.]

3. How many inches are there in

(*a*) one-fourth of a yard? (*b*) three-fourths of a yard?

[1 yard = 36 inches.]

4. How many ounces are there in

(*a*) one-quarter of a pound? (*b*) three-quarters of a pound?

[1 pound = 16 ounces.]

5. How many minutes are there in

(*a*) half-an-hour? (*b*) three-quarters of an hour?

[1 hour = 60 minutes.]

6. How many hours are there in

(*a*) three-quarters of a day? (*b*) five-sixths of a day?

[1 day = 24 hours.]

7. How many weeks are there in

(*a*) one quarter of a year? (*b*) three-quarters of a year?

[1 year = 52 weeks.]

B (7). Averages.

38. The **average** of a set of numbers or quantities is the result of dividing their sum by the number of them.

Thus, the average of 2, 3, and 7 = (2 + 3 + 7) ÷ 3 = 12 ÷ 3 = 4.

EXERCISE 27.

Find the average of the following numbers:

1. 4, 4, 4, 4.
2. 1, 7, 9, 7.
3. 4, 9, 2, 5.
4. 12, 13, 17, 18.
5. 13, 7, 23, 9.
6. 8, 25, 7, 12.
7. 1, 7, 13, 19, 20.
8. 6, 7, 8, 9, 10.
9. 13, 14, 0, 17, 1.
10. 17, 23, 10, 8, 2.
11. 8, 15, 0, 16, 0, 15.
12. 26, 13, 0, 8, 0, 19.

B (8). Multiplication and Division; Miscellaneous.

39. In any expression in which the signs $\times$, $\div$, $+$, $-$, occur **Multiplication and Division must be worked out before Addition or Subtraction, unless a bracket indicates otherwise.**

Thus $36 \div 4 + 2 = 9 + 2 = 11$;

but $36 \div (4 + 2) = 36 \div 6 = 6$.

EXERCISE 28.

Find the answers to the following:

1. $19 \times 3 - 24 \div 4 =$
2. $17 \times 3 + 24 \div 3 =$
3. $27 \div 3 - 15 \div 3 =$
4. $30 \div 3 + 24 \div 2 =$
5. $51 \div 3 - 52 \div 4 =$
6. $27 \div 3 + 48 \div 4 =$
7. $45 \div 3 + 17 \times 3 =$
8. $58 \div 2 - 32 \div 4 =$
9. $84 \div 3 + 9 \times 3 - 13 \times 4 =$
10. $14 \times 3 - 18 \times 2 + 52 \div 4 =$
11. $48 \div 3 - 48 \div 4 + 12 \times 4 =$
12. $95 \div 5 + 78 \div 6 + 87 \div 3 =$
13. $92 \div 4 + 76 \div 2 - 14 \times 4 =$
14. $16 \times 3 - 12 \times 4 + 90 \div 5 =$
15. $(15 - 3) \div 2 + 4 =$
16. $16 + (26 - 2) \div 3 =$
17. $(16 + 9) \div 5 + 17 =$
18. $(51 - 9) \div 6 + 18 =$
19. $80 - (51 - 3) \div 4 =$
20. $(29 + 35) \div 4 - 7 =$

EXERCISE 29.

(1) Express each of the following with signs and brackets, and (2) write down the answers:

1. Divide the sum of 24, 17, 28 by 3.
2. Find one-sixth of the product of 18 and 5.
3. Find how often 5 is contained in the sum of 29 and 39 added to 7.

4. Find the number which is half the difference between 95 and 17.
5. Find how many times 4 can be subtracted from the difference between 29 and 93.
6. Find the sixth part of 4 times 12.
7. Find how many times 5 can be taken from twice 30.
8. Find the result of dividing 3 times 24 by 4.
9. To 3 times 15 add the third part of 15.
10. To the sixth part of 72 add the difference between 16 and 61.

EXERCISE 30.

Verify each result.

1. There are 18 pins in a row; how many days will 4 rows last if 6 pins are used every day?

2. A farmer buys at a fair 36 sheep, 25 sheep, and 34 sheep. If he has 5 fields and puts the same number of sheep into each, how many can he put into each field?

3. There are 29 books on a shelf, 36 books in a box and 7 on the desk. If there are four classes, and I give the same number of books to each, how many do I give to each class?

4. A farmer had 50 lambs, but 13 of them died. He bought 7 more and then sold half his stock; how many has he now?

5. Smith has 58 biscuits in one tin and half as many in another tin; to how many boys can he give 3 biscuits each?

6. In a school there are three times as many boys in the lower division as in the upper division. There are 63 boys in the lower division; how many boys are there in the school?

7. Tom has 72 marbles and Fred has 27. If Tom gives one-third of his to Fred, how many will each boy have then?

8. Jones has 76 marbles. If he gives a quarter of them to Smith, who has already 29, how many more marbles than Smith will Jones then have?

9. In a school there were 65 pupils last term. One-fifth of them left at the end of the term, but 11 new pupils came after the holidays; how many pupils are now in the school?

10. I had 96 marbles. If I gave away one-sixth of them and then a quarter of the remainder, how many marbles have I now?

C. NUMBERS LESS THAN 1000.

The use of objects should be continued: the place-values of digits can be indicated by white cubes to represent 'units,' blue to represent 'tens,' red to represent 'hundreds.'

C (1). Notation and Numeration.

40. We now give examples of the Decomposition of Numbers.

EXAMPLE. *Analyse the number* 360.

$360 = 360$ units
$= 36$ tens and *no* units
$= 3$ hundreds 6 tens and *no* units.

EXERCISE 31.

(1) Analyse (*a*) into tens; (*b*) into hundreds; and (2) write in words:

1. 100.	2. 140.	3. 200.	4. 230.	5. 320.
6. 450.	7. 560.	8. 670.	9. 780.	10. 890.
11. 900.	12. 910.	13. 269.	14. 354.	15. 496.
16. 543.	17. 506.	18. 607.	19. 708.	20. 809.

EXERCISE 32.

Write in figures, placing each figure in its proper column:

1. One hundred and one.
2. One hundred and ten.
3. One hundred and eight.
4. Eight hundred and eleven.
5. Two hundred and fourteen.
6. Seven hundred and seventeen.
7. Seven hundred and seventy.
8. One hundred and eleven.
9. One hundred and twenty.
10. Eight hundred and one.
11. Two hundred and four.
12. Two hundred and forty-one.
13. Seven hundred and seven.
14. Five hundred and ten.
15. Three hundred and three.
16. Three hundred and thirteen.
17. Three hundred and thirty.
18. Three hundred and thirty-three.

C (2). Addition and Subtraction.

41. No new principle is involved in the examples which follow.

EXAMPLE. *Find the sum of* 218 *marbles,* 63 *marbles,* 155 *marbles, and* 67 *marbles.*

Arrange as in the margin.

Beginning at the bottom of the column and adding upwards, the mental steps are

1st. Adding units: 12, 15, 23; write down **3** and carry forward 2 tens.

2nd. Adding tens: 8, 13, 19, 20; write down **0** and carry forward 2 hundreds.

3rd. Adding hundreds: 3, 5; write down **5.**

The sum is **503.**

Verification. Add downwards.

Hundreds.	Tens.	Units.
2	1	8
	6	3
1	5	5
	6	7
5	**0**	**3**

EXERCISE 33.

Verify each result.

1. One boy has ninety-one marbles and another has forty-seven; how many marbles have both together?

2. One boy has one hundred and forty-seven picture post-cards, and another has two hundred and six; how many picture post-cards have they together?

3. I have three bags of nuts. One contains three hundred and twelve, another sixty-eight, and another two hundred and eighty-six; how many nuts are there in all?

4. One book has twenty-eight pages, another one hundred and ninety-two, a third ninety-six, and a fourth two hundred and fifty-six; how many pages are there in the four books?

5. A shopkeeper has two hundred and forty-seven walnuts, two hundred and fifty-six chestnuts, one hundred and sixty-nine brazil nuts, one hundred and ninety-two hazel nuts and seventy-three filberts; how many nuts has he?

6. One case of oranges contains 259, another 163, a third 247, and a fourth 303; how many oranges are there in the four cases?

7. One boat brought 207 fish, another 365 fish, a third boat 189 fish, and a fourth 198 fish; how many fish were brought in the four boats?

8. One barge brought 250 tons of coal, another 136, a third 197, and a fourth 209; how many tons were brought altogether?

9. Bob has 107 marbles, Fred 172, George 198, Tom 165, Harry 139; how many marbles have the five boys?

10. A fruiterer sells 137 pounds of gooseberries, 69 pounds of strawberries, 174 pounds of black currants, 98 pounds of raspberries, and 105 pounds of cherries; how many pounds of fruit does he sell altogether?

EXERCISE 34.

Check each result by adding (1) *upwards*, (2) *downwards.*

Find the sum of the numbers in each of the following sets:

1. 63	2. 37	3. 42	4. 54	5. 37	6. 49
26	26	37	26	59	68
17	43	59	78	15	27
28	15	16	13	23	16
7. 18	8. 36	9. 51	10. 47	11. 37	12. 49
27	47	23	56	84	98
39	28	65	75	76	27
63	79	37	94	68	36
16	15	79	33	59	75
13. 13	14. 15	15. 507	16. 480	17. 208	18. 380
209	370	46	59	392	265
77	66	198	276	16	32
380	208	30	107	160	207
19. 12	20. 470	21. 9	22. 150	23. 496	24. 92
603	169	72	63	107	573
47	82	365	219	63	19
190	39	49	75	19	8
85	9	103	309	75	273

EXERCISE 35.

Check each answer by adding (1) *from left to right*, (2) *from right to left.*

Write down the sum of the following numbers arranged as they are:

1. $47 + 74 + 98 + 100 =$
2. $284 + 102 + 112 + 326 =$
3. $9 + 411 + 29 + 321 =$
4. $306 + 603 + 15 + 61 =$
5. $425 + 9 + 103 + 234 =$
6. $92 + 108 + 251 + 215 =$
7. $177 + 77 + 107 + 7 =$
8. $360 + 63 + 36 + 106 =$
9. $6 + 666 + 200 + 18 =$
10. $644 + 25 + 119 + 5 =$

42. No new principle is involved in the examples which follow.

EXAMPLE. *How many marbles must be added to* 284 *to make* 542 *?*

Arrange the sum as in the margin.
The mental steps are
1st. 4 and **8** make 12; write down **8**;
2nd. 1 to 8 makes 9, and **5** make 14; write down **5**;
3rd. 1 to 2 makes 3, and **2** make 5; write down **2**.
The number to be added is **258.**

Verification. By addition: 284 + 258 = 542.

Hundreds.	Tens.	Units.
5	4	2
2	8	4
2	**5**	**8**

EXERCISE 36.

Verify each result.

1. There are one hundred and twenty matches in a box; how many will be left when fifty-eight of them have been used?

2. A bag contains eighty-four walnuts and one hundred and sixty-three chestnuts; how many more chestnuts than walnuts are there?

3. Tom has seventy-two pennies; how many more must he get to make up two hundred and forty?

4. The number of pupils in a school is just fifteen less than two hundred; how many pupils are there?

5. In a school of five hundred and sixty boys, two hundred and eighty-eight were in the upper school; how many boys were in the lower school?

6. Four hundred and seventy-two children were present at a pantomime; if there were two hundred and ninety-nine boys, how many girls were there?

7. A baker has two hundred and fourteen loaves in his cart, but forty-eight are stale; how many fresh loaves has he?

8. There are one hundred and ninety turkeys in the market, and ninety-nine are sold; how many are still for sale?

9. A boy has two hundred and three marbles; how many more does he want to make two hundred and fifty?

10. A boat has landed five hundred and sixty-five fish; how many is this short of six hundred and fifty-four?

EXERCISE 37.

Verify each result.

How many must be added to the first number to make the second?

1. 629, 982.
2. 238, 791.
3. 666, 874.
4. 375, 693.
5. 269, 460.
6. 159, 306.
7. 282, 400.
8. 399, 878.
9. 604, 900.
10. 509, 906.
11. 354, 642.
12. 289, 467.

What is the number denoted by x in each of the following?

13. $912 - 219 = x$.
14. $806 - 417 = x$.
15. $902 - 297 = x$.
16. $807 - x = 609$.
17. $612 - x = 216$.
18. $815 - x = 507$.
19. $x - 187 = 177$.
20. $x - 181 = 419$.
21. $x - 215 = 585$.

EXERCISE 38.

Verify each result.

In one operation take the sum of the second and following lines from the first line:

1.	2.	3.	4.	5.
600	800	900	800	700
79	325	46	8	64
108	14	159	380	275
9	186	87	18	86
346	78	506	169	171

6.	7.	8.	9.	10.
710	920	850	910	910
7	356	75	23	198
306	9	298	34	89
64	270	103	347	233
8	19	7	109	109
139	143	98	109	90

Find the number denoted by x in each of the following:

11. $(391 + 469) - 496 = x$.
12. $912 - (903 - 309) = x$.
13. $(318 + 269) - (318 - 269) = x$.
14. $712 - 207 - 155 - 317 = x$.
15. $468 + 319 - 517 + 133 = x$.
16. $603 - 407 + 311 - 208 = x$.

EXERCISE 39.

Verify each result.

1. At the beginning of the year there are 837 pupils in a school. During the year 198 leave and 99 new pupils are admitted; how many pupils are there at the end of the year?

2. George has to do an addition sum of four lines. The first line is 62, the second is 7, the third is 89, and the answer is 200; what is the fourth line?

3. Tom has to do an addition sum of four lines. The answer is 252. The second line is 73, the third line 85, and the fourth line 17; what is the top line?

4. Bob has to do an addition sum of five lines. The top line is 365, the second line 79, the fourth line 292, and the last line 39. The answer is 802; what is the middle line?

5. Six men weigh 878 pounds. Five of them weigh 137, 142, 129, 138, 133 pounds; what is the weight of the other man?

6. A boy lost 257 marbles out of 802; then he won 144, after which he lost 224; how many had he left?

7. A carpenter has 938 planks. He uses 92, 213 and 105; how many more must he use so that he may have only 67 left?

8. A school has 206 boys and 29 fewer girls than boys. A second school has 14 more boys and 19 more girls than the first school; how many children are there in the two schools?

9. In a railway train there are 925 persons. Of these 265 ride second-class and 88 fewer ride first-class. The rest ride third-class; how many ride third-class?

10. The larger of two heaps of nuts contains 253. The difference between the numbers of nuts in the heaps is 195; how many nuts are there in both heaps together?

C (3). Multiplication.

EXERCISE 40.

1. How many half-pennies are worth 96 pence?

2. 3 feet make 1 yard; how many feet long is a piece of string 69 yards long?

3. 4 quarts make 1 gallon; how many quarts of milk make 36 gallons?

4. A boy attends school 5 days per week; if he attends regularly for 39 weeks, on how many days does he attend altogether?

5. 24 hours make 1 day; how many hours are there in 6 days?

6. 7 days make 1 week; how many days are there in 52 weeks?

7. 8 pints make 1 gallon; how many pints of milk make 54 gallons?

8. 144 articles are called 1 gross; how many pens are there in half-a-dozen boxes each containing one gross?

9. 16 ounces make 1 pound; how many ounces make 11 pounds?

10. 12 pence make 1 shilling; how many pence should I receive for 45 shillings?

11. 365 days make 1 year; how many days are there in 2 years?

12. How many pounds will 3 sacks of flour weigh, if each sack contains 280 pounds?

13. How many pounds will 4 sacks of potatoes weigh, if each sack contains 168 pounds?

14. 168 hours make 1 week; how many hours make 5 weeks?

15. How many half-crowns are worth 123 sovereigns, if one sovereign is equal in value to 8 half-crowns?

EXERCISE 41.

	(*a*)	(*b*)	(*c*)		(*a*)	(*b*)	(*c*)
1. Multiply	46 by	3;	5; 7.	2. Multiply	68 by	5;	6; 7.
3. Multiply	29 by	5;	7; 4.	4. Multiply	87 by	7;	8; 9.
5. Multiply	95 by	7;	8; 9.	6. Multiply	84 by	8;	9; 11.

43. The effect of multiplying a number by 10 is clearly to change *units* into *tens* of units, and *tens* into *hundreds*; therefore to multiply by 10 we simply write down the digits of the original number in the order in which they occur in that number, and place a 0 to the right of them.

EXAMPLE. *Multiply* 37 *by* 10.

$$37 \times 10 = \underline{370}.$$

44. Again, since 20 is ten times as great as 2, to multiply by 20 we multiply by 2 and put 0 to the right of the result; in a similar manner we multiply by 30, 40, etc.

EXAMPLE. *Multiply* 37 *by* 20.

$$\begin{array}{r} 37 \\ 20 \\ \hline 740 \end{array}$$

EXERCISE 42.

Write down the following products:

1. $65 \times 10 =$	2. $11 \times 90 =$	3. $32 \times 30 =$	4. $13 \times 40 =$
5. $16 \times 50 =$	6. $13 \times 70 =$	7. $12 \times 80 =$	8. $18 \times 50 =$
9. $16 \times 40 =$	10. $28 \times 30 =$	11. $15 \times 60 =$	12. $48 \times 20 =$

EXERCISE 43.

Find the continued product of

1. 3, 5 and 7.	2. 2, 5 and 11.	3. 2, 7 and 11.
4. 3, 5 and 11.	5. 5, 5 and 5.	6. 7, 7 and 7.
7. 2, 8 and 11.	8. 4, 3 and 9.	9. 4, 4 and 11.
10. 8, 7 and 7.	11. 4, 5 and 9.	12. 8, 8 and 8.

45. If the multiplier be a number which can easily be separated into factors, each not greater than 12, we multiply by these factors in succession.

EXAMPLE. *Multiply* 37 *by* 16.

$16 = 4 \times 4$, or 8×2, and we may use either of these pairs.

$37 =$ Multiplicand	$37 =$ Multiplicand
4	8
$148 = M \times 4$	$296 = M \times 8$
4	2
$592 = M \times 4 \times 4$	$592 = M \times 8 \times 2$

Verification. In both cases the results are the same.

EXERCISE 44.

What **two** numbers, each not greater than 12, give the following results when multiplied together?

1. 14.	2. 15.	3. 21.	4. 22.	5. 25.	6. 33.
7. 35.	8. 49.	9. 55.	10. 77.	11. 42.	12. 66.
13. 27.	14. 81.	15. 32.	16. 64.	17. 28.	18. 45.
19. 63.	20. 24.	21. 56.	22. 88.	23. 54.	24. 48.
25. 36.	26. 72.	27. 18.	28. 44.	29. 84.	30. 99.

EXERCISE 45.

Verify each result.

1. Twenty-four sheets of paper make a quire; how many sheets are there in thirty-eight quires?

2. Twenty-one shillings are termed a guinea; how many shillings are equal in value to twenty-nine guineas?

3. Thirty-six inches make a yard; how many inches are there in seventeen yards of canvas?

4. Fourteen pounds are termed a stone; how many pounds are there in sixty-five stones of flour?

5. Sixteen ounces make a pound; how many ounces are there in nineteen pounds of chocolate?

6. A barrel of ale contains thirty-six gallons; how many gallons fill twenty-six barrels?

7. A cask of port wine contains twenty-eight gallons; how many gallons fill twenty-nine casks?

8. How many apples are there in a barrel, if I can empty it by taking out twenty-two apples thirty-seven times?

9. How many letters are there in a page of fifteen lines if, on an average, each line contains forty-three letters?

10. How many pages are there in forty-seven books, if each book contains eighteen pages?

EXERCISE 46.

Multiply by factors. Verify by interchanging multiplier and multiplicand.

Find the following products:

1. 16×18.	2. 16×25.	3. 63×15.	4. 18×54.
5. 45×21.	6. 35×22.	7. 24×32.	8. 27×35.
9. 25×24.	10. 64×14.	11. 35×28.	12. 15×36.
13. 40×16.	14. 30×28.	15. 60×15.	16. 20×48.

46. When a number is multiplied by itself, the product is called the **square**, or *second power*, of the number.

Thus the square of 8 is 8×8, that is, 64.

When a number is multiplied by itself and the result again by the number, the product is called the **cube**, or *third power*, of the number.

Thus the cube of 8 is $8 \times 8 \times 8$, that is, 512.

The square of 8 is written 8^2, and means 8×8. The cube of 8 is written 8^3, and means $8 \times 8 \times 8$.

The small number, which denotes the number of equal factors which are to be multiplied together, is called an **index.**

Thus in 8^2 the index is 2; in 8^3 the index is 3.

EXERCISE 47.

Write down the square of

1. 2. 2. 4. 3. 6. 4. 8. 5. 10. 6. 12.

Find the square of

7. 15. 8. 18. 9. 21. 10. 22. 11. 25.

Find the value of

12. 3^2. 13. 9^2. 14. 5^2. 15. 7^2. 16. 11^2.
17. 14^2. 18. 24^2. 19. 27^2. 20. 28^2. 21. 16^2.

Write down the cube of

22. 6. 23. 5. 24. 7. 25. 9. 26. 4.

EXERCISE 48.

Find the number denoted by x in each of the following:

1. $35 \times 4 + 37 \times 11 = x$.
2. $57 \times 9 + 34 \times 8 = x$.
3. $89 \times 9 - 72 \times 8 = x$.
4. $86 \times 9 - 68 \times 11 = x$.
5. $24 \times 5 + 19 \times 6 + 28 \times 7 = x$.
6. $39 \times 7 + 14 \times 8 + 16 \times 9 = x$.
7. $81 \times 6 + 91 \times 5 - 87 \times 9 = x$.
8. $75 \times 11 - 36 \times 8 - 47 \times 7 = x$.
9. $76 \times 12 - 89 \times 3 - 93 \times 4 = x$.
10. $84 \times 11 - 74 \times 7 + 75 \times 5 = x$.
11. $(23 + 36 + 48) \times 9 = x$.
12. $11 \times (72 - 48) = x$.
13. $(39 + 45)(40 - 31) = x$.
14. $(50 - 42)(61 - 39) = x$.
15. $(22 + 59)(30 - 19) = x$.
16. $(71 - 59)(38 + 37) = x$.

EXERCISE 49.

Verify each result.

1. Each boy has four pencils; how many pencils are there in a school of five classes, if each class has in it forty-six boys?

2. Find how many pence must be paid for eighty-nine pencils at a penny each, ninety-six rulers at twopence each, and seventy-four books at threepence each.

3. In a school there are forty-five boys in the preparatory class, six times as many boys in the lower forms, and five times as many boys in the upper forms; how many boys are there in the school?

4. A railway train consists of eight first-class carriages, seven second-class carriages, and twelve third-class carriages; in each of the first-class carriages there are six persons, in each of the second-class eight, and in each of the third-class nine; how many passengers are there altogether?

5. George had thirty-seven dozen marbles, and lost twelve dozen and seven; how many marbles has he left?

6. At a school treat there are one hundred and sixty-five boys and one hundred and twenty-three girls; how many buns will be required to give three to each girl and two to each boy?

7. From a bag containing marbles thirty-eight boys receive twenty-four each; if there are twenty-nine left, how many did the bag contain?

8. There are twenty rows of boys, with fifteen in a row. If we reduce the number of rows by one and add one boy to each row, what is the difference in the total number?

9. From fifteen rolls of cloth, each containing fifty-seven yards, a draper sold seven hundred and ninety-nine yards; how many yards of cloth has he left?

10. A fruiterer had forty-one score of walnuts; after selling three gross to one customer and thirty dozen to another, how many walnuts had he left?

C (4). Division.

EXERCISE 50.

Verify each result.

1. How many marbles will each boy receive, if 986 are equally divided between 2 boys?

2. How many apples will you receive, if I give you the fourth part of 372 apples?

3. If you have 696 oranges and give me one-eighth of them, how many shall I receive?

4. If 263 plums are equally divided among 3 boys, how many will each boy get, and how many plums will be left over?

5. There are 534 pears in 6 bags, an equal number in each; how many pears are there in each bag?

6. If 759 cherries are arranged as far as possible in 12 equal groups, how many are there in each group, and how many cherries will be left over?

7. How many inches are there in each length, when 532 inches of string are measured off into 7 equal lengths?

8. How many pence are there in a group, if 351 pennies are arranged in 9 equal groups?

9. Eleven batsmen score 352 runs in a cricket match; how many runs does each batsman average?

10. How many times can you take 5 walnuts at a time out of a bag containing originally 670 walnuts?

11. If I buy 469 apples at 7 for a penny, how many pence do I spend?

12. How often can you take 6 filberts out of a bag containing 414 filberts at first?

13. How many bricks, each 9 inches long, will form a line of bricks 603 inches long?

14. How many yards long is a piece of string 825 feet long? (3 feet make 1 yard.)

15. How many gallons are there in 596 jugs, each containing one quart of milk? (4 quarts make 1 gallon.)

16. How many gallons are there in 376 bottles, each containing one pint of milk? (8 pints make 1 gallon.)

17. How many shillings should be given in exchange for 780 pennies? (12 pence make 1 shilling.)

18. How many pounds should be given in exchange for 736 half-crowns? (8 half-crowns make 1 pound.)

EXERCISE 51.

Verify each result.

	(*a*)	(*b*)	(*c*)		(*a*)	(*b*)	(*c*)
1. Divide 864 by	6;	8;	9.	2. Divide 504 by	7;	8;	9.
3. Divide 792 by	9;	11;	12.	4. Divide 990 by	5;	9;	11.
5. Divide 497 by	3;	5;	9.	6. Divide 987 by	4;	5;	6.
7. Divide 789 by	7;	8;	9.	8. Divide 675 by	6;	7;	11.

47. When the divisor is 10, the quotient and the remainder can be written down at once; the *last* figure of the dividend being the remainder, and the others forming the quotient.

EXAMPLE. *Divide* 269 *by* 10.

269 ÷ 10 gives **26** as quotient, and **9** as remainder.

EXERCISE 52.

Write down the quotient and the remainder when each of the following numbers is divided by 10:

1. 910.	2. 821.	3. 732.	4. 643.	5. 567.
6. 456.	7. 754.	8. 327.	9. 436.	10. 812.

48. To divide by 20, 30, 40, etc. we proceed thus:

EXAMPLE. *Divide* 637 *by* 40.

The easiest factors of 40 are 10 and 4.

To divide by 10 we mark off the 7, and we have

$10 \mid \underline{637}$

63 *tens* and 7 *units* over.

Next we divide 63 by 4, the other factor, and we have

$4 \mid \underline{63}$

15 and 3 *tens* over.

The quotient is **15**; and when we add 3 *tens* of the second remainder to the 7 *units* of the first remainder, we get the true remainder **37**.

The work is usually written thus:

$40 \mid \underline{637}$

15...37

EXERCISE 53.

Verify each result.

Find the quotient and the remainder in

1. $917 \div 20$.	2. $851 \div 30$.	3. $795 \div 40$.	4. $639 \div 50$.
5. $828 \div 60$.	6. $762 \div 70$.	7. $606 \div 80$.	8. $462 \div 90$.
9. $461 \div 50$.	10. $740 \div 60$.	11. $684 \div 70$.	12. $528 \div 80$.

49. If the divisor be a number which can be easily separated into factors each not greater than 12, we divide by these factors in succession.

The following is an example of division *without remainder*.

EXAMPLE 1. *If* 592 *sticks are arranged in* 16 *equal groups, how many are there in each group?*

[PROCESS: *Divide* 592 *by* 16.]

$16 = 4 \times 4$ or 8×2, and we may use either of these pairs.

$4 \mid \underline{592}$
$4 \mid \underline{148}$ *fours*
37 *sixteens*

or

$2 \mid \underline{592}$
$8 \mid \underline{296}$ *twos*
37 *sixteens*

Verification. (1) The results are the same in both cases.

(2) Dividend = Quotient × Divisor
= 37 × 16
= 592.

The following is an example of division *with remainder.*

EXAMPLE 2. *If* 603 *sticks are arranged in groups of* 16 *sticks, how many groups are there, and how many sticks will be left over?*

[PROCESS: *Divide* 603 *by* 16.]

Compare, line for line, with Example 1.

4 | 603		2 | 603
4 | 150 *fours* + 3 *units*	or	8 | 301 *twos* + 1 *unit*
37 *sixteens* + 2 *fours.*		**37** *sixteens* + 5 *twos.*
The complete remainder		The complete remainder
= 2 *fours* + 3 *units*		= 5 *twos* + 1 *unit*
= (8 + 3) *units*		= (10 + 1) *units*
= **11** *units.*		= **11** *units.*
Quotient **37**; remainder **11.**		Quotient **37**; remainder **11.**

NOTE. The work is usually arranged thus:

$$16\left\{\begin{array}{l|l} 4 & 603 \\ \hline 4 & 150 \ldots 3 \\ \hline & \underline{37} \ldots 2 \end{array}\right\} 11$$

To get the true remainder we multiply the second remainder by the first divisor, and add the first remainder.

Verification. (1) The results are the same in both cases.

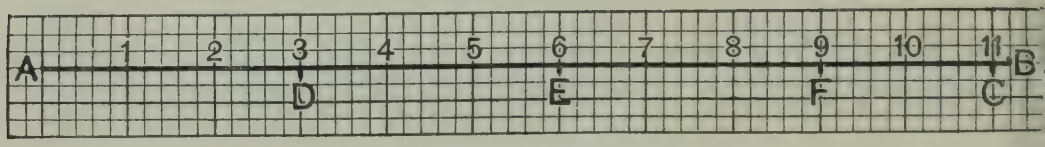
(2) Dividend = Quotient × Divisor + Remainder
= 37 × 16 + 11
= 592 + 11
= 603.

50. The method of finding the complete remainder in short division may be illustrated by using squared paper.

We will consider the division of 56 by 15, using the factors 5 and 3 in succession. In the usual way the operation would appear as in the margin.

$$15\left\{\begin{array}{l|l} 5 & 56 \\ \hline 3 & 11 \ldots 1 \\ \hline & 3 \ldots 2 \end{array}\right\} 11$$

Using squared paper we will take a line AB, 56 units in length, and we will measure off from it lengths of 5 units each. The ends of these lengths will be at the points marked 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. The last will be at the point marked 11 or C, and there will be a remainder CB containing 1 unit.



We will next count these lengths 3 at a time, and we will mark the ends of the new sections with the letters D, E, F respectively. The remainder after this operation will be the length FB, and we see that

$$\begin{aligned} FB &= \quad FC \quad + CB \\ &= 2 \textit{ lengths of } 5 \textit{ units} + 1 \textit{ unit} \\ &= 2 \times 5 \textit{ units} \quad + 1 \textit{ unit} \\ &= 11 \textit{ units.} \end{aligned}$$

EXERCISE 54.

Verify each result.

Using squared paper, perform the following divisions:

1. $40 \div 14$.	2. $50 \div 18$.	3. $50 \div 21$.	4. $60 \div 24$.
5. $60 \div 25$.	6. $70 \div 27$.	7. $80 \div 32$.	8. $80 \div 35$.
9. $100 \div 42$.	10. $100 \div 45$.		

Perform the following divisions:

11. $896 \div 32$.	12. $990 \div 45$.	13. $945 \div 27$.	14. $945 \div 63$.
15. $681 \div 35$.	16. $917 \div 49$.	17. $819 \div 24$.	18. $438 \div 66$.
19. $430 \div 18$.	20. $849 \div 56$.	21. $879 \div 33$.	22. $871 \div 48$.
23. $590 \div 36$.	24. $968 \div 84$.	25. $943 \div 72$.	26. $618 \div 99$.

EXERCISE 55.

How many are

1. One-fourth of 228 books?
2. Three-fourths of 372 pages?
3. One-eighth of 448 words?
4. Three-eighths of 520 letters?
5. Five-eighths of 624 stamps?
6. Seven-eighths of 784 post-cards?

7. One-third of 234 apples?
8. Two-thirds of 837 oranges?
9. One-sixth of 522 walnuts?
10. Five-sixths of 738 chestnuts?
11. One-twelfth of 984 bananas?
12. Five-twelfths of 984 chocolates?
13. Seven-twelfths of 984 marbles?
14. Eleven-twelfths of 984 pennies?
15. One-fifth of 115 shillings?
16. Three-fifths of 635 boys?
17. Four-fifths of 345 sailors?
18. One-tenth of 450 fish?
19. Three-tenths of 540 sheep?
20. Seven-tenths of 670 horses?

EXERCISE 56.

Find the number denoted by x in each of the following:

1. $792 \div 3 - 318 \div 2 = x.$
2. $477 \div 9 + 910 \div 7 = x.$
3. $972 \div 12 - 616 \div 11 = x.$
4. $384 \div 8 + 684 \div 6 = x.$
5. $(58 \times 12) \div 4 = x.$
6. $858 \div (11 \times 3) = x.$
7. $864 \div (8 \times 9) = x.$
8. $(11 \times 8 \times 9) \div 12 = x.$
9. $(36 \times 3 \times 5) \div 6 = x.$
10. $(280 \div 7) \times 11 = x.$
11. $(144 + 180 + 68) \div 7 = x.$
12. $(825 - 278 - 179) \div 8 = x.$

EXERCISE 57.

Verify each result.

1. There are 500 passengers in a train; 78 of these travel first-class, 87 second-class, and the rest in equal numbers in five third-class carriages; how many passengers are in each of these five carriages?

2. Tom has 6 baskets, each basket containing 137 nuts. He keeps two dozen nuts and then divides the rest equally among 7 of his companions; how many does each of these boys receive?

3. Bob has a bag containing 336 nuts. He takes 3 dozen for himself and divides the rest, as far as possible, equally among 7 of his friends, keeping those nuts which remained; how many does he keep altogether for himself, and how many does each of the other boys receive?

4. John saves 59 pence and Tom saves 8 times as much as John. Fred saves a quarter as much as Tom; how many pence are saved altogether by the three boys?

5. George has 597 cherries. If he gives one-third of his cherries to Harry, how many more than Harry will he have?

6. A grocer buys 6 boxes each containing 130 eggs; how many baskets each containing only a dozen eggs must he buy to make his eggs number 900?

7. A newsboy sells 168 papers on Saturday and half as many on each of the other week-days; how many does he sell between Monday morning and Saturday night?

8. There are 70 dozen oranges in a box and it is found that half of them are bad; how many pence can be got for the rest if they are sold at the rate of 4 a penny?

9. Jones makes the following scores in eight cricket matches: 87, 36, 24, 0, 19, 34, 76, 108; what is his average score per match?

10. In eight innings Smith makes the following numbers of runs: 31, 28, 17, 0, 68, 16, 4, 12; what is his average score? How many more runs should he have made to increase his average by 2 runs?

D. NUMBERS LESS THAN 10000.

D (1). Notation and Numeration.

EXERCISE 58.

Write in words:

1. 1001.	2. 1100.	3. 1010.	4. 1805.
5. 2040.	6. 3414.	7. 4506.	8. 5651.
9. 6105.	10. 7070.	11. 8780.	12. 9190.

EXERCISE 59.

Write in figures, placing each figure in its proper column:

1. One thousand and eleven.
2. Two thousand and one.
3. Three thousand and fifty-four.
4. Four thousand eight hundred and six.
5. Nine thousand and five.
6. Eight thousand two hundred and seventeen.
7. Nineteen hundred and twelve.
8. Seven thousand and seven.
9. Seven thousand and seventeen.
10. One thousand seven hundred.

D (2). Addition and Subtraction.

EXERCISE 60.

Verify each result.

Find the sum of

1. Four hundred and fifty-six, five thousand and thirteen, three thousand and eighty-four and sixteen.

2. Seventy-nine, seven hundred and eighty-six, one thousand and ninety, nine hundred and forty-six.

3. Two hundred and fifty, five thousand and seventy-seven, one thousand two hundred and six and sixty-five.

4. Seven, seven thousand and ninety-seven, two hundred and seventy and seventeen.

5. Four thousand three hundred and sixty, four hundred and thirty-six, forty-six and nine.

6. Nine hundred and seventy-eight, eight hundred and seventy-nine, seven hundred and eighty-nine, nine hundred and eighty-seven.

7. Two thousand and sixty, two thousand six hundred and six, two thousand six hundred and sixty-six and sixty-six.

8. Nine hundred and forty-nine, two thousand and ninety-nine, six hundred and eight, five thousand three hundred and eighty.

9. Forty-five, four thousand and forty-five, six hundred and nine and thirty-nine.

10. Seventeen hundred and one, eighteen hundred and forty, nineteen hundred and four and fourteen hundred and forty-nine.

EXERCISE 61.

Verify each result.

1. Find the difference between eight thousand and sixteen and four thousand and fifty-three.

2. What number must be added to three thousand two hundred and five to make seven thousand and eighty?

3. What number must be taken from eight thousand seven hundred and one to leave nine hundred and seventy-five?

4. What number will be left from nine thousand and seventy-four after subtracting fifteen hundred and seventy-eight?

5. The sum of two numbers is eight thousand and eighteen; one of them is eighteen hundred and eighty; what is the other number?

6. The difference between two numbers is eight hundred and seventeen; the greater number is seven thousand and two; what is the smaller?

7. By what number is three thousand one hundred and fifty greater than thirteen hundred and five?

8. By what number is six thousand two hundred and thirty-four less than nine thousand one hundred and twenty-three?

9. From the sum of five thousand two hundred and sixty-one and one thousand seven hundred and eighty-four subtract the difference between three thousand and twenty-nine and seven thousand and eighteen.

10. To the difference between two thousand and seventy-five and three thousand and fifty-three add the difference between four thousand and thirty-eight and five thousand and ninety-six.

11. What number must be added to the sum of three thousand seven hundred and nine, two thousand eight hundred and eighteen

and one thousand nine hundred and six to make nine thousand and ninety?

12. What is the difference between the sum of, and the difference between, four thousand nine hundred and seventy-two and two thousand one hundred and eighty-seven?

D (3). Multiplication.

EXERCISE 62.

Verify each result.

Find the following products:

1. 692×2.	2. 729×3.	3. 862×4.	4. 623×5.
5. 746×6.	6. 897×7.	7. 925×8.	8. 483×9.
9. 796×11.	10. 589×12.	11. 483×10.	12. 237×40.
13. 576×14.	14. 536×15.	15. 456×21.	16. 351×27.
17. 219×42.	18. 174×56.	19. 148×63.	20. 127×77.

51. The following method may be used in all cases, whatever the multiplier may be.

EXAMPLE. *Multiply* 359 *by* 23.

The multiplier is equivalent to $20+3$; hence to obtain the product required we multiply 359 by 20 and by 3, and we add the results together. The work is shown in full in (i), but is usually arranged as in (ii).

In multiplying, **always begin with the highest, i.e. the left-hand, digit of the multiplier.**

(i)
$$\begin{array}{r} 359 = \text{Multiplicand} \\ 23 \\ \hline 7180 = M \times 20 \\ 1077 = M \times 3 \\ \hline \underline{8257} = M \times 23 \end{array}$$

(ii)
$$\begin{array}{r} 359 \\ 23 \\ \hline 718 \\ 1077 \\ \hline \underline{8257} \end{array}$$

Each of the lines 7180 and 1077 is called a **partial product.**

It is customary to omit the 0's on the right of the lines in the work; but in using (ii) the pupil must be careful to place the *right-hand digit* of each partial product *under the multiplying digit which produces it.*

EXERCISE 63.

Find the following products:

1. 589×13.	2. 576×17.	3. 487×19.	4. 408×23.
5. 296×29.	6. 183×31.	7. 253×37.	8. 219×43.
9. 167×59.	10. 146×67.	11. 132×71.	12. 109×83.

SUPPLEMENTARY EXAMPLES.

Re-work Exercise 62, 13–20, by the Long Multiplication method of Art. 51.

52. If a and b represent any two numbers, the following general principles are frequently useful:

(i) $(a+b)\times(a-b)=a^2-b^2$,

and conversely, $a^2-b^2=(a+b)\times(a-b)$;

(ii) $(a+b)^2=a^2+2\times a\times b+b^2$,

and (iii) $(a-b)^2=a^2-2\times a\times b+b^2$.

The following illustrate the practical use of these principles:

EXAMPLE 1. *Find the square of* 83.

Using (ii)
$$\begin{aligned}83^2&=(80+3)^2\\&=80^2+2\times80\times3+3^2\\&=6400+480+9\\&=\underline{6889.}\end{aligned}$$

EXAMPLE 2. *Find the square of* 87.

Using (iii)
$$\begin{aligned}87^2&=(90-3)^2\\&=90^2-2\times90\times3+3^2\\&=8109-540\\&=\underline{7569.}\end{aligned}$$

EXAMPLE 3. *Find the value of* 78^2-67^2.

Using (i)
$$\begin{aligned}78^2-67^2&=(78+67)(78-67)\\&=145\times11\\&=\underline{1595.}\end{aligned}$$

EXERCISE 64.

Find the value of

1. 37^2.	2. 41^2.	3. 53^2.	4. 67^2.	5. 79^2.
6. 52^2.	7. 68^2.	8. 74^2.	9. 85^2.	10. 46^2.
11. 35^2-29^2.	12. 47^2-42^2.	13. 58^2-48^2.		
14. 87^2-78^2.	15. 54^2-44^2.	16. 79^2-65^2.		

SUPPLEMENTARY EXAMPLES.

Re-work Exercise **47**, 7–11 and 17–21, by this method.

D (4). Division.

53. The following method is a general method and may be used in all cases whatever the divisor may be; it is called **Long Division**, and is an arrangement consisting of a series of divisions and subtractions.

EXAMPLE 1. *Divide* 7859 *by* 8.

Long Division Method:

The work is shown in full in (i), but is usually arranged as in (ii).

(i) Quotient = 900 + 80 + 2 = **982**

8 | 7859
7200 = 900 times 8
659 = remainder after subtracting 900 times 8
640 = 80 times 8
19 = remainder after subtracting (900 + 80) times 8
16 = 2 times 8
3 = remainder after subtracting (900 + 80 + 2) times 8

(ii) Quotient = **982**

8 | 7859
72
65
64
19
16
3

The pupil will readily see that in "Short Division" much less is written down, whilst more is done mentally.

Short Division Method:

8 | 7859

982 *eights* + **3** *units* remainder.

EXAMPLE 2. *How many times can* 31 *sticks be taken from* 1426 *sticks?*

[PROCESS: *Divide* 1426 *by* 31.]

The work is shown in full in (i), but is usually arranged as in (ii).

(i) Quotient = 40 + 6 = **46**

31 | 1426
1240 = 40 times 31
186 = remainder after subtracting 40 times 31
186 = 6 times 31

(ii) Quotient = **46**

31 | 1426
124
186
186

Thus 31 can be subtracted **46** times from 1426.

Explanation of (ii).

1st. We begin at the left of the dividend and we take as many figures as are required to make up a number equal to, or greater than, the divisor; in this case we must take the three figures 142.

2nd. We find that 142 contains 31 *four times*, with a remainder; hence **4** is the first figure in the quotient.

This means that 31 *is contained forty times in* 1420 *with a remainder* [see (i)].

3rd. Subtracting 4 times 31 from 142, the remainder is 18.

4th. Putting at the right of this the next figure 6 of the dividend we get 186, and we find that 186 contains 31 *six times* exactly; hence the second figure of the quotient is 6, and the whole quotient is **46.**

CAUTION. The quotient should be placed over the dividend, each quotient figure immediately above that figure of the dividend which is used in obtaining it.

NOTE 1. At every step we must have a remainder LESS than the divisor.

NOTE 2. For every figure brought down from the dividend a figure must be placed in the quotient.

Verification. Dividend = Quotient × Divisor
= 46 × 31
= 1426.

EXERCISE 65.

Perform the following operations (1) by Long Division in full, (2) by Short Division:

1. $9342 \div 2$.	2. $9105 \div 3$.	3. $5636 \div 4$.	4. $8710 \div 5$.
5. $8237 \div 6$.	6. $4129 \div 7$.	7. $3991 \div 8$.	8. $7246 \div 9$.
9. $8109 \div 11$.	10. $6138 \div 12$.	11. $6319 \div 10$.	12. $9300 \div 50$.
13. $4892 \div 60$.	14. $3008 \div 70$.	15. $9792 \div 24$.	16. $9072 \div 36$.
17. $8783 \div 18$.	18. $3917 \div 48$.	19. $4038 \div 72$.	20. $3857 \div 84$.

EXERCISE 66.

Verify each result by Multiplication.

Perform the following operations by Long Division:

1. $4186 \div 91$.	2. $3763 \div 71$.	3. $5394 \div 87$.	4. $3255 \div 93$.
5. $2054 \div 79$.	6. $9782 \div 67$.	7. $9486 \div 51$.	8. $9324 \div 37$.
9 $9047 \div 83$.	10. $9384 \div 23$.	11. $8636 \div 17$.	12. $9126 \div 13$.

EXERCISE 67.

Verify each result.

1. Tom has five lots of marbles, each containing five hundred and seven, and Bob has four lots of six hundred and eight marbles; which has the most, and by how many?

2. How many will be left after taking three thousand eight hundred and nine apples from six times seven hundred and five apples?

3. Farmer Jones buys thirty-nine horses at forty-three pounds each; Farmer Smith buys twice as many cows at twenty-five pounds each; which spends the greater sum, and by how much?

4. A ship's cargo is 3829 tons of coal. During five hours in the forenoon 132 tons are emptied per hour, and during nine hours in the afternoon 173 tons are emptied per hour; how many tons remain to be emptied?

5. There are 6032 soldiers in a camp, and a quarter of them march away; how many are left?

6. A bag contains 1853 pennies; if I take out 365, how many shillings can I get for the rest? (12 pence make 1 shilling.)

7. Four books contain respectively 178, 216, 349, 457 pages; what is the average number of pages in a book?

8. During six days the number of papers sold by a newsagent are 291, 352, 390, 468, 517, 574; what is the daily average?

9. There are 16 ounces in a pound. A grocer mixes 4176 ounces of coffee with 1648 ounces of chicory; how many pounds of the mixture can he make?

10. In a factory 3960 shillings are paid weekly in wages. One-fifth of it is paid to managers and foremen, and the remainder to 132 workmen; how many shillings per week on an average will each workman get?

E. THE UNITARY METHOD.

54. In the solution of simple problems which involve both multiplication and division, we generally employ a *mode of expression* which is called the **UNITARY METHOD.** It will be seen from the following examples what the method is.

EXAMPLE 1. *If each of* 11 *books has the same number of pages, and altogether they contain* 352 *pages, how many pages will* 6 *of the books contain?*

Number of pages in 11 books = 352 ;
$\therefore$ ..... 1 book = 32 ;
$\therefore$ ..... 6 books = **192.**

EXAMPLE 2. *If each of* 18 *books has the same number of pages and altogether they contain* 432 *pages, how many of the books will contain* 312 *pages altogether?*

Number of pages in 18 books = 432 ;
$\therefore$ ..... 1 book = 24 ;
$\therefore$ Number of books which contain 312 pages $= \frac{312}{24} = 13.$

EXAMPLE 3. *If* 19 *men can complete a work in* 437 *days, in how many days will* 23 *men be able to do it?*

Time required for 19 men = 437 days ;
$\therefore$ ..... 1 man $= 19 \times 437$ days ;
$\therefore$ ..... 23 men $= \frac{19 \times 437}{23}$ days
$= 361$ days.

EXERCISE 68.

In each problem state your reasoning clearly.

1. How many shillings will 9 books cost, if 28 shillings will buy 7 of them?

2. How many pence are eggs per score, if 24 pence will buy a dozen?

3. A train travels at the rate of 128 miles in 4 hours ; at this rate how many miles will it travel in 7 hours?

4. A man walks 216 feet in 36 steps ; at this rate how many feet does he walk in 45 steps?

5. Each of 11 boys works the same number of sums and altogether they work 165; how many do 3 boys together work?

6. Each of 9 boys collects the same number of stamps and altogether they collect 648; at this rate how many will 11 boys collect?

7. Forty-two boxes of equal size contain altogether 504 eggs; how many should 13 of the boxes contain?

8. Thirty-one boxes of equal size contain altogether 744 sardines; at this rate how many should 40 similar boxes contain?

9. Each of 64 books has the same number of pages and altogether they contain 1216 pages; how many pages will 25 of them contain?

10. A train travels at the rate of 144 miles in 4 hours; at this rate how many hours will it take to travel 252 miles?

11. Each of 18 books has the same number of pages, and altogether they contain 576 pages; at this rate how many of the books will contain 416 pages?

12. Three hundred and sixty eggs are packed in 15 boxes of equal size; how many such boxes will be wanted in which to pack 600 eggs?

13. If 25 men can do a piece of work in 24 days, in how many days will 15 men do it?

14. If 18 men can do a piece of work in 10 days, in how many days will 45 men do the same work?

15. If 8 men would take 75 days to finish a piece of work, how many men would be needed to finish it in 40 days?

16. How many men in 14 days can do as much work as it takes 21 persons to do in 4 days?

17. If I can finish certain work in 16 days, working 9 hours a day, how many days should I take working 12 hours a day?

18. If a gang of workmen can do a piece of work in 8 days, working 9 hours a day, how many hours a day must they work to do it in 6 days?

19. If a certain quantity of food would be enough for 9 sheep for 28 days, how many sheep would it serve for 36 days?

20. If 40 horses eat a certain quantity of hay in 9 days, how many days will the same amount of hay last 15 horses?

F. NUMBERS LESS THAN 100000.

F (1). Notation and Numeration.

EXERCISE 69.

Write in words:

1. 10020.	2. 10202.	3. 11011.	4. 15060.
5. 50016.	6. 18008.	7. 60606.	8. 90090.
9. 70917.	10. 41040.	11. 81081.	12. 38100.

EXERCISE 70.

Write in figures, placing each figure in its proper column:

1. Ten thousand and ten.
2. Ten thousand one hundred and one.
3. Eleven thousand and one.
4. Fourteen thousand and sixteen.
5. Forty thousand and sixty.
6. Seventeen thousand and seven.
7. Fifty thousand five hundred and five.
8. Seventy thousand and seventy.
9. Sixty thousand six hundred and six.
10. Eighty-eight thousand eight hundred and eight.

F (2). Addition and Subtraction.

EXERCISE 71.

1. Find the sum of three thousand and eighteen, nine thousand six hundred and fourteen, two thousand and ten, seven hundred and eighty, one thousand and forty-two.

2. Find the total of seven hundred and one, eighty-three, eight thousand and eleven, six thousand and forty-one, nine thousand seven hundred and sixty-two.

3. Find the amount of twenty-nine, two hundred and nineteen, twenty thousand and ninety-two, seven thousand eight hundred and four, sixty thousand and twenty-nine.

4. Find the sum of two thousand and sixteen, twenty thousand and sixty-one, four thousand and ninety-four, eighteen thousand and seventy-five, thirty thousand two hundred and eight.

5. In a town there are 15269 men, 18392 women, 7263 boys, and 8965 girls; what is the total population of the town?

6. The circulation of a newspaper is 6284 copies on Monday, 7765 on Tuesday, 7659 on Wednesday, 7876 on Thursday, 7632 on Friday and 8469 on Saturday; how many copies are circulated in the six days?

7. What number must be added to nine thousand seven hundred and eight to make thirty thousand six hundred and twelve?

8. What number must be taken from eighty thousand seven hundred and three to leave eighteen hundred and ninety-four?

9. To the sum of five thousand one hundred and sixty-nine and three thousand seven hundred and eighty-six add their difference.

10. In a town containing sixteen thousand five hundred people there are seven thousand and forty-eight children and four thousand seven hundred and sixty-four women; how many men are there in the town?

F (3). Multiplication.

55. By making a preliminary rough approximation to a product the pupil should satisfy himself that his answer is at least reasonable.

EXAMPLE. *Make a rough approximation to* 698×67.

Taking round numbers,

$$700 \times 60 = 42000,$$

and $700 \times 70 = 49000$;

$\therefore$ the answer to the question must lie between **42000** and **49000**.

EXERCISE 72.

(1) *Make a preliminary rough approximation to each product; and* (2) *verify each result.*

Find the following products:

	(*a*)	(*b*)		(*a*)	(*b*)		(*a*)	(*b*)
1.	698×67;	82.	2.	765×36;	95.	3.	856×84;	72.
4.	785×59;	36.	5.	856×27;	69.	6.	948×75;	39.
7.	867×29;	84.	8.	982×76;	37.	9.	567×98;	25.
10.	943×47;	56.	11.	754×69;	43.	12.	782×35;	82.

EXERCISE 73.

1. What is the sum of twice eighty-three, three times eight hundred and six, and four times eight thousand and sixty?

2. What is the difference between nine times five thousand seven hundred and eight and eight times six thousand seven hundred and five?

3. How many trees are there in a plantation of twenty-six rows, each row containing one thousand nine hundred and eighty trees?

4. How many pounds must be paid for seven hundred and sixty-eight horses at forty-nine pounds each?

5. Sixteen ounces make a pound; how many ounces are there in eighteen hundred and ninety-four pounds?

6. What is the result if five hundred and sixty-seven be added thirteen times to four thousand and eighty?

7. A fruiterer has fifteen hundred and twenty score of plums; after selling thirteen gross to one shop and one thousand and eighteen dozen to another, how many plums will he have left?

8. A house contains fifty-four windows, and each window has eighteen panes; how many pence did the windows cost at sixteen pence each pane?

9. Twelve trains of nine carriages each run daily on a railway, and each carriage contains on an average forty-seven persons; how many people are conveyed in a week (7 days)?

10. In a book of three hundred and fifty-two pages, each page contains twenty-four lines, and each line, on an average, contains eleven words; how many words are there in the book?

F (4). Division.

EXERCISE 74.

Verify each result by Long Division.

Perform, by factors, the following divisions, obtaining the true remainder in each case:

1. $32935 \div 15$.	2. $20115 \div 16$.	3. $17842 \div 18$.
4. $92512 \div 21$.	5. $38733 \div 24$.	6. $49368 \div 30$.
7. $18421 \div 27$.	8. $21580 \div 36$.	9. $11111 \div 32$.
10. $43160 \div 42$.	11. $23696 \div 45$.	12. $33033 \div 64$.

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EXERCISE 75.

Verify each result by Multiplication.

Perform the following operations by Long Division :

1. $14196 \div 91$.	2. $17205 \div 93$.	3. $12874 \div 82$.
4. $12495 \div 85$.	5. $18031 \div 73$.	6. $19875 \div 75$.
7. $14274 \div 61$.	8. $12285 \div 65$.	9. $55067 \div 53$.
10. $45021 \div 43$.	11. $42316 \div 17$.	12. $63300 \div 19$.
13. $64318 \div 26$.	14. $80000 \div 37$.	15. $10133 \div 39$.
16. $11339 \div 46$.	17. $66606 \div 56$.	18. $74200 \div 58$.
19. $89314 \div 67$.	20. $82211 \div 69$.	21. $91540 \div 76$.
22. $80372 \div 79$.	23. $98211 \div 87$.	24. $96614 \div 96$.

EXERCISE 76.

Verify each result.

1. To 23948 add 31844, and divide the sum by 8.

2. Divide the difference between 92950 and 69878 by 7.

3. Multiply the ninth part of 65637 by 13.

4. Take 72427 from 90005, and divide the result by 34.

5. Divide the sum of 9206, 15052, 162, and 10600 by 68.

6. If we divide 8556 by 92, by what number must we multiply the quotient in order to obtain 46686?

7. A father left at his death 91644 pounds to his children. His son received a sixth of the amount, and the remainder was divided equally among his five daughters; how much did each child receive?

8. Twenty shillings are changed for a sovereign. How many sovereigns can be obtained for 11349 shillings, and how many shillings will be left over?

9. Forty-eight farthings make a shilling; how many shillings could be obtained for 20628 farthings, and how many farthings would be left over?

10. There are 17 bags each containing 3876 letters, and these are changed into 19 bags; how many letters will there be in each of these?

56. The following are further examples of simple problems involving division.

EXAMPLE 1. *Divide* 392 *nuts between Smith and Jones, so that Smith may have* 3 *shares and Jones* 4 *shares.*

We can do this by dividing 392 into (3+4) equal parts, *i.e.* 7 equal parts, and taking 3 of them for Smith and 4 of them for Jones.

$$\text{Each equal part} = 392 \div 7 = 56 ;$$
$$\therefore \text{ Smith's share} = 56 \times 3 = \mathbf{168},$$
$$\text{and Jones's share} = 56 \times 4 = \mathbf{224}.$$

Verification. $168 + 224 = 392$.

EXAMPLE 2. *Divide* 392 *nuts among* 3 *boys and* 2 *girls, giving each girl twice as many as each boy.*

$$\text{Let one boy's share} = x \text{ nuts},$$
$$\text{then one girl's share} = 2x \text{ nuts} ;$$
$$\therefore \ 3 \times x + 2 \times 2x = 392 ;$$
$$\therefore \ 3x + 4x = 392 ;$$
$$\therefore \ 7x = 392 ;$$
$$\therefore \ x = 56 ;$$

$\therefore$ a boy receives **56** nuts, and a girl receives **112** nuts.

Verification. $3 \times 56 + 2 \times 112 = 168 + 224 = 392$.

EXAMPLE 3. *Divide* 390 *nuts among* 3 *boys and* 2 *girls, giving each boy* 55 *nuts more than each girl.*

It is evident that as the 3 boys will receive 165 nuts more than the girls, this number must be first taken away, and the result of dividing the remainder into 5 equal parts will give the share of each girl.

$$\text{The share of each girl} = \frac{390 - 165}{5} \text{ nuts}$$
$$= \frac{225}{5} \text{ nuts}$$
$$= \mathbf{45} \text{ nuts},$$
$$\text{and the share of each boy} = \mathbf{100} \text{ nuts}.$$

Verification. $3 \times 100 + 2 \times 45 = 300 + 90 = 390$.

EXERCISE 77.

(1) *In each problem write down a clear account of your reasoning ; and* (2) *verify each solution.*

1. Divide 351 postage stamps between Smith and Jones, so that Smith may have 6 shares and Jones 7 shares.

2. Divide 45 marbles among A, B and C, so that A may have 3 shares, B 5 shares and C 7 shares.

3. Divide 702 nuts among A, B and C, so that A may have 2 shares, B 3 shares and C 4 shares.

4. Divide 360 nuts among 4 boys, giving A, B and C, three of the boys, three times as many as D, the fourth boy.

5. Divide 180 picture post-cards among A, B and C, so that A may have three times as many as B, and B twice as many as C.

6. Divide 54 shillings among 2 men, 3 women and 4 children, giving a man twice as many shillings as a woman, and a woman twice as many as a child.

7. One hundred and twenty apples are picked by 10 boys and 10 girls, and each boy picks twice as many as each girl; how many apples does each child pick?

8. Divide 85 picture post-cards between A and B, giving A 15 more than B.

9. Divide 1000 apples into three groups, placing 250 more in the first group than in the second or the third.

10. Divide 180 marbles among 6 boys, giving A and B, two of the boys, 15 more than each of the others.

11. Three boys together work 63 sums. A does twice as many as B, who does twice as many as C; how many sums does each work?

12. Thirty-six questions are answered by 3 boys. C answers 6 more than B, who answers 6 more than A; how many questions does each answer?

G. NUMBERS LESS THAN 1000000.

G (1). Notation and Numeration.

EXERCISE 78.

Write in words :

1. 100100.	2. 110010.	3. 101001.	4. 210012.
5. 301021.	6. 421030.	7. 500050.	8. 505015.
9. 660606.	10. 707707.	11. 808080.	12. 919909.

EXERCISE 79.

Write in figures, placing each figure in its proper column :

1. Three hundred thousand and thirty.
2. Four hundred and seventy thousand and seventy.
3. Nine hundred and nine thousand and nine.
4. Eight hundred and eighty thousand and eighty.
5. Five hundred and seventeen thousand and seventeen.
6. Six hundred and ninety thousand nine hundred.
7. Seven hundred and eighteen thousand and seventy-eight.
8. Two hundred thousand two hundred and ninety-two.
9. One hundred and ninety-two thousand and nineteen.
10. Nine hundred and nineteen thousand and nine.

G (2). Addition and Subtraction.

EXERCISE 80.

(1) *Write the answer in words as well as in figures ;*
and (2) *check your result by adding in the reverse order.*

Arrange the following sets of numbers in vertical columns, and then add :

1. $37420 + 56 + 189201 + 7502 + 342$.
2. $8476 + 29 + 134209 + 37561 + 421$.
3. $56201 + 323 + 9992 + 476500 + 1291$.
4. $37651 + 843 + 620712 + 13520 + 2159$.
5. $34184 + 804 + 380824 + 6 + 3289$.

6. 308426 + 30180 + 561489 + 362 + 10.
7. 218031 + 345608 + 21 + 26840 + 3807.
8. 184752 + 290 + 2052 + 695634 + 51.
9. 639643 + 6298 + 67493 + 67 + 698.
10. 269326 + 398 + 61932 + 6009 + 3765.

EXERCISE 81.

Check each result.

Write down the answers to the following:

1. 620394 – 123456.
2. 506047 – 446098.
3. 196740 – 124056.
4. 807065 – 298798.
5. 700604 – 654329.
6. 490023 – 134067.
7. 932082 – 369325.
8. 610324 – 309067.
9. 376004 – 160640.
10. 960804 – 308076.

G (3). Multiplication.

EXERCISE 82.

Write down the following products:

	(*a*)	(*b*)		(*a*)	(*b*)
1.	5478 × 10;	20.	2.	3765 × 30;	40.
3.	4987 × 60;	70.	4.	3698 × 80;	90.
5.	2453 × 100;	200.	6.	1579 × 300;	400.
7.	1578 × 500;	600.	8.	1298 × 40;	300.
9.	1396 × 50;	600.	10.	1345 × 80;	700.

EXERCISE 83.

(1) *Make a preliminary rough approximation to each product;* and (2) *verify each result by Long Division.*

Find the following products:

	(*a*)	(*b*)		(*a*)	(*b*)
1.	4567 × 89;	75.	2.	3892 × 98;	76.
3.	4705 × 74;	56.	4.	6809 × 63;	47.
5.	5732 × 47;	65.	6.	3985 × 39;	48.
7.	6249 × 28;	93.	8.	3967 × 39;	84.
9.	4895 × 46;	39.	10.	3972 × 37;	58.

57. No new principle is involved in the examples which follow.

EXAMPLE 1. *Multiply* 1342 *by* 324.

The multiplier 324 is equivalent to $300+20+4$; hence to obtain the product required we multiply 1342 by 300, by 20 and by 4, and we add the results together. The work is shown in full in (i), but is usually arranged as in (ii).

(Rough Approximation. $1400 \times 300 = 420000$.)

(i)

$$\begin{array}{r} 1342 = \text{Multiplicand} \\ 324 \\ \hline 402600 = M \times 300 \\ 26840 = M \times 20 \\ 5368 = M \times 4 \\ \hline \underline{434808} = M \times 324 \end{array}$$

(ii)

$$\begin{array}{r} 1342 \\ 324 \\ \hline 4026 \\ 2684 \\ 5368 \\ \hline \underline{434808} \end{array}$$

Verification. Notice the relation between the digits of this multiplier. We can arrange the work in *two* lines: line (a) is 1342×3 and line (b) is 8 times (a), but we must set down the right-hand figure of each product in its correct place.

$$\begin{array}{rl} 1342 & \\ 324 & \\ \hline 4026 & (a) \\ 32208 & (b) \\ \hline 434808 & \end{array}$$

EXAMPLE 2. *Multiply* 507 *by* 450.

If one or more of the figures of the multiplier be 0, these 0's may be neglected in forming the partial products; but the pupil must be careful to place the *right-hand digit* of each partial product *under the multiplying digit which produces it.*

(i)

$$\begin{array}{r} 507 \\ 450 \\ \hline 202800 \\ 25350 \\ \hline \underline{228150} \end{array}$$

(ii)

$$\begin{array}{r} 507 \\ 450 \\ \hline 2028 \\ 2535 \\ \hline \underline{228150} \end{array}$$

EXERCISE 84.

(1) *Make a preliminary rough approximation to each product; and* (2) *verify each result by interchanging multiplier and multiplicand.*

Find the following products:

1. 573×514.	**2.** 860×461.	**3.** 295×635.
4. 918×724.	**5.** 475×847.	**6.** 258×758.
7. 461×430.	**8.** 572×370.	**9.** 907×506.
10. 4060×178.	**11.** 2007×261.	**12.** 3090×309.

EXERCISE 85.

(1) *Make a preliminary rough approximation to each product; and* (2) *prove each result by multiplying in* **two** *lines.*

Find the following products:

1. 357×324.	**2.** 468×216.	**3.** 487×312.
4. 742×918.	**5.** 296×872.	**6.** 428×749.
7. 537×856.	**8.** 849×642.	**9.** 864×428.
10. 2761×124.	**11.** 7658×112.	**12.** 1736×321.

EXERCISE 86.

Find the continued product of

1. 12, 53, 546. **2.** 17, 25, 368. **3.** 19, 67, 482.

Find the square of

4. 752. **5.** 365. **6.** 578. **7.** 694. **8.** 869.

Find the cube of

9. 86. **10.** 47. **11.** 59. **12.** 68. **13.** 74.

58. The following process should be carefully noticed.

EXAMPLE. *Subtract in one operation* 8 *times* 23478 *from* 784592.

$$\begin{array}{r} 784592 \\ 23478 \times 8 \\ \hline \text{Difference} = \underline{596768} \end{array}$$

The mental steps by which we obtain the result at once are as follows:

1st. $8 \times 8 = 64$, and **8** make 72; write down **8**, and carry 7;
2nd. $8 \times 7 + 7 = 63$, and **6** make 69; write down **6**, and carry 6;
3rd. $8 \times 4 + 6 = 38$, and **7** make 45; write down **7**, and carry 4;
4th. $8 \times 3 + 4 = 28$, and **6** make 34; write down **6**, and carry 3;
5th. $8 \times 2 + 3 = 19$, and **9** make 28; write down **9**, and carry 2;
6th. 2, and **5** make 7; write down **5**.

EXERCISE 87.

Find in *one operation* the value of

1. $4313 - 5 \times 817$.	**2.** $2287 - 2 \times 817$.	**3.** $4690 - 7 \times 618$.
4. $3643 - 5 \times 618$.	**5.** $5532 - 8 \times 618$.	**6.** $2240 - 4 \times 489$.
7. $2840 - 5 \times 489$.	**8.** $1924 - 5 \times 342$.	**9.** $2149 - 6 \times 342$.
10. $7880 - 9 \times 861$.	**11.** $1635 - 6 \times 237$.	**12.** $2659 - 7 \times 358$.

G (4). Division.

59. To divide by 100, 200, etc., we mark off the last *two* figures, and proceed as before (Art. 47, 48).

EXERCISE 88.

Perform the following divisions:

1. $917345 \div 20$.	2. $851709 \div 30$.	3. $639528 \div 50$.
4. $828436 \div 60$.	5. $606255 \div 80$.	6. $540619 \div 90$.
7. $573982 \div 300$.	8. $417346 \div 400$.	9. $740618 \div 600$.
10. $684073 \div 700$.	11. $462863 \div 900$.	12. $629392 \div 1100$.

60. No new principle is involved in the examples which follow: the only difference being that somewhat larger numbers are now used.

EXAMPLE. *Divide* 562469 *by* 324.

We take the figures 562 to make up a number greater than the divisor. We find that 562 contains 324 *once*, with 238 remainder. Hence **1** is the first figure of the quotient.

Placing to the right of 238 the next figure in the dividend, 4, we get 2384, which contains 324 *seven* times with 116 remainder. Thus **7** is the second figure of the quotient.

Proceeding in this way until all the figures of the dividend have been used, we obtain for the complete quotient **1736**, with remainder **5**.

Quotient = **1736**

```
324 | 562469
      324
      ----
      2384
      2268
      ----
       1166
        972
        ---
        1949
        1944
        ----
Remainder =  5
```

Verification. Dividend = Quotient × Divisor + Remainder
= 1736 × 324 + 5
= 562469.

EXERCISE 89.

Verify each result by Multiplication.

Perform the following operations by Long Division:

1. $619760 \div 976$.	2. $737715 \div 935$.	3. $529248 \div 894$.
4. $282855 \div 865$.	5. $224076 \div 789$.	6. $592767 \div 679$.
7. $391535 \div 572$.	8. $372621 \div 469$.	9. $208310 \div 357$.
10. $170847 \div 246$.	11. $142493 \div 194$.	12. $141466 \div 187$.

61. When the divisor is the product of three or more factors we proceed as in the example which follows.

EXAMPLE. *Divide* 4586 *by* $3 \times 5 \times 7$, *and find the remainder.*

After division by 3 and 5 the complete remainder is 11. To find the complete remainder after the next division, we consider 15 as our first divisor and 11 as our first remainder. The complete remainder is therefore $4 \times 15 + 11$, *i.e.* 71.

$$\left.\begin{array}{l} 105 \left\{ \begin{array}{l} 15 \left\{ \begin{array}{l|l} 3 & 4586 \\ 5 & 1528 \ldots 2 \\ 7 & 305 \ldots 3 \end{array} \right. \\ \qquad 43 \ldots\ldots 4 \end{array} \right. \end{array} \right\} \begin{array}{l} 11 \\ 71 \end{array}$$

EXERCISE 90.

Verify each result by Long Division.

1. Divide 37629 by $3 \times 5 \times 7$.
2. Divide 57816 by $2 \times 7 \times 11$.
3. Divide 91976 by $5 \times 5 \times 5$.
4. Divide 57671 by $7 \times 7 \times 7$.
5. Divide 62928 by $3 \times 7 \times 12$.
6. Divide 152639 by $8 \times 8 \times 8$.
7. Divide 110893 by $9 \times 9 \times 9$.
8. Divide 122415 by $3 \times 5 \times 11$.
9. Divide 124488 by $2 \times 8 \times 9$.
10. Divide 755568 by $12 \times 12 \times 12$.

62. The Shortened form of Long Division, in which the products are omitted and only the successive differences are written down (Art. 58), is known as the **Italian Method.**

EXAMPLE. *Divide* 562469 *by* 324.

Our previous example (Art. 60) will appear thus when the Italian Method is adopted:

Quotient = **1736**

$$\begin{array}{r|l} 324 & 562469 \\ & 2384 \\ & 1166 \\ & 1949 \end{array}$$

Remainder = **5**

EXERCISE 91.

Verify each result by Multiplication.

Find, by the Italian Method, the quotient and remainder:

1. $431379 \div 817$.
2. $469032 \div 618$.
3. $224009 \div 489$.
4. $192494 \div 342$.
5. $133120 \div 237$.
6. $337103 \div 861$.
7. $218860 \div 689$.
8. $976278 \div 946$.
9. $863667 \div 358$.
10. $827625 \div 723$.
11. $978462 \div 409$.
12. $143270 \div 128$.

G (5). Special Multipliers and Divisors.

63. There are certain multipliers and divisors for which we may adopt special methods.

(1) **To multiply by 100.** We place 00 at the right of the multiplicand.

(2) **To multiply by 99.** Since $99 = 100 - 1$, we multiply by 100, and subtract the multiplicand.

EXAMPLE. *Multiply* 451 *by* 99.

$451 \times 100 = 45100$
Subtract $\quad 451$
$\underline{44649} = 99$ times 451.

(3) **To multiply by 999.** Since $999 = 1000 - 1$, we multiply by 1000, *i.e.* we place 000 at the right of the multiplicand, and then subtract the multiplicand from this.

EXAMPLE. *Multiply* 451 *by* 999.

$451 \times 1000 = 451000$
Subtract $\quad 451$
$\underline{450549} = 999$ times 451.

(4) **To multiply by such numbers as 98, 998, etc.**

EXAMPLE 1. *Multiply* 451 *by* 997.

$451 \times 1000 = 451000$
$451 \times 3 = \quad 1353$
By subtraction, $\underline{449647} = 997$ times 451.

EXAMPLE 2. *Multiply* 451 *by* 188.

$188 = 200 - 12$;
$\therefore\ 451 \times 188 = 451 \times 200 - 451 \times 12.$
$451 \times 200 = 90200$
$451 \times \ 12 = \ 5412$
By subtraction, $\underline{84788} = 188$ times 451.

EXERCISE 92.

Find the following products by the shortest method:

1. 873×99.	2. 725×98.	3. 694×95.
4. 783×101.	5. 572×102.	6. 946×105.
7. 987×999.	8. 326×997.	9. 654×994.
10. 673×199.	11. 728×299.	12. 452×399.
13. 367×194.	14. 872×296.	15. 524×498.

(5) **To multiply by 25.** Since $25 = 100 \div 4$, we place 00 at the right of the multiplicand, and we then divide the result by 4.

EXAMPLE. *Multiply* 451 *by* 25.

$$451 \times 25 = 451 \times \frac{100}{4} = \frac{45100}{4} = \underline{11275}.$$

(6) **To multiply by 125.** Since $125 = 1000 \div 8$, we place 000 at the right of the multiplicand, and we then divide the result by 8.

EXAMPLE. *Multiply* 451 *by* 125.

$$451 \times 125 = 451 \times \frac{1000}{8} = \frac{451000}{8} = \underline{56375.}$$

(7) **To divide by 25 (or 125).** We multiply the dividend by 4 (or 8), and we then divide the product by 100 (or 1000).

EXAMPLE. *Divide* 56375 *by* 125.

$$56375 \div 125 = 56375 \div \frac{1000}{8} = 56375 \times \frac{8}{1000} = \frac{451000}{1000} = \underline{451.}$$

EXERCISE 93.

Verify each result by using another method.

Perform the following operations by the shortest method:

1. 581×25.	2. 1869×25.	3. 4723×25.
4. $325 \div 25$.	5. $8575 \div 25$.	6. $96875 \div 25$.
7. 963×125.	8. 3874×125.	9. 4257×125.
10. $10875 \div 125$.	11. $46125 \div 125$.	12. $91875 \div 125$.

G (6). Missing Figures.

EXERCISE 94.

Verify each result.

Insert the missing line in the following subtraction sums:

1.	2.	3.	4.
97531	63072	.....	.....
.....	.....	117629	306085
8104	44978	285669	305984

Complete the following addition sums:

5.	6.	7.	8.
.....	4762	4562	229619
3241	35670	801	.....
15623	20134	.....	65983
105	117	48	602306
22345	.....	6024	86094
97339	83699	843737	984009

Complete the following division sums:

9. 11 | ____
25317 rem. 2.

10. 80 | ____
7578 rem. 15.

11. 400 | ____
1043 rem. 146.

12. 1200 | ____
819 rem. 695.

13. 9 | ____
2 | ...0
2378...1

14. 11 | ____
9 | 38129...2

15. 7 | ____
3 | ...4
20174...2

16. 4 | ____
4 | 146898...3

Complete the following multiplication sums:

17.
205
**
8815

18.
***
98
30086

19.
917
***
372302

20.
***
176
123904

Fill in the missing figures in the following subtraction sums:

21.
123*
**6
782

22.
8*4*
*4*8
619

23.
*45*9
94
75*22

24.
292
*1*4
7*77

Fill in the missing figures in the following multiplication sums:

25.
*8**
3
8*47

26.
3*7*
7
**7*6

27.
98
9
26**4

28.
78
11
4*6*5

Fill in the missing figures in the following division sums:

29. 8 | *84*
980 rem. 2

30. 9 | 4*5*
5*0 rem. 6

31. 5 | 6*8*6
1*5*9 rem. 1

32. 7 | *17**
73*8 rem. 3

H. FACTORS AND PRIME NUMBERS.

H (1). Introduction.

64. If two or more numbers be multiplied together the result is called the **product**, and the numbers multiplied are called **factors** of the product.

Thus 6, being the result of multiplying 2 by 3, is called the *product* of 2 and 3, and 2 and 3 are called the *factors* of 6.

All numbers are multiples of unity, and each contains itself once. Any number can therefore be divided by itself and by unity, but if it cannot be divided without remainder by *any other* number it is said to be **prime**, and is called **a prime number.**

We may therefore define a prime number as *a number which has no factor, or divisor, but itself and unity.*

Numbers which are not prime are called **composite.**

Taking the numbers in order it will be seen that 2, 3, 5, 7, 11, 13, 17, 19, etc., are prime numbers, and that 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, etc., are composite.

Numbers which are exactly divisible by 2, such as 2, 4, 6, 8, etc., are called **even** numbers.

Numbers which are *not* exactly divisible by 2, such as 1, 3, 5, 7, etc., are called **odd** numbers.

65. Such a number as 12 is the product of 6 and 2, also of 4 and 3, and also of 2, 2 and 3, but the last of these sets, namely 2, 2 and 3, has a property which the others have not, for 2, 2 and 3 are all *prime.*

Many numbers can be separated into factors in more than one way, just as 12 can be; but *no* number can be separated in more than one way into *prime* factors.

When therefore we speak of separating a number into factors, we shall as a rule mean *prime* factors.

EXAMPLE. *Separate* 12 *into prime factors.*

$$12 = 2 \times 2 \times 3$$
$$= \underline{2^2 \times 3.}$$

EXERCISE 95.

Separate each of the following numbers into prime factors:

1. 30.	2. 42.	3. 66.	4. 70.	5. 27.
6. 28.	7. 45.	8. 63.	9. 50.	10. 110.

66. It will be well to remember the following **rules of divisibility**:

(1) **2** is a factor of all numbers whose last digit can be divided by 2.

(2) **4** is a factor if the number composed of the last 2 digits can be divided by 4.

(3) **8** is a factor if the number composed of the last 3 digits can be divided by 8.

(4) **5** is a factor if the last digit be either 0 or 5.

(5) **10** is a factor if the last digit be 0.

(6) **3** is a factor if the sum of the digits can be divided by 3.

(7) **9** is a factor if the sum of the digits can be divided by 9.

(8) **6** is a factor if both 2 and 3 are factors.

(9) **12** is a factor if both 3 and 4 are factors.

(10) **11** is a factor if, when the 1st, 3rd, 5th, etc., digits are added together, and also the 2nd, 4th, 6th, etc., the difference between these sums is 0, or can be divided by 11.

(11) Divisibility by **7** can be determined best by actual division.

EXERCISE 96.

Which of the following numbers contain the factor (*a*) 2, (*b*) 3, (*c*) 4, (*d*) 5, (*e*) 6, (*f*) 8, (*g*) 9, (*h*) 10, (*i*) 12?

1. 54.	**2.** 84.	**3.** 60.	**4.** 72.	**5.** 156.
6. 195.	**7.** 520.	**8.** 480.	**9.** 696.	**10.** 948.

67. We will now apply the rules of divisibility.

EXAMPLE. *Find the prime factors of* 504.

1st. 8 is a factor, for $504 = 8 \times 63$.

2nd. 9 is a factor of 63, for $63 = 9 \times 7$.

The process may be effected briefly, thus:

$$\begin{array}{r|l} 8 & 504 \\ 9 & 63 \\ \hline & 7 \end{array}$$

$$\therefore\ 504 = 8 \times 9 \times 7$$
$$= \underline{2^3 \times 3^2 \times 7.}$$

EXERCISE 97.

Separate each of the following numbers into its prime factors:

1. 60.	**2.** 78.	**3.** 102.	**4.** 210.	**5.** 252.
6. 315.	**7.** 525.	**8.** 612.	**9.** 627.	**10.** 715.
11. 1331.	**12.** 1430.	**13.** 1456.	**14.** 3465.	**15.** 3675.
16. 4536.	**17.** 4620.	**18.** 4851.	**19.** 5250.	**20.** 7623.

H (2). Square Root by Factorisation.

68. The **square root** of a given number is a number such that when it is multiplied by itself, the product is the given number.

For example, we know that $3 \times 3 = 9$; consequently, 9 is the *square* of 3, and 3 is the *square root* of 9.

69. The square root of a number is denoted by the symbol $\sqrt{\ }$ placed in front of, and over, the number.

Thus $\sqrt{9} = 3$, and $\sqrt{16} = 4$.

EXERCISE 98.

What number multiplied by itself will give the following?

1. 4.	**2.** 9.	**3.** 16.	**4.** 25.	**5.** 36.	**6.** 49.
7. 64.	**8.** 81.	**9.** 100.	**10.** 121.	**11.** 144.	**12.** 169.

70. When a number can be easily separated into prime factors the square root may be determined by inspection.

EXAMPLE. *Find the square root of* 576.

$$576 = 2^6 \times 3^2;$$

$$\therefore \sqrt{576} = 2^3 \times 3 = 8 \times 3$$

$$= \underline{24}.$$

71. In a number which has an exact square root, *every prime factor which occurs at all must occur raised to an* **even** power.

EXAMPLE. *What is the least number by which* 735 *must be multiplied so that the product may be a perfect square?*

$$735 = 3 \times 5 \times 7^2.$$

The least multiplier which will make this into a perfect square

$$= 3 \times 5 = \underline{15}.$$

EXERCISE 99.

Factorise and find the square root of

1. 196.	2. 256.	3. 324.	4. 225.	5. 625.
6. 576.	7. 784.	8. 441.	9. 729.	10. 900.
11. 1024.	12. 1225.	13. 1936.	14. 5625.	15. 7056.
16. 11025.	17. 17424.	18. 53361.	19. 63504.	20. 99225.

Find the least multipliers which will make the following numbers into perfect squares:

21. 63.	22. 21.	23. 96.	24. 132.	25. 480.
26. 567.	27. 1573.	28. 2352.	29. 3549.	30. 1344.

H (3). Cube Root by Factorisation.

72. The **cube root** of a given number is that number which when raised to the *third* power will produce the given number.

For example, we know that $3^3=27$; consequently, 27 is the *cube* of 3, and 3 is the *cube root* of 27.

73. The symbol $\sqrt[3]{\ }$ is used to denote the cube root of a number.

Thus $\sqrt[3]{27}=3$, and $\sqrt[3]{64}=4$.

74. In every number whose cube root can be exactly determined, *every prime factor which occurs at all must occur raised to a power which is a multiple of* 3.

EXAMPLE. *Find the cube root of* 1728.

$$\begin{aligned}\sqrt[3]{1728}&=\sqrt[3]{8\times 216}\\&=\sqrt[3]{2^3\times 6^3}\\&=2\times 6\\&=\underline{12.}\end{aligned}$$

EXERCISE 100.

Factorise and find the cube root of

1. 125.	2. 216.	3. 512.	4. 343.	5. 729.
6. 2744.	7. 4096.	8. 3375.	9. 9261.	10. 10648.
11. 13824.	12. 35937.	13. 91125.	14. 166375.	15. 216000.

Find the least multipliers which will make the following numbers perfect cubes:

16. 968.	17. 11979.	18. 49392.	19. 16807.	20. 41503.

H (4). Highest Common Factor.

75. A number which is a factor of two or more numbers is said to be a **common factor**, or **common measure**, of the numbers.

Thus 2 is a common measure of 4 and 6.

76. The *greatest* number which will divide each of two or more numbers is called their **Highest Common Factor**, or **Greatest Common Measure**, and is denoted by the letters **H.C.F.** or **G.C.M.**

Thus 4 is the highest common factor of 8 and 12.

77. In many cases the H.C.F. of two or more numbers may readily be determined by separating each number into its prime factors.

EXAMPLE. *Find the* H.C.F. *of* 36, 60 *and* 84.

We find that $36 = 2 \times 2 \times 3 \times 3,$

$60 = 2 \times 2 \times 3 \times 5,$

and $84 = 2 \times 2 \times 3 \times 7.$

The factor 2 occurs *twice* in each number, and the factor 3 occurs *once*; hence the H.C.F. $= 2 \times 2 \times 3 = 2^2 \times 3 = \mathbf{12}$, for this number is the product of all the prime factors common to the three numbers.

NOTE. When the prime factors have been found, it will be seen that the H.C.F. must contain *all* the factors which are *common* to *all* the numbers, and the *index* of any factor in the H.C.F. must be that of the *lowest* power to which it is raised in any of the numbers.

EXERCISE 101.

Find the H.C.F. of

1.	4 and 6.	**2.**	6 and 8.	**3.**	6 and 9.	**4.**	8 and 12.
5.	9 and 12.	**6.**	6 and 14.	**7.**	8 and 14.	**8.**	10 and 15.
9.	8 and 18.	**10.**	12 and 18.	**11.**	10 and 24.	**12.**	10 and 25.
13.	15 and 25.	**14.**	18 and 27.	**15.**	21 and 28.	**16.**	14 and 22.
17.	49 and 77.	**18.**	45 and 63.	**19.**	27 and 42.	**20.**	24 and 36.
21.	52 and 68.	**22.**	65 and 85.	**23.**	54 and 72.	**24.**	84 and 98.

78. We will work another example with somewhat larger numbers.

EXAMPLE. *Find the* H.C.F. *of* 196, 350 *and* 924.

$196 = 4 \times 49 = 2^2 \times 7^2,$

$350 = 2 \times 175 = 2 \times 7 \times 25 = 2 \times 5^2 \times 7,$

and $924 = 4 \times 231 = 2^2 \times 11 \times 21 = 2^2 \times 3 \times 7 \times 11.$

The highest power of 2 which will divide each of these numbers is 2 itself.

The highest power of 7 which will do so is 7 itself.

Of the other factors 3, 5, 11 there are none common to all. Hence

H.C.F. $= 2 \times 7 = \underline{14}.$

EXERCISE 102.

Verify each result.

Find by factorisation the H.C.F. of

1. 132 and 364.
2. 112 and 176.
3. 324 and 720.
4. 315 and 330.
5. 675 and 891.
6. 308 and 392.
7. 528 and 608.
8. 693 and 819.
9. 792 and 924.
10. 42, 112 and 140.
11. 72, 108 and 180.
12. 121, 154 and 165.
13. 144, 162 and 198.
14. 264, 360 and 600.
15. 102, 136 and 170.

H (5). Least Common Multiple.

79. A number which can be divided without remainder by two others, or more, is a **common multiple** of them.

Thus 12 is a common multiple of 2, 3, 4 and 6.

80. The *least* number which can be so divided is called their **Least Common Multiple,** and is denoted by the letters **L.C.M.**

81. The L.C.M. may be determined very readily when the numbers can be separated into prime factors.

EXAMPLE. *Find the* L.C.M. *of* 9, 10, 15, 16, 20.

$9 = 3 \times 3$,
$10 = 2 \times 5$,
$15 = 3 \times 5$,
$16 = 2 \times 2 \times 2 \times 2$,
and $20 = 2 \times 2 \times 5$.

Hence the L.C.M. must contain the prime factors 2, 3 and 5. Now, 2 occurs *four* times in one number, 3 occurs *twice*, and 5 occurs only *once* in any of them ;

$\therefore$ L.C.M. $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$
$= 2^4 \times 3^2 \times 5$
$= \underline{720.}$

NOTE. When the prime factors have been found, it will be seen that the L.C.M. must have for its factors all the different factors which occur in any of the numbers, and the *index* of each factor must be that of the *highest* power to which it is raised in any of them.

We will now consider another form in which the work can be arranged. Having written the numbers in a row, we see that 10 is a factor of 20 ; hence all numbers divisible by 20 must also be divisible by 10. We may consequently cross out 10 thus :

9, ~~10~~, 15, 16, 20

and we state the work as follows :

2	9, ~~10~~, 15, 16, 20
2	9, 15, 8, 10
3	9, 15, 4, ~~5~~
	3, 5, 4.

Hence the L.C.M. $= 2 \times 2 \times 3 \times 3 \times 5 \times 4$
$= 2^4 \times 3^2 \times 5$
$= \underline{720.}$

Explanation.—The divisors 2, 2 and 3 are prime factors which are common to two or more of the numbers. In any case we divide by such *prime* factors as will divide two or more of the numbers; we begin with the smallest prime factor, and we take them in order. We write down the quotients below such numbers as can be divided, and bring down unaltered the numbers which cannot be divided. In the third line we strike out 5, because it is a factor of 15, and may therefore be neglected; in the fourth line there are three numbers, no two of which have a common factor.

EXERCISE 103.

Find the L.C.M. of

1. 2 and 4.	**2.** 4 and 8.	**3.** 2 and 6.
4. 2, 3 and 6.	**5.** 2, 3 and 4.	**6.** 3, 5 and 6.
7. 3, 4 and 5.	**8.** 3, 5 and 7.	**9.** 2, 3 and 5.
10. 2, 5 and 7.	**11.** 2, 6 and 5.	**12.** 2, 6 and 8.
13. 3, 4 and 10.	**14.** 5, 8 and 10.	**15.** 5, 12 and 15.
16. 6, 12 and 24.	**17.** 7, 10 and 14.	**18.** 6, 10 and 18.

EXERCISE 104.

Verify each result.

Find the L.C.M. of

1. 8, 12 and 20.	**2.** 18, 20 and 30.
3. 12, 15 and 16.	**4.** 36, 60 and 72.
5. 14, 20, 28 and 30.	**6.** 5, 12, 36 and 45.
7. 12, 28, 35 and 40.	**8.** 10, 15, 24 and 32.
9. 12, 15, 18 and 27.	**10.** 12, 20, 36 and 54.
11. 18, 24, 32 and 48.	**12.** 15, 21, 28 and 35.
13. 12, 36, 72 and 120.	**14.** 12, 18, 96 and 144.
15. 8, 12, 20, 30 and 80.	**16.** 4, 6, 16, 20 and 24.
17. 8, 10, 12, 15, 18 and 20.	**18.** 9, 12, 16, 18, 24 and 27.

EXERCISE 105.

Verify each result.

1. I have two pieces of string: one measures 8 inches and the other 14 inches; what is the length of the longest piece which I can cut from each an exact number of times?

2. Three lengths of string measure 14 inches, 21 inches, and 35 inches; what is the length of the longest tape which will measure each of them an exact number of times?

3. Tom has 16 marbles in one pocket and 24 in another; what is the greatest number he can take an exact number of times from each pocket?

4. A box contains 63 oranges and 84 lemons, and I wish to arrange them in bags, putting the same number in every bag; what is the greatest number I can put in a bag, oranges and lemons being kept separate?

5. Tom has 30 walnuts, 18 chestnuts and 12 filberts, and he wants to arrange them in heaps putting the same number in each heap without mixing the three kinds; what is the greatest number of nuts he can have in each heap?

6. What is the least number of inches in a piece of string if it can be cut exactly into lengths of 2, 4, 6 or 8 inches?

7. What is the smallest number of trees that can be planted in rows of 5, 6, 8 or 12?

8. How long is the shortest piece of string which can be cut exactly into lengths of 12 inches or 20 inches?

9. What is the smallest number of counters which can be arranged exactly in 14, 21, 28 or 35 rows?

10. The books in a case can be arranged in rows of 15, 16, 18, 20 or 24; what is the least possible number of books in the case?

J. SUMS OF MONEY LESS THAN £1.

It is assumed that pupils have committed the **Pence Table** *to memory.*

J (1). Introduction.

82. The value of an article is estimated in British money as a certain number of pounds, shillings, and pence.

A sum of money, such as 1 pound 12 shillings and 6 pence is written £1. 12*s*. 6*d*.

The symbol £ stands for the Latin word *libra*, the letter *s*. for *solidus*, and the letter *d*. for *denarius*; these words were the names of certain Roman coins or sums of money.

NOTE. The coin which represents 2*s*. is called a *florin*, that which represents 5*s*. is called a *crown*, and that which represents 2*s*. 6*d*. is called a *half-crown*.

83. A **compound** quantity is a quantity in the expression of which there are terms of more than one denomination. For example, £1. 12*s*. 6*d*. is a compound quantity with terms of three different *denominations*.

J (2). Addition.

84. When we add together several sums of money, we arrange them as in the following example.

EXAMPLE. *Find the total of* 6*s*. 10*d*., 2*s*. 6*d*., 8*d*., 5*s*. 9*d*.

We begin at the right hand and add upwards.
The mental steps are
1st. Adding pence: 9*d*., 1*s*. 5*d*., 1*s*. 11*d*., 2*s*. 9*d*.; write down **9d.** and carry forward 2*s*.
2nd. Adding shillings: 7*s*., 9*s*., 15*s*.; write down **15s.**
The result is **15s. 9d.**

s.	*d*.
6 .	10
2 .	6
	8
5 .	9
15 .	**9**

NOTE. The pence column may be checked thus: 17*d*., 23*d*., 33*d*., *i.e.* 2*s*. 9*d*.

Verification. Add downwards.

EXERCISE 106.

Check each result.

Find the total of the following payments:

1. Apples 5*d*., nuts 4*d*., oranges 8*d*., cherries 10*d*.
2. Buns 9*d*., chocolates 6*d*., cake 1*s*., lemonade 3*d*.
3. Potatoes 1*s*. 4*d*., cabbages 3*d*., apples 8*d*., lemons 3*d*.
4. Notepaper 10*d*., envelopes 9*d*., postcards 8*d*., stamps 7*d*.

Find the total of the following payments:

5. Milk 9*d.*, eggs 10*d.*, cream 5*d.*, butter 1*s.* 2*d.*
6. Tea 3*s.* 9*d.*, coffee 1*s.* 8*d.*, sugar 11*d.*, cocoa 1*s.* 6*d.*
7. Foolscap paper 4*s.* 6*d.*, ink 7*d.*, pens 1*s.* 3*d.*, pencils 2*s.* 1*d.*
8. Books 4*s.* 9*d.*, chalk 11*d.*, pencils 2*s.* 7*d.*, paper 3*s.* 4*d.*
9. Gloves 4*s.* 10*d.*, buttons 9*d.*, braid 1*s.* 5*d.*, silk 7*s.* 9*d.*
10. Bacon 5*s.* 1*d.*, ham 3*s.* 4*d.*, butter 2*s.* 11*d.*, cheese 1*s.* 11*d.*

J (3). Subtraction.

85. To subtract one sum of money from another we proceed as in the following examples.

EXAMPLE 1. *Out of* 15*s.* 10*d.* *in my purse I spend* 4*s.* 6*d.*; *how much have I left?*

The mental steps are
1st. 6*d.* and **4d.** make 10*d.*; write down **4d.**
2nd. 4*s.* and **11s.** make 15*s.*; write down **11s.**
The result is **11s. 4d.**

s.	*d.*
15	10
4	6
11	**4**

EXAMPLE 2. *I give* 5*s.* *to a shopkeeper to settle a bill for* 3*s.* 9*d.*; *how much change does he give me?*

The mental steps are
1st. 9*d.* and **3d.** make 1*s.*; write down **3d.** and carry forward 1*s.*
2nd. 1*s.* and 3*s.* make 4*s.*, and **1s.** makes 5*s.*; write down **1s.**
The result is **1s. 3d.**

s.	*d.*
5	0
3	9
1	**3**

EXAMPLE 3. *I have* 7*s.* 9*d.* *left out of* 10*s.* 8*d.*; *how much do I spend?*

The mental steps are
1st. 9*d.* and **11d.** make 1*s.* 8*d.*; write down **11d.** and carry forward 1*s.*
2nd. 1*s.* and 7*s.* make 8*s.*, and **2s.** make 10*s.*; write down **2s.**
The result is **2s. 11d.**

s.	*d.*
10	8
7	9
2	**11**

Verification. Add the last two lines.

EXERCISE 107.

Verify each result.

1. Tom has 7*d.* in his money-box; how much must he add to make up a florin?

2. Fred has 1*s.* 6*d.* in his purse; how much must he add to make up 5*s.*?

3. Bob has 4*s.* 9*d.* in his purse; how much is this amount less than 8*s.*?

4. George had 12*s.* in his pocket and spent 5*s.* 8*d.*; how much remains?

5. I give 10*s.* to a shopkeeper to settle a bill for 8*s.* 4*d.*; how much change does he give me?

6. A boy writes down 9*s.* 1*d.* for 1*s.* 9*d.*; by what amount is this wrong?

7. I have 6*s.* 9*d.* left out of 9*s.* 6*d.*; how much did I spend?

8. Two purses together contain 11*s.* 3*d.*, and one of them contains 3*s.* 11*d.*; how much does the other purse contain?

9. I have 15*s.* 2*d.*; how much can I spend on a cricket bat to leave 4*s.* 8*d.*?

10. A boy has 18*s.* 4*d.* and his sister has 14*s.* 8*d.*; how much more has he than his sister?

86. The method of procedure in the following example should be noticed.

EXAMPLE. *Find in one operation how much is left out of* 18*s.* 7*d.* *after spending* 6*s.* 4*d.* *and* 8*s.* 6*d.*

The mental steps are

1st. 6*d.* and 4*d.* make 10*d.*, and **9d.** make 1*s.* 7*d.*; write down **9d.** and carry forward 1*s.*

2nd. 1*s.*, 8*s.* and 6*s.* make 15*s.*, and **3s.** make 18*s.*; write down **3s.**

The result is **3s. 9d.**

s.	*d.*
18	7
6	4
8	6
3	**9**

Verification. Add the last three lines.

EXERCISE 108.

Verify each result.

1. Harry has a florin in his purse; how much will be left after paying 5*d.* for postcards and 9*d.* for stamps?

2. Tom has half-a-crown in his pocket; how much will be left after spending 7*d.* on chocolates and 1*s.* 3*d.* on cakes?

3. George had 8*s.* in his money-box. He has taken out at various times 1*s.* 7*d.*, 1*s.* 10*d.* and 1*s.* 9*d.*; how much money remains in the box?

4. What change shall I receive out of 10*s.*, after paying 10*d.* for newspapers, 1*s.* 6*d.* for magazines and 3*s.* 4*d.* for a book?

5. A woman goes to pay bills with 17*s.* 2*d.* She pays 2*s.* 7*d.* at one shop, 5*s.* 6*d.* at a second and 7*s.* 8*d.* at a third; how much has she remaining?

6. Bob wants 18*s.* 6*d.* with which to buy a cricket bat. His father gives him 7*s.* 6*d.*, his uncle 4*s.* 9*d.* and he has saved 2*s.* 10*d.*; how much more does he want?

7. Three persons together owe me 18*s.* 3*d.* The first owes 7*s.* 10*d.* and the second 8*s.* 10*d.*; what does the third owe?

8. Jones had 8*s.* 4*d.* in his purse. He paid away 4*s.* 10*d.* to Smith, but afterwards received 5*s.* 3*d.* from Brown. How much has he now?

9. Tom has 13*s.* 2*d.* and Bob has 12*s.* 9*d.* If Tom gives Bob 3*s.* 6*d.*, how much will Bob have more than Tom?

10. John has 7*s.* 6*d.* and George has 10*d.* less than John has; how much have they together?

J (4). Multiplication.

87. In multiplying a sum of money we proceed as in the following example.

EXAMPLE. *How much must be paid for* 5 *railway tickets at* 3*s.* 4*d. each?*

The mental steps are

1st. 5 times 4*d.* = 20*d.* = 1*s.* 8*d.*; write down **8d.** and carry forward 1*s.*

2nd. 5 times 3*s.* = 15*s.*; which becomes **16s.** when we add the 1*s.* brought forward.

The result is **16s. 8d.**

s.	*d.*
3	4
	5
16	8

Verification. 3*s.* 4*d.* + 3*s.* 4*d.* + 3*s.* 4*d.* + 3*s.* 4*d.* + 3*s.* 4*d.* = 16*s.* 8*d.*

EXERCISE 109.

Verify each result.

1. How much must be paid for 5 books at 1*s.* 10*d.* each?

2. What is the cost of 6 toys at 2*s.* 7*d.* each?

3. What is the value of 4 chickens at 3*s.* 8*d.* each?

4. What must be paid for 3 railway tickets at 4*s.* 9*d.* each?

5. If I have 7*s.* 11*d.* in my purse and you have twice as much in yours, how much have you?

6. What will be the change from 12*s.* after paying for 6 pots of jam at 1*s.* 9*d.* each?

7. What is the total cost of 5 cakes at 2*s.* 9*d.* each and 4 boxes of chocolate at 1*s.* 4*d.* each?

8. What is the difference between the total cost of 3 penknives at 5*s.* 6*d.* each and the total cost of 4 penknives at 3*s.* 11*d.* each?

9. How much change will there be from 19*s.* 6*d.* after paying for 2 books at 5*s.* 8*d.* each and 3 books at 2*s.* 5*d.* each?

10. I buy 5 hares at 1*s.* 3*d.* each and 3 chickens each double the price of a hare; how much change do I receive out of 15*s.*?

11. If 2 pairs of gloves cost 3*s.* 11*d.*, what will half-a-dozen pairs cost?

12. If 3 railway tickets to a certain place cost 8*s.* 9*d.*, what will 6 tickets for the same journey cost?

J (5). Division.

88. The process of separating a sum of money into a given number of equal parts, and finding the value of each part will be seen from the following example.

EXAMPLE. *Divide* 16*s.* 8*d.* *into* 5 *equal shares.*

We begin at the left hand as in simple division.

1st. If we divide 16 by 5, the quotient is 3 and the remainder is 1; hence, if we divide 16*s.* by 5, the quotient is **3s.** and the remainder is 1*s.*; or, arranging 16 shillings in 5 equal groups, we have 3*s.* in each, and 1 shilling left.

	s.	*d.*
5	16	8
	3	**4**

2nd. We change the 1*s.* into 12 pence and add to it the 8*d.* in the dividend: we thus get 20*d.* altogether. Dividing 20*d.* by 5 the quotient is **4d.**; or, arranging 20 pennies in 5 equal groups, we have 4 pennies in each group.

The result is **3s. 4d.**

Verification.

	s.	*d.*
5 groups, containing 3*s.* each, $= 3s. \times 5 =$	15	0
5 groups, containing 4*d.* each, $= 4d. \times 5 =$	1	8
$\therefore 3s.\ 4d. \times 5 =$	**16**	**8**

EXERCISE 110.

Verify each result.

1. If I divide 3*s.* 8*d.* equally between two boys, how much will each receive?

2. If 3 boys receive 7*s.* 9*d.* in equal shares, how much is each share?

3. If 4 railway tickets for the same journey cost 15*s.* 8*d.*, what is the price of each ticket?

4. If five copies of a book cost 18*s.* 9*d.*, what is the value of one copy?

5. Each of 6 boys has the same sum of money, and the total amount is 11*s.*; how much has each boy?

6. What is the difference in the price of eggs per dozen, if 5 dozen new-laid eggs can be bought for 4*s.* 7*d.* and 6 dozen cooking eggs for 4*s.* 6*d.*?

7. What is the total cost of half-a-yard of silk at 7*s.* 6*d.* per yard and a quarter-yard of lace at 10*s.* 8*d.* per yard?

8. In each of 5 boxes there is the same sum of money, and the total amount is 17*s.* 11*d.* In each of 6 purses there is a certain sum of money and the total amount is 11*s.* 6*d.* How much more is there in each box than in each purse?

9. Three boys spend respectively 2*s.* 2*d.*, 6*s.* 4*d.* and 4*s.* 9*d.*; what is the average per boy?

10. I buy four books for 4*s.* 6*d.*, 3*s.* 8*d.*, 4*s.* 3*d.* and 2*s.* 11*d.*; what is the average cost of each?

11. I pay four accounts as follows: baker 3*s.* 9*d.*, grocer 4*s.* 6*d.*, milkman 2*s.* 1*d.*, newsagent 1*s.* 4*d.*; what is the average amount paid?

12. If 6 copies of a magazine cost 5*s.*, what ought I to pay for 5 copies?

13. Railway fare for 5 boys costs 4*s.* 2*d.*; how much would it be for 3 boys?

14. If I buy 3 copies of a book for 4*s.* 9*d.*, how much shall I have to pay for 5 copies?

15. If 5 railway tickets to the same place cost 11*s.* 8*d.*, what will be the cost of 6 tickets for the journey?

K. SUMS OF MONEY NOT GREATER THAN £10.

K (1). Reduction.

89. When we change a quantity from one denomination to another, we are said to **reduce** it, and the process is called **Reduction.**

Thus 20 shillings become 1 pound, when *reduced* to pounds, and become 240 pence, when *reduced* to pence.

> MEMORIZE:
> £1 = 240 pence
> = 480 halfpence
> = 960 farthings.

90. To reduce a sum of money to a lower denomination.

EXAMPLE. *Reduce* 19*s.* $6\frac{1}{2}$*d.* *to farthings.*

We multiply the number of *shillings* by 12, and add to the product the number of odd *pence*; we then multiply the number of pence by 4, and add the number of odd *farthings.*

s.	*d.*
19	. $6\frac{1}{2}$
12	
234	= number of *pence.*
4	
938	= number of *farthings.*

Shortened Form. 19*s.* $6\frac{1}{2}$*d.* = $234\frac{1}{2}$ *pence* = **938** *farthings.*

Verification. 19*s.* = 19 × 48 = 912 *farthings,*
and $6\frac{1}{2}$*d.* = 26 *farthings;*
∴ 19*s.* $6\frac{1}{2}$*d.* = **938** *farthings.*

NOTE. The symbol for 1 farthing = $\frac{1}{4}$*d.*,
..... 3 farthings = $\frac{3}{4}$*d.*,
and ..... 2 farthings = $\frac{1}{2}$*d.*

EXERCISE 111.

Verify each result.

Reduce to shillings:

1. £1. 12*s.*	2. £1. 16*s.*	3. £2. 2*s.*	4. £3. 5*s.*
5. £2. 17*s.*	6. £7. 13*s.*	7. £9. 10*s.*	8. £5. 18*s.*
9. £4. 14*s.*	10. £8. 15*s.*	11. £6. 19*s.*	12. £9. 19*s.*

Reduce to pence:

13. £1. 1*s.* 2*d.* 14. £1. 9*s.* 10*d.* 15. £1. 17*s.* 5*d.*
16. £2. 8*s.* 7*d.* 17. £3. 5*s.* 4*d.* 18. £3. 13*s.* 3*d.*
19. £3. 10*s.* 11*d.* 20. £3. 19*s.* 1*d.* 21. £2. 3*s.* 6*d.*
22. £2. 9*s.* 8*d.* 23. £2. 19*s.* 5*d.* 24. £4. 3*s.* 3*d.*

Reduce to halfpence:

25. 16*s.* 26. 18*s.* 27. 12*s.* 28. 21*s.*
29. £1. 12*s.* 30. £1. 16*s.* 31. £1. 9*s.* 6*d.* 32. £2. 1*s.* 3*d.*

Reduce to farthings:

33. 11*s.* 6*d.* 34. 13*s.* $4\frac{1}{4}d.$ 35. 15*s.* $7\frac{3}{4}d.$ 36. 17*s.* $10\frac{1}{2}d.$

91. To reduce a sum of money to a higher denomination.

EXAMPLE. *Reduce* 938 *farthings to shillings, etc.*

[NOTE. This is also a Verification of the Example in Art. 90.]

Since a penny contains 4 farthings, changing farthings into pence consists in arranging the farthings in groups of 4 each, in other words, in dividing by 4. Thus 938 farthings contain 234 pence and **2** *farthings.*

Similarly, to change pence into shillings we divide by 12. Thus 234 pence contains **19** *shillings* and **6** *pence.*

4 | 938
12 | 234 *pence*, and **2** *farthings* over,
19 *shillings*, and 6 *pence* over;

∴ 938 farthings = **19s. $6\frac{1}{2}$d.**

Verification. 19*s.* = 912 *farthings*,
and $6\frac{1}{2}d.$ = 26 *farthings*;
∴ 19*s.* $6\frac{1}{2}d.$ = **938** *farthings.*

EXERCISE 112.

Verify each result.

Express in shillings and pence:

1. 126 pence. 2. 148 pence. 3. 194 pence. 4. 236 pence.

Express in pounds and shillings:

5. 126 shillings. 6. 173 shillings. 7. 148 shillings.

Express in £. *s.* *d.*:

8. 999 pence. 9. 379 pence. 10. 798 pence. 11. 673 pence.

Express in shillings and pence:

12. 823 farthings. 13. 792 farthings. 14. 469 farthings.
15. 397 halfpence. 16. 269 halfpence. 17. 455 halfpence.

EXERCISE 113.

1. How many sixpences can you get for £5. 15*s.*?
2. How many threepences are worth £8. 13*s.*?
3. How many half-crowns should be given in exchange for £5?
4. How many people can receive half-a-crown each from a purse containing £6. 17*s.* 6*d.*?
5. How many books each worth half-a-crown can be bought with £8. 12*s.* 6*d.*?
6. How many boys can receive threepence each from a bag containing £1. 15*s.* 9*d.*?
7. What is the cost of 313 magazines at sixpence each?
8. What is the cost of 109 bottles of lemonade at threepence each?
9. What is the cost of 79 cakes at half-a-crown each?
10. What is the cost of 8 dozen boxes of chocolate at a florin each box?

K (2). Addition and Subtraction.

92. No new principle is involved in the examples which follow.

EXAMPLE 1. *Find the sum of* 6*s.* $10\frac{1}{2}d.$, £2. 12*s.* $6\frac{1}{4}d.$, £4. 15*s.* $4\frac{3}{4}d.$

The mental steps are

1st. Adding the farthings we have 3, 4, 6 farthings, that is $1\frac{1}{2}d.$; write down **$\frac{1}{2}$d.**, and carry forward 1*d.*

2nd. Adding the pence we have 5*d.*, 11*d.*, 1*s.* 9*d.*; write down **9d.**, and carry forward 1*s.*

3rd. Adding the shillings we have 16, 28, 34 shillings, that is £1. 14*s.*; write down **14s.**, and carry forward £1.

4th. Adding the pounds we have **£7.**

The result is **£7. 14s. 9$\frac{1}{2}$d.**

£	*s.*	*d.*
	6 .	$10\frac{1}{2}$
2 .	12 .	$6\frac{1}{4}$
4 .	15 .	$4\frac{3}{4}$
7 .	**14 .**	**$9\frac{1}{2}$**

Verification. Add downwards.

EXERCISE 114.

Check each result.

Find the sum of

	s.	d.
1.	10	6
	17	$4\frac{1}{2}$
	2	$9\frac{3}{4}$
	6	$8\frac{1}{2}$
	13	6
	19	3

	s.	d.
2.	17	$6\frac{1}{4}$
	18	9
	12	$3\frac{1}{2}$
	16	$10\frac{3}{4}$
	7	$8\frac{1}{2}$
	10	4

	s.	d.
3.	13	$6\frac{1}{2}$
	12	$3\frac{3}{4}$
	6	$2\frac{1}{2}$
	15	$10\frac{1}{2}$
	6	8
	13	$7\frac{1}{2}$

	s.	d.
4.	16	2
	13	$6\frac{1}{2}$
	7	$8\frac{3}{4}$
	19	$11\frac{1}{4}$
	6	$8\frac{1}{2}$
	7	$3\frac{1}{4}$

	s.	d.
5.		$10\frac{3}{4}$
	9	$4\frac{1}{2}$
	16	7
	18	$0\frac{1}{4}$
	1	$9\frac{1}{2}$
	11	10

	£	s.	d.
6.		11	$9\frac{1}{4}$
	5	17	$0\frac{3}{4}$
			$10\frac{1}{2}$
	2	13	$11\frac{1}{4}$

	£	s.	d.
7.	2	7	$0\frac{1}{2}$
	2	12	$11\frac{1}{4}$
	1	19	$8\frac{3}{4}$
	2	10	6

	£	s.	d.
8.	1	11	$11\frac{1}{2}$
		19	$11\frac{3}{4}$
			$11\frac{3}{4}$
	6	17	$9\frac{1}{4}$

	£	s.	d.
9.		19	$9\frac{1}{2}$
	1	19	$9\frac{1}{4}$
	1	19	$9\frac{3}{4}$
		19	9

	£	s.	d.
10.	4	15	7
	2	14	$8\frac{3}{4}$
		10	$11\frac{3}{4}$
			$7\frac{1}{4}$
		7	$2\frac{1}{2}$

	£	s.	d.
11.			$6\frac{1}{2}$
	6	16	$0\frac{1}{2}$
		16	$6\frac{1}{4}$
	1	14	$7\frac{3}{4}$
		1	$8\frac{1}{2}$

	£	s.	d.
12.		11	$7\frac{1}{4}$
	2	18	$6\frac{3}{4}$
	1	19	$10\frac{1}{2}$
	2	12	$8\frac{3}{4}$
		17	$2\frac{3}{4}$

	£	s.	d.
13.	2	15	7
		11	$6\frac{1}{2}$
			$9\frac{1}{4}$
		12	$10\frac{1}{2}$
	3	18	11

EXERCISE 115.

Check each result.

Write down the answers, adding (1) in vertical columns (1-8), (2) in horizontal rows (9-26):

	s.	d.
1.	1	4
	3	8
	2	7
		11
	2	4
	4	10
		9
	8	6
	.....	

	s.	d.
2.		8
	1	6
	4	10
	2	4
	9	7
	3	5
	3	9
		10
	.....	

	s.	d.
3.	2	5
		6
	1	9
	4	6
		11
	5	10
	4	6
	2	8
	.....	

	s.	d.
4.	4	6
	2	9
		9
	1	10
	1	10
		9
	7	11
	5	7
	.....	

.....	9.
.....	10.
.....	11.
.....	12.
.....	13.
.....	14.
.....	15.
.....	16.
.....	17.

	s.	d.		s.	d.		s.	d.		s.	d.		
5.	4	6	**6.**		9	**7.**	7	8	**8.**	1	9	.....	18.
	1	8		3	7			8		6	10	.....	19.
	5	9		1	10		3	9		4	11	.....	20.
		10		6	11		1	11		3	11	.....	21.
	1	8			10		3	11		8	8	.....	22.
	5	10		1	6		2	9		4	3	.....	23.
	3	5		6	11		1	7		2	10	.....	24.
	1	11		2	6		7	9		1	11	.....	25.
	.....			.....			.....			.....		.....	26.

IMPORTANT. Answers **17** and **26** must be the same both vertically and horizontally.

EXAMPLE 2. *What have I spent out of* £9. 10*s.* 8*d.*, *if I have* £6. 17*s.* 9*d. left?*

The mental steps are

1st. 9*d.* and **11d.** make 1*s.* 8*d.*; write down **11d.**, and carry forward 1*s.*

2nd. 1*s.* and 17*s.* make 18*s.*, and **12s.** make £1. 10*s.*; write down **12s.**, and carry forward £1.

3rd. £1 and £6 make £7, and **£2** make £9; write down **£2.**

The result is **£2. 12s. 11d.**

£	s.	d.
9	10	8
6	17	9
2	**12**	**11**

Verification. Add the last two lines.

EXERCISE 116.

Verify each result.

In each of the following sums the upper line is the money I had in my purse, and the lower line is the money I spent; what money remains unspent?

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
1.	1	18	9	**2.**	2	10	6	**3.**	3	5	7	**4.**	2	16	0
		19	10		1	9	10		1	14	8		1	16	$6\frac{1}{2}$
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
5.	4	0	10	**6.**	5	12	1	**7.**	5	19	5	**8.**	6	13	$8\frac{1}{2}$
	3	4	$10\frac{1}{4}$		3	17	$3\frac{3}{4}$		3	19	$7\frac{1}{2}$		1	18	9
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
9.	7	17	11	**10.**	8	15	$0\frac{1}{2}$	**11.**	9	9	9	**12.**	5	0	0
		17	$11\frac{1}{4}$		7	17	$5\frac{1}{4}$			19	$9\frac{1}{2}$		1	11	$1\frac{1}{2}$

EXAMPLE 3. *In one operation take the sum of* £5. 16*s.* 4*d.* *and* £2. 8*s.* 5*d.* *from* £9. 16*s.* 7*d.*

The mental steps are

1st. 5*d.* and 4*d.* make 9*d.*, and **10d.** make 1*s.* 7*d.*; write down **10d.**, and carry forward 1*s.*

2nd. 1*s.*, 8*s.* and 16*s.* make 25*s.*, and **11s.** make £1. 16*s.*; write down **11s.**, and carry forward £1.

3rd. £1, £2 and £5 make £8, and **£1** make £9; write down **£1.**

£	*s.*	*d.*
9	16	7
5	16	4
2	8	5
1	**11**	**10**

The result is **£1. 11s. 10d.**

Verification. Add the last three lines.

EXERCISE 117.

Verify each result.

1. What change shall I have out of £2 after paying 8*s.* 3*d.* to a baker, 7*s.* 9*d.* to a grocer and 6*s.* 8*d.* to a butcher?

2. I had £4 in my purse, and I spent 17*s.* 11*d.*, 13*s.* 1*d.* and 14*s.* 5*d.*; how much money have I remaining?

3. A woman had £5. 5*s.* in her purse. She first spent £1. 11*s.* 6*d.*, and afterwards she spent £1. 19*s.*; how much money has she left?

4. What change shall I have out of £9, after paying £5. 15*s.* for rates, 16*s.* 2*d.* for water and 17*s.* 9*d.* for gas?

5. What change shall I receive out of £3, after paying away £1. 10*s.* 9*d.*, 8*s.* 4*d.* and 19*s.* 10*d.*?

6. I had £5. 10*s.* in my pocket; how much was left after paying away £1. 11*s.* 6*d.*, £2. 18*s.* 3*d.* and 5*s.* 8*d.*?

7. Three boys have £2. 12*s.* 1*d.* among them. The first has 17*s.* 5*d.* and the second has 18*s.* 9*d.*; how much has the third?

8. Three persons spend £3. 15*s.* 4*d.* among them. The first spends £1. 7*s.* 8*d.* and the second 17*s.* 11*d.*; how much does the third spend?

9. Tom has £2. 15*s.* 4*d.*; if he had 17*s.* 10*d.* less he would have 18*s.* 11*d.* more than Bob; how much has Bob?

10. John has £1. 12*s.* 6*d.*; he gives 16*s.* 8*d.* to George; George gives 4*s.* 11*d.* to Dick; how much now has John more than George?

EXERCISE 118.

1. I have in my purse 3 sixpences, 3 florins, 3 half-crowns, 3 half-sovereigns and 3 sovereigns; what is the total value of the contents of my purse?

2. I have in my pocket-book 27 sixpenny stamps, 14 threepenny stamps, 29 penny stamps and 32 halfpenny stamps; what is their total value?

3. I buy 18 half-crown books, 33 shilling books, 31 sixpenny books and 37 threepenny books; what is the total cost?

4. I use in one day 500 penny stamps, 250 halfpenny stamps, 100 three-halfpenny stamps and 80 twopenny stamps; what is their total value?

5. I pay A 13*s.* 1*d.*, B 1*s.* 4*d.* more than A, and C 3*s.* 6*d.* more than B; what is the total of the three payments?

6. I took £6. 17*s.* 10½*d.* with me to pay a bill, but found I had too little by 7*s.* 8*d.*; what is the amount of the bill?

7. I owed £6. 13*s.* 3*d.* and received £3. 16*s.* 9*d.* out of a £10 bank-note; was this the correct change, and if not, was it too much or too little, and by how much?

8. I have £1. 16*s.* 8*d.*, my brother has 8*s.* 9*d.* more, and my sister has 18*s.* 9*d.* less; how much have we all three together?

9. A purse and its contents are worth £7. 5*s.* 2*d.* and the purse is worth 15*s.* 6*d.*; what is the amount of money in it?

10. A has 10*s.* 9*d.* more than B. If A has £2. 8*s.* 6*d.*, how much have A and B together?

11. C has £2. 15*s.* 4*d.* and D has 16*s.* 6*d.* less than C; how much have C and D together?

12. How much is the amount of a grocer's bill for £3. 7*s.* 3*d.* greater than the difference between a butcher's bill for £3. 17*s.* 9*d.* and a draper's bill for £4. 17*s.* 3*d.*?

K (3). Multiplication.

EXAMPLE. *Find* 3 *times* £1. 14*s.* 6¾*d.*

The mental steps are

1st. 3 times 3 farthings = 9 farthings = 2¼*d.*; write down **¼d.**, and carry forward 2*d.*

2nd. 3 times 6 pence = 18*d.*, which becomes 20*d.* when we add the 2*d.* brought forward; 20*d.* = 1*s.* 8*d.*; write down **8d.**, and carry forward 1*s.*

3rd. 3 times 14*s.* = 42*s.*, which becomes 43*s.* when we add the 1*s.* brought forward; 43*s.* = £2. 3*s.*; write down **3s.**, and carry forward £2.

4th. 3 times £1 = £3, which becomes £5 when we add the £2 brought forward; write down **£5.** The result is **£5. 3s. 8¼d.**

£.	*s.*	*d.*
1	14	6¾
		3
5	3	8¼

Verification.

	£.	*s.*	*d.*
£1 × 3 =	3	0	0
14*s.* × 3 =	2	2	0
6*d.* × 3 =		1	6
$\frac{3}{4}d.$ × 3 =			$2\frac{1}{4}$
∴ £1. 14*s.* $6\frac{3}{4}d.$ × 3 =	5	3	$8\frac{1}{4}$

EXERCISE 119.

Verify each result.

Find the value of

		(*a*)	(*b*)
1.	10*s.* 9*d.* × 2 ;	3.	
2.	14*s.* $6\frac{1}{4}d.$ × 3 ;	4.	
3.	13*s.* $4\frac{3}{4}d.$ × 4 ;	5.	
4.	12*s.* $10\frac{1}{2}d.$ × 5 ;	6.	
5.	18*s.* $5\frac{3}{4}d.$ × 6 ;	7.	
6.	15*s.* $4\frac{1}{2}d.$ × 7 ;	8.	
7.	16*s.* $9\frac{1}{4}d.$ × 8 ;	9.	

		(*a*)	(*b*)
8.	£3. 5*s.* 6*d.* × 2 ;	3.	
9.	£2. 4*s.* $9\frac{1}{4}d.$ × 3 ;	4.	
10.	£1. 18*s.* $6\frac{1}{2}d.$ × 4 ;	5.	
11.	£1. 9*s.* $8\frac{3}{4}d.$ × 5 ;	6.	
12.	£1. 5*s.* $10\frac{1}{2}d.$ × 6 ;	7.	
13.	£1. 0*s.* $11\frac{3}{4}d.$ × 7 ;	8.	
14.	£1. 1*s.* $10\frac{1}{4}d.$ × 8 ;	9.	

93. If the multiplier be a number which can easily be separated into factors each not greater than 12, we multiply by these factors in succession.

EXAMPLE. *Find the cost of* 16 *articles at* 3*s.* 7*d. each.*

16 = 4 × 4 or 8 × 2, and we may use either of these pairs.

£.	*s.*	*d.*	
	3	7	
		4	
	14	4	= cost of 4 articles
		4	
2	17	4	= cost of 16 articles

£.	*s.*	*d.*	
	3	7	
		8	
1	8	8	= cost of 8 articles
		2	
2	17	4	= cost of 16 articles

Verification. In both cases the results are the same.

EXERCISE 120.

Verify each result.

1. What will a workman earn in 49 hours at $10\frac{1}{2}d.$ an hour?

2. What is the cost of 24 copies of a book at 4*s.* $1\frac{1}{2}d.$ per copy?

3. What is the value of 72 pairs of gloves at 1*s.* $11\frac{3}{4}d.$ per pair?

4. What is the cost of 20 pairs of slippers at 3*s.* 11*d.* per pair?

5. What must be paid for 18 books at 6*s.* 5*d.* each?

6. What is the value of 16 cricket bats at 10*s.* 9*d.* each?

7. What will a workman earn in 32 days at 4*s.* 9*d.* per day?

8. What must be paid for 27 cricket balls at 4*s.* 8*d.* each?

9. What is the cost of 48 boxes of chocolates at 2*s.* 2*d.* per box?

10. What is the value of 42 school caps at 2*s.* $7\frac{1}{2}d.$ each?

EXERCISE 121.

Verify each result.

Find the value of

1. 1*s.* 11*d.* × 14.	2. 1*s.* $11\frac{3}{4}d.$ × 60.	3. 1*s.* $8\frac{1}{2}d.$ × 63.
4. 2*s.* 10*d.* × 28.	5. 2*s.* $9\frac{1}{4}d.$ × 54.	6. 2*s.* $6\frac{3}{4}d.$ × 56.
7. 3*s.* 9*d.* × 42.	8. 3*s.* $11\frac{1}{2}d.$ × 49.	9. 3*s.* $7\frac{3}{4}d.$ × 50.
10. 4*s.* 9*d.* × 25.	11. 4*s.* $8\frac{3}{4}d.$ × 42.	12. 4*s.* $5\frac{1}{2}d.$ × 40.

94. The following rules should be remembered.

(*a*) **To find the cost of a dozen articles at a given price per article**: Reckon a shilling for every penny, and 3*d.* for every farthing, in the price.

EXAMPLE 1. *What is the cost of* 1 *dozen books at* 1*s.* 4*d. each?*

1*s.* 4*d.* = 16*d.*;

∴ cost = **16s.**

EXAMPLE 2. *What is the cost of* 1 *gross of pencils at* $1\frac{1}{2}d.$ *each pencil?*

NOTE. A gross = a dozen times a dozen.

Cost of 1 dozen at $1\frac{1}{2}d.$ = 1*s.* 6*d*;

∴ cost of 1 gross = cost of 1 dozen at 18*d.* = **18s.**

(*b*) **To find the cost of a score of articles at a given price per article**: Reckon the given cost of one article in shillings, and call the shillings pounds.

EXAMPLE. *What is the cost of* 1 *score of articles at* 2*s.* 9*d. each?*

2*s.* 9*d.* × 20 = £2 + 9*d.* × 20

= **£2. 15s.**

EXERCISE 122.

(The price given in each example is the price of *one* article.)

Find the cost of 1 dozen articles at

1. $4d.$ **2.** $5\frac{1}{4}d.$ **3.** $6\frac{1}{2}d.$ **4.** $8\frac{3}{4}d.$ **5.** $10\frac{1}{2}d.$

6. $1s.\ 2d.$ **7.** $2s.\ 6d.$ **8.** $3s.\ 9d.$ **9.** $4s.\ 10d.$ **10.** $5s.\ 3d.$

Find the cost of 1 gross of articles at

11. $3d.$ **12.** $7d.$ **13.** $10d.$ **14.** $2\frac{1}{2}d.$ **15.** $4\frac{1}{4}d.$ **16.** $5\frac{3}{4}d.$

Find the cost of 1 score of articles at

17. $5s.\ 6d.$ **18.** $4s.\ 3d.$ **19.** $7s.\ 9d.$ **20.** $3s.\ 1\frac{1}{2}d.$ **21.** $8s.\ 2d.$

Find the cost of

22. 2 dozen at $1\frac{1}{2}d.$ **23.** 3 dozen at $2\frac{1}{4}d.$ **24.** 5 dozen at $1\frac{3}{4}d.$

25. 48 at $3\frac{3}{4}d.$ **26.** 72 at $6\frac{1}{4}d.$ **27.** 84 at $4\frac{1}{2}d.$

28. 2 gross at $6d.$ **29.** 5 gross at $1\frac{3}{4}d.$ **30.** 3 gross at $5\frac{1}{2}d.$

31. 40 at $3s.\ 9d.$ **32.** 60 at $2s.\ 3d.$ **33.** 80 at $1s.\ 6d.$

K (4). Invoices.

95. An **Invoice** is a written list of the articles bought, with a statement of the cost of them attached, sent by a seller to a buyer. Each separate entry in the list is called an **item**. An invoice should always be headed after the manner shown here. When the money is paid, the person receiving the money should write at the bottom "Received," with his name and the date of receipt. When the sum received is £2 or upwards, a penny stamp must be affixed to the bill by the receiver, who must write his name and the date across it.

BRIGHTON, 22*nd June*, 1912.

MRS. DOWNS

Bought of C. BREEZE & Co.,

Drapers.

	£.	s.	d.
5 *yds. Muslin* @ 1/2½ *per yd.*		6	0½
30 *yds. Merino* @ 4/3¼ *per yd.*	6	8	1¼
3 *doz. Buttons* @ 4¾*d. per doz.*		1	2¼
16 *yds. Sheeting* @ 1/3½ *per yd.*	1	0	8
Received	£7	16	0¼

C. BREEZE & Co.

June 28*th*, 1912.

EXERCISE 123.

Make out the following invoices, with names and dates:

(In connection with invoices **1–5**, say how much change there will be out of £5; in invoices **6–10** out of £10.)

1. 8 loaves at $2\frac{3}{4}d.$ each, 11 lb. of ginger-nuts at $8d.$ per lb., 30 cheese cakes at $\frac{3}{4}d.$ each, 14 bags of flour at $3d.$ per bag, 7 tea cakes at $5d.$ each.

2. 6 lb. of tea at $2s.$ $6d.$ per lb., 9 lb. of rice at $3d.$ per lb., 8 lb. of currants at $4d.$ per lb., 8 lb. of coffee at $1s.$ $8d.$ per lb., 10 lb. of sugar at $2\frac{1}{2}d.$ per lb.

3. 14 lb. of beef at $9\frac{1}{2}d.$ per lb., 10 lb. of pork at $7\frac{1}{2}d.$ per lb., 12 lb. of mutton at $8\frac{1}{2}d.$ per lb., 8 lb. of veal at $9d.$ per lb., 9 lb. of lamb at $11d.$ per lb.

4. Loin of veal, 18 lb. at $6\frac{1}{2}d.$ per lb., aitchbone of beef, 12 lb. at $7\frac{1}{2}d.$ per lb., sirloin of beef, 28 lb. at $8d.$ per lb., leg of mutton, 10 lb. at $8\frac{1}{2}d.$ per lb., shoulder of mutton, 7 lb. at $10d.$ per lb.

5. 14 lb. of tea at $1s.$ $10d.$ per lb., 36 lb. of rice at $2\frac{3}{4}d.$ per lb., 18 lb. of raisins at $4\frac{1}{4}d.$ per lb., 28 lb. of currants at $4\frac{1}{2}d.$ per lb., 16 lb. of biscuits at $11\frac{3}{4}d.$ per lb.

6. 6 hats at $3s.$ $6\frac{1}{2}d.$ each, 14 ties at $11\frac{3}{4}d.$ each, 18 collars at $5\frac{3}{4}d.$ each, 8 shirts at $4s.$ $6\frac{1}{2}d.$ each, 8 dozen handkerchiefs at $7\frac{1}{2}d.$ each.

7. 3 dozen knives at $11s.$ $6d.$ per dozen, 2 sets of carvers at $29s.$ $6d.$ per set, 18 plated forks at $1s.$ $11d.$ each, 12 spoons at $6\frac{1}{2}d.$ each, 5 sets of fire-irons at $10s.$ $9d.$ per set.

8. 5 dozen copy-books at $2d.$ each, 18 boxes of steel pens at $1s.$ $5d.$ per box, 4 dozen slates at $5\frac{1}{2}d.$ each, 3 gross of lead pencils at $1s.$ $9d.$ per dozen, 5 dozen drawing books at $15s.$ per dozen.

9. 14 lamps at $8s.$ $6\frac{1}{2}d.$ each, 25 gallons of oil at $1s.$ $1\frac{1}{2}d.$ per gallon, 400 boxes of matches at $\frac{1}{4}d.$ each, 36 pairs of hinges at $3\frac{1}{2}d.$ per pair, 3 coal vases at $8s.$ $9d.$ each.

10. 64 yds. of sheeting at $1s.$ $4\frac{1}{2}d.$ per yd., 20 yds. of calico at $5\frac{3}{4}d.$ per yd., 15 yds. of flannel at $1s.$ $6d.$ per yd., 21 yds. of velvet at $2s.$ $4\frac{3}{4}d.$ per yd., 12 yds. of linen at $1s.$ $8\frac{3}{4}d.$ per yd.

K (5). Division.

96. No new principle is involved in the examples which follow.

EXAMPLE 1. *Divide* £5. 3*s.* $8\frac{3}{4}$*d.* *by* 3.

	£.	*s.*	*d.*
3	5 .	3 .	$8\frac{3}{4}$
	1 .	**14 .**	$\mathbf{6\frac{3}{4}}$

Remainder = 2 farthings.

We begin at the left hand.

1st. We divide £5 by 3: the quotient is **£1**, and the remainder is £2.

2nd. £2 = 40 shillings, and 40*s.* + 3*s.* = 43*s.* We divide 43*s.* by 3: the quotient is **14s.**, and the remainder is 1*s.*

3rd. 1*s.* = 12 pence, and 12*d.* + 8*d.* = 20*d.* We divide 20*d.* by 3: the quotient is **6d.**, and the remainder is 2*d.*

4th. 2*d.* = 8 farthings, and 8 farthings + 3 farthings = 11 farthings. We divide 11 farthings by 3: the quotient is **3 farthings**, and the remainder is 2 farthings.

The result is **£1. 14s. $6\frac{3}{4}$d.**, with a remainder 2 farthings.

NOTE. £1. 14*s.* $6\frac{3}{4}$*d.* × 3 = £5. 3*s.* $8\frac{1}{4}$*d.*, that is, a sum of money 2 farthings short of the dividend.

Verification.

	£.	*s.*	*d.*
£1 × 3 =	3 .	0 .	0
14*s.* × 3 =	2 .	2 .	0
6*d.* × 3 =		1 .	6
$\frac{3}{4}$*d.* × 3 =			$2\frac{1}{4}$
Rem. =			$\frac{1}{2}$
∴ £1. 14*s.* $6\frac{3}{4}$*d.* × 3 + Rem. =	**5 .**	**3 .**	$\mathbf{8\frac{3}{4}}$

EXERCISE 124.

Verify each result.

Divide

	(*a*)		(*b*)		(*c*)		
	s.	*d.*	*s.*	*d.*	*s.*	*d.*	
1.	4 .	$8\frac{1}{4}$;	8 .	$6\frac{3}{4}$;	5 .	$5\frac{1}{4}$;	by 3.
2.	6 .	3 ;	10 .	5 ;	7 .	3 ;	by 4.
3.	9 .	$9\frac{1}{2}$;	13 .	$7\frac{3}{4}$;	18 .	$0\frac{1}{4}$;	by 5.
4.	5 .	$10\frac{1}{2}$;	9 .	9 ;	18 .	$7\frac{1}{2}$;	by 6.
5.	5 .	$11\frac{3}{4}$;	14 .	$5\frac{1}{4}$;	12 .	$6\frac{1}{2}$;	by 7.
6.	13 .	10 ;	15 .	0 ;	10 .	10 ;	by 8.
7.	8 .	$9\frac{3}{4}$;	12 .	$11\frac{1}{4}$;	16 .	$10\frac{1}{2}$;	by 9.
8.	17 .	1 ;	13 .	$6\frac{1}{2}$;	18 .	$1\frac{1}{2}$;	by 10.
9.	16 .	$11\frac{1}{2}$;	18 .	$6\frac{3}{4}$;	14 .	$5\frac{1}{4}$;	by 11.
10.	19 .	6 ;	18 .	9 ;	11 .	9 ;	by 12.

EXERCISE 125.

Verify each result.

Divide

	(a)			(b)			(c)			
	£.	s.	d.	£.	s.	d.	£.	s.	d.	
1.	2	18	$6\frac{1}{2}$;	2	11	$0\frac{1}{2}$;	5	10	$8\frac{1}{2}$;	by 2.
2.	3	18	$6\frac{3}{4}$;	6	10	$0\frac{3}{4}$;	7	10	$6\frac{3}{4}$;	by 3.
3.	4	16	8 ;	1	12	6 ;	3	0	9 ;	by 4.
4.	5	15	$2\frac{1}{2}$;	2	5	$1\frac{1}{4}$;	8	0	$3\frac{3}{4}$;	by 5.
5.	1	16	$7\frac{1}{2}$;	7	10	9 ;	4	5	$1\frac{1}{2}$;	by 6.
6.	1	1	$1\frac{3}{4}$;	7	11	$4\frac{1}{2}$;	9	2	$3\frac{1}{2}$;	by 7.
7.	9	13	4 ;	3	5	0 ;	6	1	6 ;	by 8.
8.	9	19	6 ;	9	16	$10\frac{1}{2}$;	1	12	$5\frac{1}{4}$;	by 9.
9.	1	15	5 ;	9	12	$8\frac{1}{2}$;	8	17	1 ;	by 10.
10.	7	12	$4\frac{3}{4}$;	7	18	$4\frac{1}{4}$;	9	1	$11\frac{1}{2}$;	by 11.

EXAMPLE 2. *Find three-quarters of* £1. 7*s.* 6*d.*

One-quarter of £1. 7*s.* 6*d.* = £1. 7*s.* 6*d.* ÷ 4
= 6*s.* $10\frac{1}{2}$*d.*;

∴ three-quarters ..... = 6*s.* $10\frac{1}{2}$*d.* × 3
= **£1. 0s. $7\frac{1}{2}$d.**

EXERCISE 126.

1. What is the cost of half a pound of tea at 3*s.* 8*d.* per pound?

2. What is the cost of one-quarter of a pound of tea at 2*s.* 10*d.* per pound?

3. What is the cost of three-quarters of a pound of tobacco at 9*s.* 2*d.* per pound?

4. What is the value of one-eighth of a yard of cloth at 5*s.* 2*d.* per yard?

5. What is the value of three-eighths of a yard of ribbon at 3*s.* 4*d.* per yard?

6. What is the value of five-eighths of a yard of silk at 2*s.* 8*d.* per yard?

7. What is the price of seven-eighths of a yard of lace at 19*s.* 8*d.* per yard?

8. What is the price of three-quarters of a yard of satin at 19*s.* 4*d.* per yard?

Make out the following invoices, with names and dates:

9. $10\frac{1}{2}$ lb. of rice at $2\frac{1}{2}d.$ per lb., $8\frac{1}{2}$ lb. of sugar at $3\frac{1}{2}d.$ per lb., $9\frac{1}{2}$ lb. of tea at 1*s.* 10*d.* per lb., $5\frac{1}{2}$ lb. of coffee at 1*s.* 9*d.* per lb., $3\frac{1}{2}$ lb. of chocolate at 2*s.* 8*d.* per lb.

10. $10\frac{1}{4}$ lb. of raisins at 7*d.* per lb., $12\frac{1}{4}$ lb. of cheese at 9*d.* per lb., $9\frac{1}{4}$ lb. of coffee at 1*s.* 10*d.* per lb., $5\frac{1}{4}$ lb. of tea at 3*s.* 6*d.* per lb., $7\frac{1}{4}$ lb. of butter at 1*s.* 3*d.* per lb.

11. $3\frac{3}{4}$ lb. of cheese at 11*d.* per lb., $8\frac{3}{4}$ lb. of lard at 9*d.* per lb., $7\frac{3}{4}$ lb. of butter at 1*s.* 4*d.* per lb., $10\frac{3}{4}$ lb. of pork at 8*d.* per lb., $11\frac{3}{4}$ lb. of bacon at 10*d.* per lb.

12. $10\frac{1}{8}$ yds. of silk at 2*s.* 4*d.* per yd., $9\frac{3}{8}$ yds. of calico at 10*d.* per yd., $12\frac{5}{8}$ yds. of lace at 1*s.* 2*d.* per yd., $11\frac{7}{8}$ yds. of satin at 3*s.* 10*d.* per yd., $11\frac{3}{4}$ yds. of ribbon at 9*d.* per yd.

97. If the divisor be a number which can easily be separated into factors, each not greater than 12, we divide by these factors in succession.

EXAMPLE. *Divide* £2. 17*s.* 4*d.* *into* 16 *equal shares.*

$16 = 4 \times 4$ or 8×2, and we may use either of these pairs.

	£.	s.	d.
4	2	17	4
4		14	4
		3	7

or

	£.	s.	d.
8	2	17	4
2		7	2
		3	7

Verification. In both cases the results are the same.

EXERCISE 127.

Verify by Multiplication.

Find the value of

1. £1. 6*s.* 10*d.* ÷ 14.
2. £4. 3*s.* 9*d.* ÷ 15.
3. £7. 9*s.* 4*d.* ÷ 16.
4. £7. 17*s.* 6*d.* ÷ 42.
5. £8. 13*s.* 4*d.* ÷ 32.
6. £3. 19*s.* 4*d.* ÷ 28.
7. £5. 18*s.* 9*d.* ÷ 60.
8. £7. 14*s.* 6*d.* ÷ 24.
9. £7. 19*s.* $4\frac{1}{2}d.$ ÷ 18.
10. £8. 14*s.* $6\frac{3}{4}d.$ ÷ 49.
11. £3. 18*s.* $7\frac{1}{4}d.$ ÷ 77.
12. £3. 18*s.* 9*d.* ÷ 108.

In the following cases give the remainders in pence:

13. £4. 14*s.* 8*d.* ÷ 64.
14. £7. 3*s.* 6*d.* ÷ 56.
15. £9. 12*s.* 5*d.* ÷ 144.
16. £2. 9*s.* 6*d.* ÷ 35.

In the following cases give the remainders in farthings:

17. £4. 16*s.* 8*d.* ÷ 84.
18. £5. 19*s.* 6*d.* ÷ 54.
19. £9. 16*s.* 8*d.* ÷ 99.
20. £5. 14*s.* 3*d.* ÷ 45.

EXERCISE 128.

In each problem state your reasoning clearly.

1. The cost of 4 dozen pairs of gloves is £9. 8*s.*; what is the average price per pair?

2. If 4 men earn £5. 2*s.* in 6 days, what are the average daily wages per man?

3. If I pay £7. 13*s.* for 3 lengths of cloth, each measuring 9 yards, what is the average price per yard?

4. I buy 12 toys for £1. 18*s.* 4*d.* and 4 of them cost 3*s.* 9*d.* each; what is the average cost of each of the others?

5. I buy 7 cakes for 18*s.* 8*d.* and 9 boxes of chocolate for 16*s.* 6*d.*; how much dearer is a cake than a box of chocolate?

6. If I pay £3. 2*s.* 8*d.* for 8 postcard albums and one of them costs 10*s.* 9*d.*, what is the average price of the others?

7. If a bookseller buys 11 copies of a book for £3. 4*s.* 2*d.* and sells them so as to gain 1*s.* 8*d.* on each book, what is the selling price of each?

8. If a shopkeeper buys 36 dozen new-laid eggs for £1. 19*s.* and sells them so as to gain 9*s.*, what is the selling price per dozen?

K (6). Problems.

98. The following are further examples of problems with money.

EXAMPLE 1. *I buy* 4 *books at* 3*s.* 4*d. each and* 5 *books at* 4*s.* 10*d. each; what is the average price per book?*

The total cost = 3*s.* 4*d.* × 4 + 4*s.* 10*d.* × 5
= 13*s.* 4*d.* + 24*s.* 2*d.*
= 37*s.* 6*d.*;
∴ the average cost = 37*s.* 6*d.* ÷ 9
= **4s. 2d.**

EXAMPLE 2. *If* 9 *copies of a book cost* £1. 17*s.* 6*d.*, *what must be paid for* 11 *copies?*

Cost of 9 copies = £1. 17*s.* 6*d.*;
∴ ..... 1 copy = 4*s.* 2*d.*;
∴ ..... 11 copies = **£2. 5s. 10d.**

EXERCISE 129.

In each problem state your reasoning clearly.

1. I buy 2 books at 12*s.* 6*d.* each and 1 book for 18*s.*; what is the average price per book?

2. Each of 4 boys has 4*s.* 7½*d.* in his pocket, each of 5 others has 5*s.* 3½*d.*, and another boy has 3*s.* 4½*d.*; what is the average per boy?

3. The cost of 18 books is £2. 10*s.* The cost of 6 of them is 3*s.* 4*d.* each; what is the average cost of the others?

4. The average price of 9 pictures is £1. 2*s.* The price of 4 of them is 15*s.* 9*d.* each; what is the average price of the others?

5. If 10 eggs cost 1*s.* 3*d.*, what is the price of a dozen?

6. What is the price of a case containing 100 oranges, if oranges are sold at 16 for 1*s.*?

7. If 50 packets of envelopes cost 12*s.* 6*d.*, what will 11 dozen packets cost?

8. If 36 men earn £3 for a day's work, what would 25 men earn?

9. What will 40 yards of silk cost, when 6 yards cost £1. 7*s.* 9*d.*?

10. If 12 yards of cloth cost £8. 19*s.*, what will 7 yards cost?

11. What will 36 copies of a book cost, when 25 copies cost £4. 7*s.* 6*d.*?

12. If 33 boxes of chocolate are worth £1. 6*s.* 1½*d.*, what are 8 boxes worth?

L. SUMS OF MONEY GREATER THAN £10.

L (1). Addition and Subtraction.

99. No new principle is involved in the examples which follow.

EXERCISE 130.

Check each result.

1. I pay a butcher £3. 10*s.* 6*d.*, a grocer 15*s.* 7*d.* more than the butcher, and a baker £1. 12*s.* 5*d.* more than the grocer; what money do I pay away altogether?

2. I have 5*s.* left after paying £16. 10*s.* for a horse, £2. 18*s.* for a cart, £1. 17*s.* 6*d.* for harness, and £1. 15*s.* 9*d.* for hay; what money had I at first?

3. A grocer pays £11. 4*s.* for sugar, £18. 17*s.* 6*d.* for tea, £10. 13*s.* 9*d.* for coffee, £15. 14*s.* 10*d.* for cocoa, and £5. 6*s.* 4*d.* for jam; how much does he pay away altogether?

4. A man buys a horse for £32. 7*s.*, a carriage for £25. 15*s.* 6*d.*, a set of harness for £6. 18*s.* 4*d.*, and a whip for 12*s.* 3*d.*; how much does he spend altogether?

5. A man goes to market to buy a horse, a cow, and a pig. The horse costs £60. 15*s.* 9*d.*, the cow £12. 17*s.* 10*d.*, the pig £2. 12*s.* 6*d.*; how much money does he spend altogether?

6. A shopkeeper pays for tea £31. 13*s.* 6*d.*, for bacon £15. 17*s.* 4*d.*, for coffee £20. 18*s.* 10*d.*, for cheese £10. 17*s.* 9*d.*, for sugar £8. 18*s.* 6*d.*; what does he pay away altogether?

7. In a shop there are 5 carpets; the first is worth £20. 15*s.* 4*d.*, the second £39. 16*s.* 9*d.*, the third £40. 15*s.* 8*d.*, the fourth £25. 19*s.* 10*d.*, and the fifth £60. 7*s.* 6*d.*; how much are the five worth?

8. A builder pays every week in wages £37. 15*s.* 8*d.* to foremen, £13. 14*s.* 10*d.* to clerks, £84. 16*s.* 11*d.* to joiners, £60. 17*s.* 4*d.* to bricklayers, and £19. 15*s.* 2*d.* to labourers; what is his weekly expenditure in wages?

9. A man puts in the bank in January £37. 17*s.*, in February £15. 16*s.* 10*d.*, in March £3. 19*s.* 9*d.*, in April £50. 16*s.* 4*d.*, in May £9. 7*s.* 11*d.*, in June £12. 17*s.* 6*d.*; how much does he put in the bank during the six months?

10. A merchant's clerk pays away the following sums: £65. 17*s.* 4*d.*, £92. 13*s.* 6*d.*, £22. 13*s.* 9*d.*, £13. 16*s.* 2*d.*, £10. 10*s.* 11*d.*, £9. 16*s.* 9*d.*, and £3. 15*s.* 7*d.*; how much does he pay away altogether?

EXERCISE 131.

Verify each result.

1. In copying down his sum a boy writes 13*s.* 6*d.* instead of £13. 6*s.*; how much is his error?

2. A tradesman allows me 17*s.* off a bill for £17. 16*s.* 8*d.*; how much do I pay?

3. I buy a horse for £15. 15*s.* 6*d.* and sell it immediately for £18. 5*s.* 4*d.*; what do I gain?

4. Two men's wages for a month are £12. 15*s.* 8*d.*; of this one receives £5. 18*s.* 10*d.*; what does the other receive?

5. A farmer sells a horse for £32. 15*s.* and buys a cow for £15. 17*s.* 6*d.*; what money does he take home?

6. If I exchange a horse worth £19. 18*s.* 4*d.* for one worth £30. 15*s.*, how much money must I pay?

7. I wish to pay a bill for £25. 10*s.* 6*d.*, but find I have not enough by £3. 19*s.* 11*d.*; how much money have I?

8. A shopkeeper takes £75. 12*s.* 5*d.* on Saturday as against £25. 18*s.* 11*d.* on Friday; how much more does he take on the one day than on the other?

9. A clerk adds up an account and makes the amount come to £250. 17*s.* 3*d.*; another makes the amount £249. 18*s.* 4*d.*; what is the difference between their results?

10. A cow and a calf cost £21. 15*s.*; the calf costs £3. 17*s.* 6*d.*; how much more does the cow cost than the calf?

11. A pony and harness cost £23. 2*s.* 3*d.*; the harness costs £4. 10*s.* 6*d.*; how much more does the pony cost than the harness?

12. A horse, carriage, and harness cost £100. 16*s.* 8*d.* The horse costs £50. 15*s.*, and the carriage £38. 17*s.* 9*d.*; what is the cost of the harness?

13. A clerk copies down in an account 7*s* 6*d.* for £7. 6*s.* and £10. 9*s.* for 10*s.* 9*d.*; by what amount is the account wrong?

14. I owe £6. 13*s.* 3*d.*, and receive £3. 16*s.* 9*d.* out of a £10 note; is this the correct change, and if not, is it too much or too little, and by how much?

15. Jones has £17. 9*s.*; Robinson has £5. 17*s.* 6*d.* more than Jones; Smith has £10. 10*s.* 6*d.* less than Jones; and Brown has as much as all the others together. How much has Brown?

L (2). Multiplication.

EXERCISE 132.

Verify each result.

Find the value of

	£.	*s.*	*d.*	(*a*)	(*b*)		£.	*s.*	*d.*	(*a*)	(*b*)
1.	10	14	6	×2;	9.	2.	12	13	8	×3;	8.
3.	15	15	4	×4;	7.	4.	16	12	9	×5;	11.
5.	13	17	3	×6;	10.	6.	18	10	$5\frac{3}{4}$	×7;	12.
7.	11	11	$11\frac{1}{4}$	×5;	9.	8.	14	18	$4\frac{1}{4}$	×4;	11.
9.	17	16	$7\frac{1}{4}$	×3;	10.	10.	19	17	$2\frac{1}{2}$	×6;	9.

EXERCISE 133.

Verify each result.

Find the value of

	£.	*s.*	*d.*	(*a*)	(*b*)		£.	*s.*	*d.*	(*a*)	(*b*)
1.	3	4	9	×25;	72.	2.	4	8	6	×15;	28.
3.	7	5	$7\frac{1}{2}$	×32;	49.	4.	2	8	$10\frac{1}{4}$	×63;	64.
5.	3	13	$8\frac{1}{2}$	×56;	60.	6.	5	17	$9\frac{3}{4}$	×77;	80.
7.	8	16	$3\frac{1}{2}$	×14;	81.	8.	3	19	$7\frac{1}{4}$	×42;	45.
9.	6	12	$5\frac{1}{4}$	×16;	35.	10.	9	15	$11\frac{3}{4}$	×18;	77.

100. When the multiplier is not the product of factors each less than 12.

All examples of multiplication by a number containing two digits can be performed by considering the multiplier as a multiple of 10, added to a number less than 10.

EXAMPLE 1. *Multiply* £1. 10*s.* $3\frac{1}{2}$*d.* *by* 17.

(Rough Approximation. The answer must lie between £1×17 and £2×17, *i.e.* between £17 and £34.)

We multiply by 10 and by 7, and add the results together.

£.	*s.*	*d.*	
1	10	$3\frac{1}{2}$	= Multiplicand
		10	
15	2	11	$= M \times 10$
10	12	$0\frac{1}{2}$	$= M \times 7$
25	14	$11\frac{1}{2}$	$= M \times 17$

Verification. Multiply by 12 and also by 5, and add the products together.

EXAMPLE 2. *Multiply* £1. 10*s.* $3\frac{1}{2}$*d.* *by* 29.

(Rough Approximation. The answer must lie between £29 and £58.**)** We multiply by 20 and by 9, and add the results together.

£.	*s.*	*d.*	
1 .	10 .	$3\frac{1}{2}$	= Multiplicand
		10	
15 .	2 .	11	$= M \times 10$
		2	
30 .	5 .	10	$= M \times 20$
13 .	12 .	$7\frac{1}{2}$	$= M \times 9$
43 .	**18 .**	$\mathbf{5\frac{1}{2}}$	$= M \times 29$

Verification. Multiply by $4 \times 7 + 1$, or by $10 \times 3 - 1$, etc.

NOTE 1. The above is the method in general use, but it is well to exercise a little judgment in selecting the factors to be used.

Thus in Example 2 we may multiply by 4×7 and *add* in the top line, or we may multiply by 30 (*i.e.* by 10×3) and *subtract* the top line from the result, for $29 = 28 + 1$ and $= 30 - 1$.

NOTE 2. When a sum of money contains odd farthings, we should, if possible, use 4 or 8 or 12 as a first multiplier : there will then be *no* farthings to deal with at the next step, and error may be thus avoided.

NOTE 3. When a sum of money contains odd pence, we should, if possible, use 12 as a first multiplier.

EXERCISE 134.

(1) *Make a preliminary rough approximation to each product, and*
(2) *verify each result by using another method.*

Selecting any suitable method, find the following products :

	£.	*s.*	*d.*	(*a*)	(*b*)		£.	*s.*	*d.*	(*a*)	(*b*)
1.	6 .	14 .	3	$\times 13$;	23.	2.	7 .	15 .	4	$\times 17$;	29.
3.	8 .	17 .	$8\frac{1}{4}$	$\times 19$;	29.	4.	9 .	13 .	$10\frac{1}{2}$	$\times 37$;	65.
5.	10 .	12	$6\frac{1}{2}$	$\times 41$;	87.	6.	12 .	11 .	$2\frac{3}{4}$	$\times 43$;	95.
7.	15 .	15 .	$10\frac{1}{4}$	$\times 47$;	93.	8.	17 .	18 .	$7\frac{1}{2}$	$\times 52$;	89.
9.	18 .	19 .	$4\frac{3}{4}$	$\times 67$;	75.	10.	20 .	12 .	$11\frac{1}{4}$	$\times 69$;	73.

EXERCISE 135.

1. How much more will be paid for 19 books at 2*s.* 4*d.* each than for 23 books at 1*s.* 8*d.* each ?

2. A labourer's wages are 12*s.* 3*d.* per week for one half of the year, and 10*s.* 4*d.* per week for the other half ; what is the total amount of his wages in a year ?

3. If I spend 1*s.* 9*d.* a day for a railway return ticket, travelling six days in the week, how much should I save in a year (52 weeks) by paying £25 for a season ticket ?

4. A dealer bought a pig for 17*s.* 8*d.*, and sold it for £1. 5*s.* 6*d.*; what would he gain on the sale of 36 pigs?

5. If a man saves 3*s.* 6*d.* a week for a year, and at Christmas has two guineas given him, how much more must he save to have £20?

6. A man earns £3. 10*s.* a week. It costs him a guinea for food and 7*s.* 6*d.* for lodgings per week; how much can he save in 19 weeks?

7. A farmer sells 3 horses for sixty-two guineas each, and 16 lambs for three half-crowns each; he then buys 17 cows for £15. 16*s.* 4*d.* each; how much money will he have to pay away?

8. A dealer bought 28 horses at £28. 15*s.* each; he sold 13 of them at £32. 5*s.* each, and the remainder at £35. 17*s.* 6*d.* each; what was his total gain?

101. When the multiplier is a number greater than 100, which cannot easily be separated into simple factors.

EXAMPLE. *Multiply* £12. 4*s.* 9½*d.* *by* 365.

(**Rough Approximation.** The answer must lie between £12 × 365 and £13 × 365, *i.e.* between £4380 and £4745.)

The multiplier is equivalent to 300 + 60 + 5; we therefore multiply £12. 4*s.* 9½*d.* by 300, 60, and 5, and add the results together.

To multiply by 300, we multiply by the factors, 10, 10, and 3 in turn.

To multiply by 60, we multiply by 10 and 6.

£.	*s.*	*d.*	
12 .	4 .	9½	= Multiplicand
		10	
122 .	7 .	11	= $M \times 10$
		10	
1223 .	19 .	2	= $M \times 100$
		3	
3671 .	17 .	6	= $M \times 300$
734 .	7 .	6	= $M \times 10 \times 6$
61 .	3 .	11½	= $M \times 5$
4467 .	**8 .**	**11½**	= $M \times 365$

Verification.

2 | 365 halfpennies.

182½*d.*	= ½*d.* × 365
3285 *d.*	= 9*d.* × 365

12 | 3467½*d.*

288*s.* 11½*d.*	= 9½*d.* × 365
1460*s.*	= 4*s.* × 365

20 | 1748*s.* 11½*d.*

£87. 8*s.* 11½*d.*	
£4380	= £12 × 365
£4467. 8s. 11½d.	= $M \times 365$

102. To find the cost of 240 articles at a given price per article :—

Reckon a sovereign for every penny, and 5*s.* for every farthing, in the price.

EXAMPLE 1. *What is the cost of* 240 *articles at* 1*s.* 6*d.* *each?*

1*s.* 6*d.* × 240 = (18*d.* × 240)

= **£18.**

EXAMPLE 2. *What is the cost of* 240 *articles at* 1*s.* 4½*d. each?*

$$1s.\ 4\tfrac{1}{2}d. \times 240 = (16d. \times 240) + (\tfrac{1}{2}d. \times 240)$$
$$= \underline{\text{£16. 10s.}}$$

103. To find the yearly wages (365 days) when the daily wages are given:—

Reckon as many sovereigns, as many half-sovereigns, and five times as many pence as there are pence in the daily wages.

EXAMPLE. *What are the yearly wages of an office boy who earns* 1*s.* 6*d. a day?*

$$1s.\ 6d. \times 365 = (18d. \times 240) + (18d. \times 120) + (18d. \times 5)$$
$$= \text{£18} + \text{£9} + 7s.\ 6d.$$
$$= \underline{\text{£27. 7s. 6d.}}$$

EXERCISE 136.

(1) *Make a preliminary rough approximation to each product; and* (2) *verify each result.*

Find the value of

	£.	s.	d.	(a)	(b)		£.	s.	d.	(a)	(b)
1.	1	4	6	×130;	203.	2.	2	13	9	×103;	330.
3.	4	5	10½	×114;	241.	4.	6	10	8¼	×141;	314.
5.	3	12	9¼	×138;	275.	6.	7	14	7½	×183;	357.
7.	5	13	8¾	×129;	432.	8.	8	15	6¼	×192;	543.
9.	6	16	4½	×165;	607.	10.	3	18	5¾	×156;	570.

Write down the answers to the following:

11. $6\frac{1}{4}d. \times 240$.
12. $1s.\ 1\frac{1}{2}d. \times 240$.
13. $1s.\ 11\frac{3}{4}d. \times 240$.
14. $2s.\ 3d. \times 240$.
15. $10d. \times 365$.
16. $1s.\ 4d. \times 365$.
17. $2s.\ 6d. \times 365$.
18. $6s. \times 365$.
19. $7s.\ 9d. \times 365$.

EXERCISE 137.

1. How much must I have in my pocket to pay 110 men 3*s.* 6*d.* each and 123 women 2*s.* 9*d.* each?

2. A bricklayer earns 3*s.* 2*d.* a day and his labourer 1*s.* 8*d.*: how much will they both earn in 130 days?

3. How much profit would be made by buying 250 tons of coal at 11*s.* 10*d.* a ton and selling it at £1. 7*s.* a ton?

4. If I buy 114 shares in a certain company for £3. 2*s*. 6*d*. each and sell them all for £4. 15*s*. each, what shall I gain?

5. A grazier went to market with £500, and he bought 165 sheep for £2. 15*s*. each; how much money had he left?

6. A man's expenses are 12*s*. 11*d*. a day for a year (365 days), and he saves £25. 17*s*. 10*d*. during the year; what is his yearly income?

7. A man earns 4*s*. 6*d*. a day and his two sons 3*s*. 4*d*. each; how much do they all earn in a year (313 working days)?

8. A dealer bought 129 chickens at 1*s*. 4*d*. each and 113 at 1*s*. 6*d*. each. If he sold the lot for £20, what did he gain?

L (3). Division.

104. No new principle is involved in the examples which follow.

EXERCISE 138.

Verify each result by Multiplication.

Find the value of

	£.	*s*.	*d*.		£.	*s*.	*d*.
1.	27	14	9 ÷ 2.	2.	43	11	$3\frac{3}{4}$ ÷ 3.
3.	98	10	3 ÷ 4.	4.	78	3	$0\frac{1}{4}$ ÷ 5.
5.	16	8	6 ÷ 6.	6.	26	10	10 ÷ 7.
7.	29	0	4 ÷ 8.	8.	41	4	$5\frac{1}{4}$ ÷ 9.
9.	46	17	$3\frac{1}{2}$ ÷ 10.	10.	60	14	$1\frac{1}{2}$ ÷ 11.
11.	62	19	$3\frac{1}{4}$ ÷ 11.	12.	82	13	6 ÷ 12.

EXERCISE 139.

Verify each result.

Using factors, find the value of

	£.	*s*.	*d*.		£.	*s*.	*d*.
1.	336	7	0 ÷ 16.	2.	581	9	$1\frac{1}{2}$ ÷ 18.
3.	132	18	0 ÷ 24.	4.	167	8	$5\frac{1}{4}$ ÷ 25.
5.	247	5	4 ÷ 32.	6.	369	7	$7\frac{1}{2}$ ÷ 42.
7.	457	6	0 ÷ 48.	8.	531	3	$11\frac{1}{2}$ ÷ 50.
9.	741	2	8 ÷ 64.	10.	501	19	$3\frac{3}{4}$ ÷ 63.
11.	318	4	$10\frac{1}{2}$ ÷ 66.	12.	412	12	$8\frac{1}{2}$ ÷ 70.

105. When the divisor cannot easily be separated into simple factors.

The work must be stated in full as below.

EXAMPLE. *Divide* £1148. 5*s.* $0\frac{3}{4}$*d.* *by* 83.

Quotient = **£13. 16s. $8\frac{1}{4}$d.**

83 | £1148. 5*s.* $0\frac{3}{4}$*d.*

83

318
249

69
20

1385 *shillings*
83

555
498

57
12

684 *pence*
664

20
4

83 *farthings*
83

Rough Approximation.

The answer is about £1148 ÷ 83,
i.e. between £13 and £14.

Verification.

£13. 16*s.* $8\frac{1}{4}$*d.* × 83
= £1148. 5*s.* $0\frac{3}{4}$*d.*

EXERCISE 140.

(1) *Make a preliminary rough approximation to each quotient; and* (2) *verify each result by Multiplication.*

Perform the following operations by Long Division:

	£.	s.	d.		£.	s.	d.
1.	954	10	$5\frac{1}{2} \div 61$.	2.	275	5	$3\frac{1}{2} \div 37$.
3.	462	12	$9\frac{1}{4} \div 83$.	4.	326	8	$1 \div 38$.
5.	146	14	$10\frac{1}{2} \div 53$.	6.	153	1	$3\frac{3}{4} \div 29$.
7.	1040	19	$7\frac{1}{2} \div 59$.	8.	1422	17	$3 \div 76$.
9.	2065	19	$6 \div 23$.	10.	1279	13	$8\frac{3}{4} \div 31$.
11.	1368	2	$9\frac{3}{4} \div 47$.	12.	6304	15	$11\frac{1}{2} \div 97$.
13.	7197	12	$4\frac{1}{2} \div 183$.	14.	7636	18	$9\frac{3}{4} \div 219$.
15.	2375	5	$11\frac{1}{4} \div 135$.	16.	1246	16	$6\frac{3}{4} \div 397$.
17.	16501	8	$4\frac{1}{4} \div 197$.	18.	39133	7	$0 \div 456$.
19.	16196	10	$2\frac{1}{2} \div 325$.	20.	41614	14	$1\frac{1}{2} \div 702$.

EXERCISE 141.

Verify each result.

1. A farmer bought 37 sheep for £108. 13*s.* 9*d.*, and sold them at £3. 16*s.* 3*d.* each; how much did he gain on each sheep?

2. A bookseller bought 19 books for £1. 11*s.* 6*d.* each, and sold them at a total loss of £7. 7*s.* 3*d.*; what did he sell each book for?

3. Eighty-three gallons of wine were bought at 16*s.* 9*d.* per gallon, and the whole sold at a profit of £8. 19*s.* 10*d.*; what was the selling price per gallon?

4. A grocer buys 59 lb. of tea at 1*s.* 8*d.* per lb., and 34 lb. for £6. 14*s.* 2*d.*, and mixes them; what must he sell each lb. of the mixture for to get back exactly what he spends?

5. If 5 dozen yards of cloth cost 10 guineas, for how much a yard must it be sold to make a profit of £4. 5*s.* on the whole?

6. How many more halfpenny stamps can I buy with £2. 4*s.* 3*d.* than penny stamps with £1. 12*s.* 8*d.*?

7. In a collection £2. 7*s.* 6*d.* is taken in threepenny pieces, £1. 18*s.* 6*d.* in sixpences, £5. 10*s.* in shillings, and £2. 12*s.* 6*d.* in half-crowns; how many coins are there?

8. From London to Birmingham is 112 miles, and the third-class railway fare is one penny per mile; what sum would the booking clerk receive for 59 third-class tickets?

9. Three thousand workmen strike because an increase of a halfpenny per hour is not given; what is the additional sum that would be required weekly if their wages were increased, taking the working day at nine hours?

10. A collection consisted of sixty-four threepenny pieces, thirteen half-sovereigns, seventeen half-crowns, fourteen florins, seventy-one sixpences, and two hundred and eighty-nine shillings; how much did it amount to?

11. At a certain post-office eight hundred and twenty stamps were sold in a day; thirty-three were sixpenny stamps, sixty-five were threepenny stamps, ninety-seven were halfpenny stamps, and the rest were penny stamps; how much money was taken for stamps that day?

12. On which day is most money taken at an exhibition, and by how much: a half-crown day when nineteen thousand people pay, or a shilling day when forty-eight thousand people pay?

13. A man went to the post-office with two florins and three half-crowns in his pocket. He bought a hundred halfpenny stamps, three dozen penny stamps, and seven sixpenny packets of postcards; what change did he receive?

14. How much less is the silver than the gold in a cash box which contains eleven sovereigns, nineteen half-sovereigns, thirteen half-crowns, twenty-three florins, ninety-two shillings, and twenty-seven sixpences?

15. If I divide the sum of ten half-crowns, fifteen florins, seventeen sixpences, and nineteen threepenny pieces, equally amongst thirteen boys, what will be the sum of money received by each?

106. To find how many times one sum of money is contained in another sum, we proceed as in the following example.

EXAMPLE. *How many times is* £4. 7*s.* 6*d. contained in* £100. 12*s.* 6*d.*?

We first reduce both sums of money to simple quantities of the same name; it is sufficient here to reduce each sum to *half-crowns.*

£	*s.*	*d.*
4	7	6
8		
35 *half-crowns*		

£	*s.*	*d.*
100	12	6
8		
805 *half-crowns*		

$$35\left\{\begin{array}{l|l} 5 & 805 \\ 7 & 161 \\ \hline & 23 \end{array}\right.$$

∴ the number of times = **23.**

Verification. £4. 7*s.* 6*d.* × 23 = £100. 12*s.* 6*d.*

EXERCISE 142.

Verify each result.

How many times is

1. 1*s.* 7*d.* contained in 17*s.* 5*d.*?
2. 2*s.* $9\frac{1}{4}$*d.* contained in 8*s.* $3\frac{3}{4}$*d.*?
3. 16*s.* 3*d.* contained in £9. 15*s.*?
4. 5*s.* 4*d.* contained in £31. 4*s.*?
5. 8*s.* $1\frac{1}{2}$*d.* contained in £5. 13*s.* 9*d.*?
6. 7*s.* 6*d.* contained in £19. 2*s.* 6*d.*?
7. £1. 12*s.* 6*d.* contained in £78?

8. £2. 4*s.* 6*d.* contained in £80. 2*s.*?

9. £3. 6*s.* 8*d.* contained in £43. 6*s.* 8*d.*?

10. £2. 17*s.* 6*d.* contained in £54. 12*s.* 6*d.*?

EXERCISE 143.

Verify each result.

1. How many cakes worth $8\frac{1}{2}d.$ each can be bought for £12. 18*s.* $6\frac{1}{2}d.$?

2. The sum of £22. 9*s.* 2*d.* is divided among a number of old people, and each person receives 1*s.* 10*d.*; how many people are there?

3. How many footballs worth 18*s.* 4*d.* each can be bought for £69. 13*s.* 4*d.*?

4. A grocer buys jam in pots at $5\frac{1}{2}d.$ each; how many dozen pots can he buy for £3. 11*s.* 6*d.*?

5. If I pay 3*s.* 9*d.* for a hundred bundles of wood, how many bundles can I buy for £9. 7*s.* 6*d.*?

6. At a factory the work-people receive £81. 0*s.* 8*d.* in wages every week; how many work-people are employed, supposing the average wages are £1. 11*s.* 2*d.* each?

7. A person going to Paris with 20 sovereigns requires French money; how many francs can he obtain, reckoning a franc at $9\frac{1}{2}d.$, and how much English money has he remaining?

8. The sum of £161. 18*s.* is divided among equal numbers of old men and women, and each person receives eleven guineas; how many persons are there, and how much money remains over?

9. I buy a number of cricket balls at 6*s.* 5*d.* each for £14. 8*s.* 9*d.*, and a number of cricket bats at 10*s.* 6*d.* each for £27. 6*s.* Of which article do I buy the greater number?

10. A draper spends £12. 18*s.* 8*d.* in gloves, at 2*s.* 8*d.* the pair. Some of them fade and are unsaleable; the rest he sells for £14. 13*s.* 10*d.*, at 3*s.* 5*d.* the pair; how many pairs fade?

L (4). Problems.

107. The following are further examples of problems involving both multiplication and division, and solved by the **Unitary Method.**

EXAMPLE 1. *If 6 copies of a book cost £3. 16s. 6d., how many copies can be bought for £8. 5s. 9d.?*

The cost of 6 books = £3. 16s. 6d.;

∴ ..... 1 book = 12s. 9d.;

∴ number of books bought for £8. 5s. 9d. $= \frac{£8.\ 5s.\ 9d.}{12s.\ 9d.} = \frac{663}{51}$

$= \underline{13}$.

EXAMPLE 2. *How many boxes of chocolate at 5d. each are worth the same as 36 tins of biscuits at 1s. 8d. each?*

The cost of 36 tins of biscuits $= 20d. \times 36 = 720d.$,

and ..... 1 box of chocolate $= 5d.$;

∴ the number of boxes of chocolate $= 720 \div 5$

$= \underline{144}$.

EXERCISE 144.

(1) *In each problem state your reasoning clearly;* *and* (2) *verify each solution.*

1. How many eggs can be bought for 2s. 6d., if 56 can be bought for 11s. 8d.?

2. How many cricket balls can be bought for £5. 15s., if 13 can be bought for £3. 5s.?

3. For 12s. I can buy 16 copies of a magazine; how many copies can I buy for 18s.?

4. For £3. 3s. I can buy 36 copies of a book; how many copies can I buy for £1. 2s. 9d.?

5. If 60 boxes of chocolate cost £4. 5s., how many boxes can be bought for £6. 3s. 3d.?

6. How many books worth 4s. 6d. each will cost as much as 84 books at 7s. 6d.?

7. How many pounds of sugar at 3d. per pound are worth 9 pounds of tea at 2s. 4d. per pound?

8. How many lambs worth £1. 2s. 6d. should be given in exchange for 15 sheep at £2. 2s. each?

9. A shopkeeper exchanged 60 yards of silk worth 11s. 3d. a yard for 48 yards of velvet; what was the price to him of the velvet per yard?

10. How much does wine cost per bottle, if 6 dozen bottles of wine are worth as much as 5 barrels of ale at 78s. a barrel?

108. The following are further examples of problems involving division with money.

EXAMPLE 1. *Divide* 24*s.* 6*d.* *between Smith and Brown, so that Smith may have* 3 *shares and Brown* 4 *shares.*

We can do this by dividing 24*s.* 6*d.* into 3+4 equal parts, *i.e.* 7 equal parts, and taking 3 of them for Smith and 4 of them for Brown.

Each equal part = 24*s.* 6*d.* ÷ 7 = 3*s.* 6*d.* ;

∴ Smith's share = 3*s.* 6*d.* × 3 = **10s. 6d.**,

and Brown's share = 3*s.* 6*d.* × 4 = **14s.**

EXAMPLE 2. *Divide* 3 *guineas among* 3 *boys and* 2 *girls, giving each girl twice as much as each boy.*

Let one boy's share = x shillings,

then one girl's share = $2x$ shillings ;

$\therefore\ 3 \times x + 2 \times 2x = 63$;

$\therefore\ 3x + 4x = 63$;

$\therefore\ 7x = 63$;

$\therefore\ x = 9$;

∴ a boy receives **9s.**, and a girl receives **18s.**

EXAMPLE 3. *Divide* £6. 5*s.* *among* 5 *boys and* 4 *girls, giving each boy* 2*s.* 6*d.* *more than each girl.*

It is evident that the 5 boys will receive 12*s.* 6*d.* more than the 4 girls. This amount must therefore be first taken away, and the result of dividing the remainder into 9 equal parts will give the share of each girl.

$$\text{The share of each girl} = \frac{\text{£6. 5s.} - \text{12s. 6d.}}{9}$$

$$= \frac{\text{£5. 12s. 6d.}}{9}$$

$$= \textbf{12s. 6d.};$$

and the share of each boy = **15s.**

EXAMPLE 4. *A bill of* £19. 16*s.* 6*d.* *was paid by giving a certain number of half-crowns and twice as many florins ; how many of each coin were there ?*

1 half-crown + 2 florins = 2*s.* 6*d.* + 4*s.* = 6*s.* 6*d.* ;

$$\therefore \text{ the number of half-crowns} = \frac{\text{£19. 16s. 6d.}}{\text{6s. 6d.}}$$

$$= \frac{793 \textit{ sixpences}}{13 \textit{ sixpences}}$$

$$= 61;$$

and the number of florins = **122.**

EXERCISE 145.

(1) *In each problem state your reasoning clearly;*
and (2) *verify each solution.*

1. Divide 5*s.* between 2 boys, giving one boy twice as much as the other.

2. Divide 3*s.* 9*d.* between 2 girls, giving one girl four times as much as the other.

3. Divide £10 among A, B and C, so that A may have 4 shares, B 5 shares and C 6 shares.

4. Divide £3. 18*s.* among A, B and C, giving A twice as much as each of the others.

5. Divide £4. 5*s.* 10*d.* among A, B and C, giving A three times as much as B, and C twice as much as A.

6. Divide £97. 7*s.* 2*d.* among A, B and C, so that A may get twice as much as B, and B twice as much as C.

7. Divide £2. 10*s.* among 2 boys and 2 girls, giving each boy twice as much as a girl.

8. Divide £10. 10*s.* among 5 men and 6 women, so that each man may have three times as much as each woman.

9. Divide 5*s.* between two boys, giving one boy 6*d.* more than the other.

10. Divide 4*s.* 6*d.* between 2 girls, giving one girl 10*d.* more than the other.

11. Divide 16*s.* between 2 boys, giving one boy 2*s.* 4*d.* less than the other.

12. Divide 3 guineas between 2 girls, giving one girl 4*s.* 6*d.* less than the other.

13. Divide 20 guineas among A, B and C, giving B 17*s.* more than A, and C 26*s.* more than B.

14. Divide 3*s.* between A and B, so that for every 3*d.* A gets B may have 6*d.*

15. Divide 4*s.* 6*d.* between C and D, so that for every 1*s.* C gets D may have 6*d.*

16. Divide £2. 15*s.* among A, B and C, so that for every 1*s.* A gets, B will get a florin, and C half-a-crown.

17. A bag contains £4. 10*s.* made up of equal numbers of half-crowns and florins; find the number of each coin.

18. A bag contains £12. 4*s.* 6*d.* made up of equal numbers of shillings and sixpences; how many are there of each coin?

19. A purse contains 9*s.* made up of twice as many shillings as threepenny pieces; how many of each coin are there?

20. A bill of £13. 7*s.* 6*d.* was paid by giving a certain number of sixpences and twice as many shillings; how many of each coin were there?

21. A bill of £8. 3*s.* 4*d.* was paid with a certain number of sovereigns, three times as many shillings, and four times as many pennies; find the number of each coin used.

22. A bag contains £18. 15*s.* 10*d.* made up of half-crowns, shillings and pennies; there are six times as many half-crowns as pennies, and twice as many shillings as pennies; find the number of each coin.

23. A bag contains 19 guineas in half-crowns, florins, and shillings; there are twice as many florins, and three times as many shillings, as half-crowns; find how many of each coin the bag contains.

24. Equal numbers of postmen and telegraph messengers received £109. 4*s.* for 4 weeks' services; the men were paid 30*s.* a week, and the boys had 15*s.* 6*d.*; how many were there of each?

M. MEASURES OF TIME, WEIGHT, CAPACITY, AND LENGTH.

109. The various processes are identical with the corresponding operations for Money.

M (1). Measures of Time.

110. The chief unit of time is a **day**, which is the average time in which the earth turns round on its axis.

The time in which the earth travels round the sun is called a **solar year**, and is equal to about 365 days 6 hours.

The **common year** consists of 365 days. We take account of the difference between a common year and a solar year by adding a whole day to every *fourth* common year (with certain exceptions).

The years which have the extra day are called **leap years** (366 days).

111. A year is divided into 12 **calendar months.**

The following rhyme with reference to the days in the calendar months should be memorised:

Thirty days hath September,
April, June, and November;
All the rest have 31,
Excepting February alone,
Which hath 28 days clear,
And 29 in each Leap Year.

EXERCISE 146.

1. How many hours are there in the month of March?

2. How many minutes are there in the month of February in a leap year?

3. How many days are there from March 22nd to July 4th, including one only of the two days mentioned?

4. How many weeks are there from August 1st to October 31st?

5. If the sun rises at 3.51 a.m. and sets at 8.17 p.m., for how many minutes is it above the horizon?

6. A train arrived at 10 minutes after 11 o'clock instead of at half-past 10; how many seconds late was it?

7. The large hand of a clock goes round once an hour; how many times does it turn round from Monday morning at 9 o'clock to the following Saturday at the same hour?

8. The minute hand of a clock passes over 60 spaces in 1 hour; over how many spaces will it pass between 9.0 a.m. on Monday and 6.45 p.m. on Tuesday?

9. A clock ticks 43200 times in a day; how many ticks is that per minute?

10. At 12.0 I wind up a clock, which ticks once in a second; at what time will it make the 1000th tick?

11. If 31620 persons entered an exhibition between 10 a.m. and 6.30 p.m., how many would you say entered per minute?

12. A man works during six days of the week as follows: 11 hrs. 19 min. 30 sec., 9 hrs. 19 min., 7 hrs. 50 min. 50 sec., 8 hrs., 8 hrs. 40 sec., and $10\frac{1}{2}$ hrs.; how many hours altogether does he work in the week?

13. A Cunard steamer can run from Fishguard to New York in 4 days 20 hrs. 7 min., and can return in 4 days 15 hrs. 57 min. How long does it take to go to New York and back?

14. A traveller arrived at a station 20 min. 45 sec. past 4 o'clock, but found the train did not start until 6 o'clock; how long had he to wait?

15. One ship took 70 days 10 hrs. 35 min. to complete a voyage. Another ship did the voyage in 68 days 12 hrs. 40 min.; how much longer did the first one take than the second?

16. If a man works 13 hrs. 20 min. 25 sec. every day, how many hours, etc., will he work between Monday morning and Saturday night?

17. What period of time is the seventh part of 20 days 20 hrs. 10 min.?

18. It took 7 days 50 min. to do some work in three portions. The first portion took $2\frac{1}{4}$ days, and the second took 3 days 18 hrs.; how long did it take to do the other portion?

19. A man worked from 7.30 a.m. until 2.15 p.m. on Monday and from 9.15 a.m. to 1.10 p.m. on Tuesday; how much longer did he work on Monday than on Tuesday?

20. A ship takes 8 days 13 hrs. 30 min. on its outward journey, and 7 days 12 hrs. 20 min. on its homeward journey; how long should it take to complete 4 voyages, outward and homeward?

21. Certain work has to be done in 200 hours. If a number of men are employed for 7 days for 13 hrs. 20 min. 20 sec. each day, how much longer will the work take?

22. 373 hrs. 25 min. have already been spent upon certain work, and if a man now works steadily for 8 hrs. 20 min. every day for 12 days the work will be completed; how long will the work have lasted altogether?

23. Brown worked 11 hrs. 30 min. 30 sec. every day for 6 days, and Smith worked 8 hrs. 10 min. 40 sec. every day for 7 days; how much longer did Brown work than Smith?

24. If a large wheel revolves once in 1 min. 15 sec., how many times will it revolve in 24 hours?

M (2). Measures of Weight.

112. The weights of all substances, except gold, silver, and jewellery (and medicines under certain circumstances) are in this country measured by **Avoirdupois Weight.**

113. The unit of weight in the British system is the **Imperial standard pound** (7000 grains).

The sixteenth part of a pound is called an **ounce** (*oz.*). Larger weights are the **stone** (*st.* 14 lb.), the **quarter** (*qr.* 28 lb.), the **hundredweight** (*cwt.* 112 lb.), and the **ton** (2240 lb.).

NOTE. 3 pennies weigh 1 *ounce.*

EXERCISE 147.

1. If a baker uses 1 lb. of flour in making a dozen buns, how many buns can be made from a sack of flour ($2\frac{1}{2}$ cwt.)?

2. How many quarter-pounds of tea can be weighed from $1\frac{1}{4}$ cwt.?

3. How many half-ounces of tea are there in a chest containing $1\frac{1}{2}$ cwt.?

4. Three pennies weigh an ounce; express in hundredweights the weight of 18816 pennies.

5. How many tons of sugar will 32 bags contain, each bag weighing 140 lb. when full?

6. Bags filled with potatoes weigh 2 cwt. 18 lb., 3 cwt. 3 qrs., 1 cwt. 27 lb., 2 cwt. 2 qrs. 20 lb., and $1\frac{1}{2}$ cwt.; find the total weight of the potatoes.

7. A quantity of bricks was removed in five carts. The first cart carried 1 ton 3 qrs.; the second, 2½ tons; the third, 2 tons 15 cwt. 1 qr.; the fourth, 18 cwt. 56 lb.; and the last, 1 ton 10 cwt. 3 qrs. 14 lb.; find the total weight of the bricks.

8. A grocer takes 6 lb. 14 oz. of tea from a canister containing 20 lb. 12 oz.; how much tea is left in the canister?

9. A provision dealer has 2 qrs. 20 lb. of bacon; if he sells 1 qr. 22 lb. 4 oz., what weight will he have remaining?

10. A merchant has 102 cwt. 1 qr. 6 lb. of flour; if he sells 12 cwt. 2 qrs. 16 lb., what weight will be left?

11. What is the difference in weight between a ton of coal and 13 cwt. 2 qrs. 19 lb. 7 oz. of sugar?

12. Out of a stock of 46 tons 12 cwt. of coal 6 tons 12 cwt. 3 qrs. remain; how much has been used?

13. If a parcel of groceries weighs 16 lb. 10 oz., what will be the weight of 5 such parcels?

14. If a fire consumes 3 cwt. 2 qrs. 12 lb. of coal daily, how much coal will be wanted during a week (7 days)?

15. Find the total weight of 16 bags of potatoes, each weighing 2 qrs. 20 lb. 14 oz.

16. Find the total weight of 23 van-loads of goods, if each van carries 3 tons 10 cwt. 1 qr. 16 lb.

17. If 3 horses pull a load weighing 9 tons 19 cwt. 2 qrs., what is the average weight pulled by each horse?

18. If 19 cwt. 2 qrs. 16 lb. of coal be distributed equally among 20 poor persons, what weight will each receive?

19. A dealer delivers 41 tons 14 cwt. 1 qr. of coal to 28 customers; what is the average weight delivered to each?

20. If 17 truck-loads of fish weigh 10 tons 4 cwt. 3 qrs. 2 lb. 1 oz., what is the average weight of each truck-load?

EXERCISE 148.

1. A grocer buys 3 cwt. 3 qrs. 23 lb. of sugar from one merchant, and 1 cwt. 2 qrs. 6 lb. from another; how much more must he buy to bring his stock of sugar up to one ton?

2. From a truck of coal containing 15 tons 3 cwt. 56 lb. two cart-loads are taken. If each cart carries 3 tons 3 qrs. 24 lb. of coal, how much is left in the truck?

3. An empty truck weighs 18 cwt. 3 qrs. If 9 bags each containing 2 cwt. 3 qrs. 16 lb. of potatoes are placed in it, what is the total weight of the truck and the bags of potatoes?

4. A truck when full of coal weighs 45 cwt. 1 qr. 13 lb., and the truck itself weighs 13 cwt. 2 qrs. 16 lb.; what is the amount of coal in seven such trucks?

5. A wagon contains 60 sacks; each sack weighs 7 lb. 10 oz. and contains 84 lb. of grain; what is the weight of the whole load in tons, etc.?

6. A butcher has 20 tons 19 cwt. 80 lb. of beef, and, after selling 20 tons 17 cwt. 4 lb. of it, the remainder is equally divided among 40 people; how much does each receive?

7. In my cellar I had $8\frac{1}{2}$ cwt. of coal. I bought $2\frac{1}{2}$ tons more, and the whole lasted me 91 days; how many lb. of coal did I burn daily?

8. An empty truck weighs a ton and a half, and when loaded with 99 packages of equal weight its weight is 5 tons 12 cwt. 23 lb.; what is the weight of each package?

9. A carrier delivers three parcels; the first weighs 12 cwt. 2 st. 8 lb., the second half as much again, and the third twice as much as the second; what is the total weight of the three?

10. How many pieces of cheese each weighing 3 lb. 6 oz. can be cut from a cheese weighing 43 lb. 14 oz.?

11. A quantity of fruit weighs 1 ton 9 cwt., and is supplied in boxes, each containing 12 lb. 11 oz.; how many boxes are there?

12. A cargo, weighing 484 tons 6 cwt. 1 qr., is landed from a vessel and placed in railway wagons, each of which holds 7 tons 13 cwt. 3 qrs.; how many wagons will be required?

13. A van loaded with parcels weighs 2 tons 2 cwt. The van itself weighs $18\frac{3}{4}$ cwt. and each parcel weighs 12 lb.; how many parcels are there in the van?

14. The weights of four men are 9 st. 9 lb., 7 st. 13 lb., 8 st. 12 lb., 7 st. 1 lb.; what is their average weight?

15. A farmer has 7 pigs. Their weights are 16 st., $16\frac{1}{2}$ st., 18 st. 11 lb., 21 st. 12 lb., 19 st. 6 lb., 22 st. 2 lb., and 23 st. 4 lb.; what is their average weight?

16. A boat's crew of 8 men weigh on the average 10 st. 6 lb. each. If one man is left out, the average of the others is 10 st. 7 lb. What is the weight of the man left out?

17. A provision dealer has 9 cheeses, each weighing 10 lb. 6 oz., and 7 cheeses, each weighing 9 lb. 12 oz.; what is their average weight?

18. The average weight of a crew of 8 men is 11 st. 6 lb. 6 oz. each. Three of them together weigh 36 st. 4 lb. What is the average weight of the others?

19. 10 lb. 1 oz. of tea is divided among 16 men and 14 women, so that each man's share is twice as much as a woman's share; how much does each man and each woman receive?

20. $1\frac{1}{4}$ cwt. of beef is divided among 10 men and 8 women, so that each woman receives half as much as a man; how much does each person receive?

M (3). Measures of Capacity.

114. The British unit of measure for liquids is the **Imperial standard gallon** (which is the space occupied by 10 lb. of water at a temperature of 62° Fahrenheit).

The fourth part of a gallon is called a **quart** (*qt.*), and one-half of a quart is called a **pint.**

It is well to remember that

A pint of pure water
Weighs a pound and a quarter.

EXERCISE 149.

1. How many half-pint bottles can be filled from a cask of ale containing four-and-a-half gallons?

2. A cow gives $1\frac{1}{2}$ gallons of milk daily; how many pints will she give in 24 days?

3. A child drinks a pint of milk every day; how many gallons, etc., is that for one year (365 days)?

4. At a school treat each of 512 children had a pint of tea; how many urns, each containing 2 gallons, were required?

5. A milkman sells the following quantities of milk: 30 gall. 3 qts., 29 gall. 1 pt., 17 gall. 2 qts. 1 pt., and 50 gall. 1 qt. 1 pt.; how much milk does he sell altogether?

6. A cistern contained $30\frac{1}{2}$ gall. of water; then 13 gall. 1 pt. was added, and again 7 gall. 3 qts.; how much water does the cistern now contain?

7. When 3 qts. $1\frac{1}{2}$ pt. of milk has been sold out of a can which contains 4 gall. 2 qts. 1 pt.; how much milk will be left?

8. A barrel of ale when full contains 36 gall.; how much ale has been drawn off, if there now remains 12 gall. 1 qt. $1\frac{3}{4}$ pt.?

9. If a barrel of oil contains 17 gall. 3 qts. 1 pt., how much oil is contained in 5 such barrels?

10. If a cow gives 1 gall. $2\frac{1}{2}$ qts. of milk daily, how much milk will she give at this rate in 24 days?

11. If a cow gives 19 gall. 1 qt. of milk in a week (7 days), how much milk does she give on an average each day?

12. If a household uses 30 gall. of milk in 16 days, what is the average daily consumption?

EXERCISE 150.

1. From a tank holding 10 qts. of water I fill a pint jug 15 times. How much water is left in the tank?

2. A tank holding $4\frac{1}{2}$ gallons is two-thirds full, how many pints are required to fill it?

3. A milkman mixes $34\frac{1}{2}$ gallons of milk with 4 gall. 3 qts. 1 pt., and then sells 28 gall. 2 qts.; how much milk has he left?

4. A cistern containing 19 gall. 2 qts. of water can be emptied by two taps. If we run off 3 gall. 1 pt. from one tap and 9 gall. 2 qts. from the other, how much water will remain in the cistern?

5. A cask contains 30 gall. 3 qts. of oil. If we fill a can three times from it, and the can holds 1 gall. 1 qt. 1 pt., how much will be left in the cask?

6. In a boarding school 13 gall. 1 pt. of milk was used every day for a week, and 15 gall. 1 qt. every day for the next week. How much milk was consumed in the fortnight?

7. I wish to fill a vessel which will hold 2 gall. 1 qt. 1 pt. of water; how many journeys must I make to it, if I can only carry 1 quart at a time?

8. From a cask of ale containing 26 gall. 1 qt. there is drawn off daily 3 qts. 1 pt.; in how many days will the cask be emptied?

9. What is the average content of water tanks holding the following quantities: 3 containing 51 gall. 1 qt. each, 2 containing 17 gall. 3 qt. 1 pt. each, and 3 containing 28 gallons each?

10. The average content of 16 barrels is $8\frac{1}{2}$ gallons and 12 of them have an average of 10 gallons; what is the average content of the remaining 4 barrels?

11. In 25 minutes 2523 gall. 2 qts. of water passed through a pipe. For 7 of those minutes the water flowed at the rate of 101 gall. 3 qts. per minute; what quantity per minute flowed uniformly through the pipe during each of the remaining 18 minutes?

12. I bottle off 126 gallons of cider into equal numbers of quart, pint and half-pint bottles; how many dozens of each size do I fill?

M (4). Measures of Length.

115. The British unit of length, by which we measure all others, is the **Imperial standard yard.**

The third-part of a yard is called a **foot** (*ft.*), and one-twelfth of a foot is called an **inch** (*in.*).

NOTE. The diameter of a halfpenny is 1 *inch.*

Engineers, builders, etc., express *feet* by ′, and *inches* by ″.

Thus 2 ft. 3 in. is written 2′ 3″.

EXERCISE 151.

1. A piece of string is $45\frac{1}{2}$ yards long. What is its length in inches?

2. A bridge is 765 yds. 2 ft. long. What is this measurement in feet?

3. How many yards are there in 74304 inches of string?

4. A telegraph wire is 32951 feet long, what is this length in yards and feet?

5. A horse is 16 hands high, what is this height in feet? (A hand = 4 inches.)

6. Find the total length of 3 pieces of stair-carpet if they measure respectively 20 yds. 2 ft. 11 inches, 7 yds. 6 inches and 3 yds. 1 ft. 9 inches.

7. What length of railing will be required for the fronts of three new houses? No. 1 will require 7 yards 1 ft. 10 inches; No. 2, 8 yards 2 ft.; and the front of No. 3 is equal in length to that of No. 1.

8. Tom is 1 ft. 11 in. shorter than his father, who is 5 ft. 9 in. in height; what is Tom's height?

9. A garden is 30 yds. 1 ft. 4 in. long and 12 yds. 2 ft. 8 in. wide. If I walk round it once what distance do I walk?

10. By how many yards, etc., does the length exceed the width of the garden in 9?

11. What length of wall must be added to 31 yards 1 ft. 10 inches to make 63 yards?

12. What is the total length of 25 rolls of carpet if each roll measures 25 yds. 10 in.?

13. To enclose a garden allotment a man requires 19 yds. 2 ft. 7 in. of wire; how much wire will be required to enclose 40 similar plots?

14. A piece of tape 76 yds. 8 in. in length is divided equally amongst 56 girls; how much does each girl receive?

15. A snail crawls $292\frac{1}{2}$ yards in 30 hours; at this rate, how far does it get every hour on an average?

EXERCISE 152.

1. A draper has a roll of cloth measuring 29 yds. 6 in. He cuts off, and sells, two pieces measuring 8 yds. 1 ft. and 5 yds. 2 ft. 7 in.; how much cloth has he left unsold?

2. A coat takes $3\frac{1}{2}$ yds. of cloth, the waistcoat $\frac{3}{4}$ yd. and the trousers 2 yds. 9 in. After making 12 suits of clothes how much cloth will be left out of a roll measuring 7 dozen yards?

3. The wheel of a car is $4\frac{1}{2}$ feet round. If it turns 160 times every minute, how many yards does the car travel in $4\frac{1}{2}$ minutes?

4. Tom walks 90 yards in a minute and Harry walks 75 yards in a minute. If they start at 10 a.m. to walk from A to B, how far are they apart at $\frac{1}{4}$ to 11 a.m.?

5. A man's step being 2 ft. 6 in., how many steps will he take in going 12 yds. 1 ft. 6 in.?

6. How many pieces of cloth, each 2 ft. $5\frac{1}{2}$ in. long, can be cut from a roll measuring 59 yards?

7. Twenty-three shrubs are planted in a row 2 yds. 2 ft. 6 in. apart; how long is the row, there being one shrub at the beginning and one at the end of the row?

8. There are 5 brothers. Their heights are respectively 5 ft. 11 in., 4 ft. 8 in., 4 ft. 6 in., 3 ft. 11 in. and 4 ft. 4 in.; what is their average height?

9. There are 12 boys in a class. Four of them are each 4 ft. $9\frac{1}{2}$ in. high, 3 more are exactly 5 ft., and the remainder are 5 ft. 2 in.; what is the average height?

10. What is the average length of 8 streets which are respectively 417, 206, 319, 408, 371, 209, 428 and 567 yards in length?

11. In 4 steps I can walk 3 yds. 2 ft. 6 in.; at this rate, how far can I go in 72 steps?

12. If 3 dresses require $22\frac{1}{2}$ yds. of material, what length of stuff will be required for 18 dresses?

116. In measuring land, surveyors use a **chain** which is 22 yards long (the length of a cricket pitch). This chain is divided into 100 equal parts, each of which is called a **link.**

100 **links** = 22 *yards* = 1 **chain.**
10 **chains** = 220 *yards* = 1 **furlong.**
8 **furlongs** = 1760 *yards* = 1 **mile.**

EXERCISE 153.

1. A road is $4\frac{1}{2}$ miles long; what is its length (i) in yards, and (ii) in chains?

2. A road is 1565 chains long; what is its length in miles?

3. A road is 18 miles 5 furlongs 5 chains long; what is its length in yards?

4. If telegraph posts are placed at a distance of 55 yards apart, and along a road from X to Y there are 321 posts (there being one at the beginning and one at the end of the road), how many miles is it from X to Y?

5. How many miles off is a thunder-cloud when a flash of lightning is seen 15 seconds before the thunder is heard. (Sound travels at a velocity of 1130 feet per second.)

6. A boy walked 935 yds. 1 ft. 10 in. How much further must he walk to complete one mile?

7. The real distance from P to Q is three-quarters of a mile; a surveyor measures the distance and makes the length $60\frac{1}{4}$ chains; what is the error in yards?

8. The four sides of a field measure: 32 chains 18 yds., 29 chains 12 yds., 17 chains 10 yds., 35 chains 15 yds.; what is the total distance round the field?

9. From A to D, viâ B, the distance is 45 chains, and, viâ C, the distance is 4 furlongs 20 yds.; which is the longer route, and by how many yards?

10. If a motor-car in a race runs 78 miles 1440 yds. in an hour; how far will it run in 3 hours, assuming there is no break-down?

11. A road is 224 miles 4 fur. 100 yds. long, and one-sixth of it is being repaired; what length is this?

12. How many times will the driving wheel of an engine 22 feet in circumference turn round in travelling 57 miles?

13. If a carriage wheel makes 78 revolutions in travelling 1 furlong 170 yds., how many revolutions will it make in travelling a mile?

14. From P to Q is 4 miles 319 yards. In how many steps will a man, who covers $\frac{3}{4}$ yard at a step, walk the whole distance?

15. In a mile race Jones gained on Smith at the rate of 2 ft. 3 in. in 110 yards; how far was Smith behind when Jones was at the winning post?

16. If a train travels at the rate of 45 miles an hour, how many feet does it travel in a second?

17. An electric tram-car travels 220 yards in 36 seconds; how many miles per hour is the car travelling?

18. The telegraph posts on a railway are 66 yards apart, and a train was observed to pass from one to another in 3 seconds; how many miles per hour is the train travelling?

M (5). Problems.

117. The following are further examples of problems involving measures.

EXERCISE 154.

1. What is the price of beef per lb., when 4 cwt. may be bought for 16 guineas?

2. The price of coal is raised 1*d.* per cwt.; what will be the extra cost of $13\frac{1}{2}$ tons?

3. If 1 lb. of cheese cost $9\frac{1}{2}d.$, how many hundredweight can be bought for £9. 19*s.* 6*d.*?

4. A shopkeeper buys $19\frac{1}{2}$ cwt. of currants at £1. 12*s.* 8*d.* per cwt., and sells them at 4*d.* per lb.; how much does he gain?

5. A debt of £3. 17*s.* 6*d.* is paid in farthings; how many are required? And what will be their total weight, if 8 farthings weigh 1 oz.?

6. A man agreed to buy a haystack, paying £10 for each wagon-load. He found his wagon held 1 ton 3 cwt. 2 qrs., and that the whole stack weighed 10 tons 11 cwt. 2 qrs.; how much had he to pay?

7. If a pint of milk costs 2*d.*, how many gallons can be bought for £1. 16*s.*?

8. If a milkseller buys 12 gallons 2 quarts of milk from a farmer for 16*s.* 8*d.*, how much is the milk per quart?

9. A milkseller buys 80 gallons of milk at 9*d.* per gallon and sells it at three-halfpence per pint; how much does he gain?

10. A man's wages being 3*s.* 5*d.* per day, how much will he earn from March 22nd to July 4th, inclusive of both days, but deducting 15 Sundays?

118. The following are further examples of problems solved by the **Unitary Method.**

EXAMPLE. *If 2 cwt. 24 lb. of cheese cost £8. 5s. 4d., find the cost of 3 cwt. 10 lb.*

By reduction, 2 cwt. 24 lb. = 248 lb.; 3 cwt. 10 lb. = 346 lb.; £8. 5*s.* 4*d.* = 1984*d.*

The cost of 248 lb. = 1984*d.*;

$\therefore$ ..... 1 lb. $= \frac{1984d.}{248} = 8d.$;

$\therefore$ ..... 346 lb. $= 8d. \times 346 = 2768d. = 230s.\ 8d.$

$= \underline{\textbf{£11. 10s. 8d.}}$

EXERCISE 155.

1. If 5 lb. 10 oz. of chocolate cost 11*s.* 3*d.*, what is the cost of 4 lb. 11 oz.?

2. What is the cost of 17 lb. of toffee, if 2 lb. 2 oz. cost 4*s.* 3*d.*?

3. If $2\frac{1}{2}$ cwt. of salt cost £2. 18*s.* 4*d.*, what will 42 lb. cost?

4. What is the cost of 3 cwt. 42 lb. of butter, if 33 lb. cost £1. 18*s.* 6*d.*?

5. If 4 cwt. of cheese cost £5. 16*s.* 8*d.*, what is the cost of 7 tons 5 cwt.?

6. If 15 yards of netting cost £1. 13*s.* 9*d.*, how much will 20 yards 1 foot cost?

7. If 19 gall. 3 qts. of milk cost £1. 6*s.* 4*d.*, what is the cost of 12 gall. 2 qts.?

8. A man spends £51. 10*s.* 3*d.* in 3 weeks and 5 days; how much is that per week? (1 week = 7 days.)

9. If butter can be bought for 1*s.* 4*d.* per lb., how many cwt. can be bought for £11. 4*s.*?

10. If potatoes can be bought at 7*d.* per stone, how many tons can be bought for £18. 13*s.* 4*d.*?

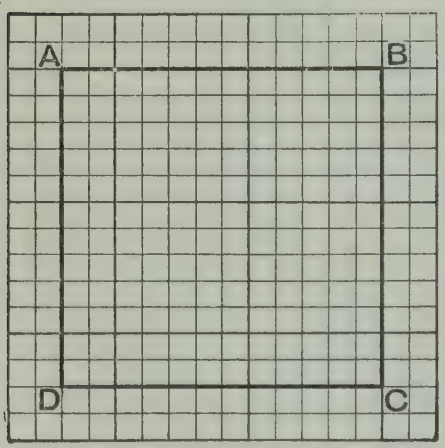
N. MEASURES OF AREA AND VOLUME.

N (1). General Measures of Area, or Square Measure.

119. The **area** of a figure is the amount of surface included within the lines which bound it.

120. The British unit of area is a square which has each of its sides equal to the unit of length, one yard. It is therefore called a **square yard**

121. In the margin is drawn on squared paper a square ABCD, such that each side contains 12 small units of length. Each of the small squares, whose sides are equal to this unit of length, is a unit of area. Thus the number of units of area in the square ABCD is equal to the number of small squares which it contains.



It will be seen that the whole square is made up of 12 strips, and that each strip contains 12 small squares; hence the number of small squares in the whole area $= 12 \times 12 = 12^2$. Thus a square, whose side contains 12 units of length, contains 12^2 or 144 units of area.

If the pupil will draw a square whose side contains 3 units of length, he can satisfy himself in the same way that this square contains 3×3 small squares, *i.e.* 3^2 or 9 units of area. Whatever lengths he may take, he will find that the number of units of area in a square is always equal to the square of the number of units of length in one side.

122. It is easy with the aid of the above principle to deduce a table of Square measures from a table of Linear measures.

Thus 12^2, or 144, **square inches** = 1 **square foot**,

and 3^2, or 9, **square feet** = 1 **square yard.**

EXERCISE 156.

Check your results by calculation.

On squared paper draw squares having sides as given below, and find their area by counting squares:

1. 2 inches. 2. 4 inches. 3. 6 inches. 4. 8 inches.
5. 10 inches. 6. 15 inches. 7. 20 inches. 8. 22 inches.

EXERCISE 157.

1. How many square inches are there in 5 sq. ft. of canvas?

2. How many square feet are there in 5 sq. yds. of paper?

3. The area of a carpet is 17 sq. yds. 8 sq. ft. 71 sq. in., how many square inches is this?

4. The area of a floor is 32146 sq. in.; how many sq. feet and sq. inches is this?

5. The area of a garden is 474 sq. ft.; how many sq. yards and sq. feet is this?

6. The area of a floor is 17875 sq. in.; how many sq. yards, sq. feet and sq. inches is this?

7. The areas of 4 pieces of linoleum are respectively 8 sq. yds. 8 sq. ft. 112 sq. in., 7 sq. yds. 6 sq. ft. 106 sq. in., 3 sq. yds. 7 sq. ft. 84 sq. in., 4 sq. yds. 7 sq. ft. 96 sq. in.; what is the total area?

8. The area of the floor of a room is 45 sq. yds. 5 sq. ft. 80 sq. in., and the area of the floor of another room is 35 sq. yds. 7 sq. ft. 100 sq. in.; what is the difference in area?

9. The area of a piece of canvas is 18 sq. yds. 7 sq. ft. 108 sq. in.; what is the total area of 6 such pieces?

10. A piece of canvas whose area is 49 sq. yds. 4 sq. ft. 20 sq. in. is divided into ten equal parts; what is the area of each part?

11. How many pieces of turf, each measuring 6 sq. ft. 36 sq. in., are required for a lawn 347 sq. yds. 2 sq. ft. in area?

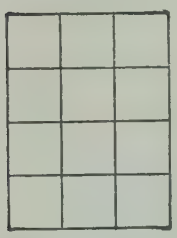
12. A gardener digs the following numbers of square feet of ground per hour, in eight hours: 38, 69, 41, 28, 37, 42, 57, 41; what is the average area which he digs per hour?

N (2). Measures of Rectangular Areas.

123. A **rectangle** is a four-sided figure, whose opposite sides are parallel, and whose angles are right angles.

The floor, ceiling and walls of a room are generally rectangular.

124. Suppose that a rectangular surface, such as the floor of a room, is 4 yards long and 3 yards wide; we can divide the sides into 4 equal parts and 3 equal parts respectively, each part being 1 yard. If now through the points of division we draw lines parallel to the sides in the manner shown in the margin, it is clear that we divide the surface into 4 rows of smaller areas, and that there are 3 of the smaller areas in each row. Each of the smaller areas is a square yard, and therefore the area of the surface is 3×4 square yards.



Hence, to find the number of *square yards* in a rectangular area, we multiply the number of *yards* in the length by the number of *yards* in the breadth. If the length and breadth be given in *feet* (or *inches*), the product of the two numbers will give the number of *square feet* (or *square inches*) in the area.

EXERCISE 158.

Check your results by calculation.

On squared paper draw the rectangles, the measurements of which are given below, and find their area by counting squares:

1. 4 in. by 2 in.	2. 5 in. by 3 in.	3. 7 in. by 4 in.
4. 6 in. by 5 in.	5. 8 in. by 9 in.	6. 13 in. by 7 in.
7. 10 in. by 12 in.	8. 11 in. by 13 in.	9. 12 in. by 11 in.

125. Using the words *area*, *length* and *breadth*, or the letters A, l and b, to denote the *number of units* in each respectively, the relation between them may be stated thus:

$$\text{Area} = \text{length} \times \text{breadth}, \text{ or } A = l \times b;$$

$$\therefore \text{ length} = \frac{\text{area}}{\text{breadth}}, \qquad \text{or } l = \frac{A}{b},$$

$$\text{and} \qquad \text{breadth} = \frac{\text{area}}{\text{length}}, \qquad \text{or } b = \frac{A}{l};$$

hence, if any two of the three are known, the third can be determined.

EXAMPLE 1. *Find the area of the floor of a room, 15 ft. in length and 12 ft. in breadth.*

$$\text{Area of floor} = 15 \times 12 \text{ sq. ft.}$$
$$= \underline{\mathbf{180 \text{ sq. ft.}}}$$

EXAMPLE 2. *How many feet wide is a mat, which is 23 sq. ft. 72 sq. in. in area and 7 ft. 10 in. in length?*

$$\text{Area} = (23 \times 144 + 72) \text{ sq. in.} = 3384 \text{ sq. in.},$$
$$\text{and length} = (7 \times 12 + 10) \text{ in.} = 94 \text{ in.};$$
$$\therefore \text{ width} = \frac{3384}{94} \text{ in.} = 36 \text{ in.} = \underline{\mathbf{3 \text{ ft.}}}$$

EXAMPLE 3. *How many turfs, 3 ft. by 2 ft., will be required for a lawn, 15 ft. by 12 ft.?*

$$\text{Area of lawn} = 15 \times 12 \text{ sq. ft.} = 180 \text{ sq. ft.},$$
$$\text{and area of one turf} = 3 \times 2 \text{ sq. ft.} = 6 \text{ sq. ft.};$$
$$\therefore \text{ number of turfs} = \frac{180 \text{ sq. ft.}}{6 \text{ sq. ft.}} = \underline{\mathbf{30.}}$$

EXERCISE 159.

Find the area of the floor of a room whose measurements are

1. 15 ft. by 10 ft.
2. 14 ft. by 12 ft.
3. 16 ft. by 14 ft.
4. 26 ft. by 25 ft.
5. 49 ft. by 38 ft.
6. 67 ft. by 39 ft.

Find the other dimension, having given

7. Area, 225 sq. ft.; breadth, 15 ft.
8. Area, 252 sq. ft.; breadth, 12 ft.
9. Area, 525 sq. ft.; length, 25 ft.
10. Area, 360 sq. ft.; length, 24 ft.

11. A classroom is 18 ft. long and 16 ft. wide; how many pupils can be accommodated, allowing 8 sq. ft. to each pupil?

12. How many turfs, each 3 ft. in length and 1 ft. in width, will be required for a plot of ground 120 ft. long and 100 ft. wide?

13. How many turfs, 3 ft. by 2 ft., will be required for a lawn 24 ft. long and 21 ft. wide, and what will they cost at 6*d.* each?

14. What is the cost of covering a room, 20 yds. long and 4 yds. wide, with linoleum at 4*s.* 9*d.* per sq. yard?

15. What will be the cost of varnishing a floor 30 ft. long and 27 ft. wide at 6*d.* per sq. yard?

EXERCISE 160.

1. Find, in square feet, the area of a board 24 in. by 30 in.

2. Find, in sq. feet and sq. inches, the area of a piece of canvas 42 in. by 36 in.

3. Find, in sq. feet and sq. inches, the area of a piece of linoleum 32 in. by 22 in.

4. Find, in square yards, the area of a paddock 71 ft. long and 63 ft. wide.

5. Find, in sq. yards, the area of a corridor 6 ft. wide and $46\frac{1}{2}$ ft. long.

6. Find, in sq. yards and sq. feet, the area of a table 8 ft. long and $2\frac{1}{2}$ ft. broad.

7. Find, in sq. yards and sq. feet, the area of a garden 29 ft. long and 25 ft. wide.

8. Find, in sq. yards and sq. feet, the area of an allotment 38 ft. long and 16 ft. wide.

9. Find, in feet, the breadth of the top of a table whose area is 11 sq. ft. and whose length is 3 ft. 8 in.

10. Find, in feet, the length of a carpet whose area is 27 sq. ft. 72 sq. in. and whose breadth is 5 ft. 6 in.

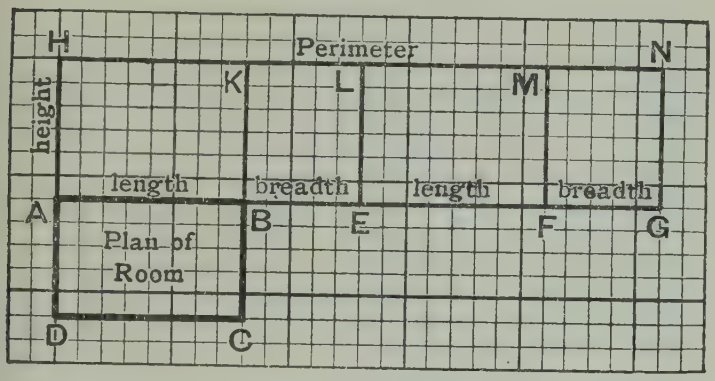
126. To find the area of a rectangular surface we multiply the number of units in the length by the number of units in the breadth, and to find the area of a wall we multiply the number of units in the length of the wall by the number of units in the height.

In such rooms as we can consider there are 4 walls, those which are opposite to one another being equal in area.

EXAMPLE 1. *Find the area of the walls of a room,* 16 *ft. long,* 10 *ft. wide and* 12 *ft. high.*

In the figure in the margin we take a side of a small square to represent 2 feet. ABCD then represents the floor of the room and AH the height; ABKH will represent the area of the wall whose lower

edge is AB; also BE, EF and FG are equal respectively to BC, CD and DA. Consequently BELK, EFML and FGNM represent the areas of the other three walls, and HAGN represents the area of all the four walls.



Thus, the area of the four walls
= area of AGNH
= AG × AH
= 2(AB + BC) × AH
= 2 × 26 × 12 sq. ft.
= **624 sq. feet.**

127. It will be evident that if l, b and h represent the number of units of length in the length, breadth and height respectively, then

the area of the four walls $= 2(l + b) \times h$ units,

the area of the floor, or ceiling $= l \times b$ units,

and the distance round the room $= 2(l + b)$ units.

The distance round the room is called the **perimeter.**

NOTE. The expression, $2(l + b) \times h$, gives the area of the walls, including that occupied by windows, doors and fireplaces.

EXAMPLE 2. *Find the area of the walls of a room* 12 *ft.* 6 *in. long,* 10 *ft.* 6 *in. wide and* 10 *ft. high.*

$$\begin{aligned}\text{Area} &= 2(l + b) \times h \text{ sq. ft.}\\ &= 2(12\tfrac{1}{2} + 10\tfrac{1}{2}) \times 10 \text{ sq. ft.}\\ &= 2 \times 23 \times 10 \text{ sq. ft.}\\ &= \underline{\textbf{460 sq. ft.}}\end{aligned}$$

EXAMPLE 3. *If in the room in Ex.* 2 *there is one door,* 7 *ft. by* 3 *ft., and one window,* 8 *ft. by* $4\frac{1}{4}$ *ft., what would it cost to paint the walls at* 2*s. per sq. yard?*

Area of walls $= 460$ sq. ft.,

area of door $= 7 \times 3$ sq. ft. $= 21$ sq. ft.,

and area of window $= 8 \times 4\frac{1}{4}$ sq. ft. $= 34$ sq. ft.;

$\therefore$ area to be painted $= (460 - 21 - 34)$ sq. ft. $= 405$ sq. ft. $= 45$ sq. yds.;

$\therefore$ cost of painting $= 2s. \times 45 = \underline{\textbf{£4. 10s.}}$

EXERCISE 161.

Find, in sq. feet, the area of the walls of a room whose dimensions are

	Length.	Breadth.	Height.
1.	20 ft.	12 ft.	10 ft.
2.	19 ft.	16 ft.	9 ft.
3.	24 ft.	18 ft.	15 ft.
4.	18 ft.	15 ft.	13 ft.
5.	12 ft. 8 in.	10 ft. 4 in.	10 ft.
6.	21 ft. 5 in.	18 ft. 7 in.	10 ft.
7.	19 ft. 3 in.	15 ft. 9 in.	12 ft.
8.	14 ft. 6 in.	11 ft. 6 in.	10 ft.

9. A room is 22 ft. long, 17 ft. wide and 12 ft. high; what would it cost to paint the walls at 1*s.* 5*d.* per sq. yard?

10. A room is 16 ft. 6 in. long, 13 ft. 6 in. broad and 10 ft. high; what would it cost to paint the walls at 9*d.* per sq. yard?

11. A room is 15 ft. long, 10 ft. broad and 9 ft. high; what would it cost to paint the walls and ceiling at 1*s.* 9*d.* per sq. yard?

12. A room is 15 ft. long, 15 ft. broad and 15 ft. high; deducting a door, 6 ft. high and 4 ft. wide, and a window, 7 ft. high and 3 ft. wide, what would it cost to paint the walls at 2*s.* 6*d.* per sq. yard?

128. In the manner explained in Art. 121, we can deduce a table of Square measures, for use in measuring land, from the table of Linear measures (Art. 116).

Thus

10000 (or 100^2) **sq. links** = 484 (or 22^2) *sq. yards* = 1 **square chain**;

10 **sq. chains** = 4840 *sq. yards* = 1 **acre**;

and 640 **acres** = 6400 (or 80^2) *sq. chains* = 1 **square mile.**

Large areas, such as the areas of countries, are expressed in *square miles*; smaller areas, such as the areas of farms, etc., are expressed in *acres.*

EXERCISE 162.

Find the acreage of a piece of land whose length and breadth are respectively

1. 220 yds. and 220 yds.
2. 330 yds. and 176 yds.
3. 440 yds. and 154 yds.
4. 363 yds. and 240 yds.
5. 484 yds. and 360 yds.
6. 605 yds. and 400 yds.

7. My garden covers three-quarters of an acre; what is the area in sq. yards?

8. A surveyor gives the area of an allotment as 5 sq. chains; what is the area in square yards?

9. The area of a farm is 2420 sq. chains; what is the area in acres?

10. The area of a certain island is $149\frac{1}{2}$ sq. miles; what is the area in acres?

11. The area of a certain county is 74240 acres; what is the area in square miles?

12. The area of a rectangular piece of ground is 6 acres, and the length of one side is 165 yards; how many yards long is the adjacent side?

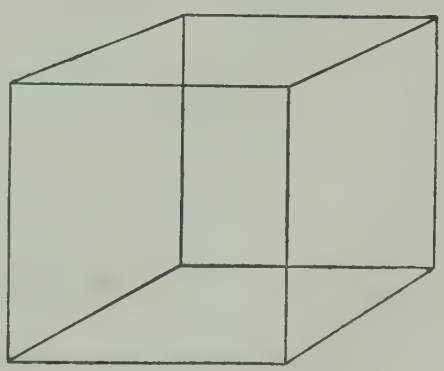
13. A rectangular piece of land, 25 acres in area, is to be cut up into smaller plots each 250 yards by 121 yards; how many plots can be made?

N (3). Measures of Volume and Space.

129. The **volume** of a body is the whole amount of space which the body occupies.

130. The principal measure of volume is a solid similar to that described in the margin. Its length, breadth and height are each *one yard*, and the solid is called a **cubic yard.**

The drawing in the margin represents an exact **cubic inch**, its length, breadth and height being each *one inch.*



If we arrange 144 *cubic inches* in 12 rows, each containing 12 cubes, we form a solid, one inch high, with a top surface whose area is a square foot.

If again we form 12 such solids, and place them one above another, we build up a cube whose length, breadth and height are each one foot. It will therefore be a **cubic foot**; and since, in making it, we have used 12×144 cubes, each of whose sides is one inch, it is clear that

12^3, or 1728, **cubic inches** = 1 **cubic foot.**

Similarly it may be shown that

3^3, or 27, **cubic feet** = 1 **cubic yard.**

EXERCISE 163.

1. A block of stone contains 9 c. ft. 722 c. in.; how many cubic inches of stone does it contain?

2. A builder has 102 c. yds. 9 c. ft. of gravel; how many cubic feet of gravel has he?

3. A labourer has excavated 2943 c. feet of earth; how many cubic yards has he excavated?

4. Another labourer has dug out 32832 c. inches of earth; how many cubic feet has he dug out?

5. A pit contains 138297 c. inches of gravel; how many cubic yards, etc., is this?

6. What is the total solid content of six blocks of stone containing respectively 10 c. yds. 25 c. ft., 50 c. yds. 180 c. in., 26 c. yds. 26 c. ft., 2 c. yds. 1080 c. in., 24 c. ft. 364 c. in., 3 c. yds. 7 c. ft. 104 c. in.?

7. What is the difference in volume between the cubic content of a block of stone containing 10 c. yds. 16 c. ft. 1000 c. in., and a block containing 9 c. yds. 20 c. ft. 1200 c. in.?

8. What is the cubic content of 9 blocks of marble, each containing 2 c. yds. 21 c. ft. 1000 c. in.?

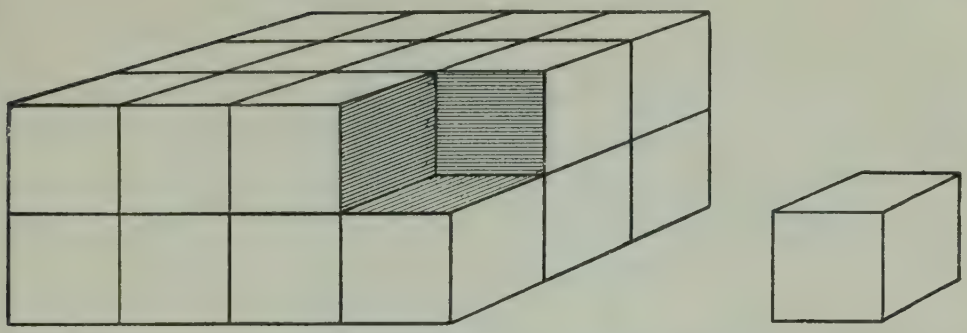
9. If the total volume of 6 blocks of granite of equal solid content is 23 c. yds. 20 c. ft. 960 c. in., what is the solid content of each block?

10. How many bricks are there in a stack measuring 5 c. yards, supposing each brick occupies 108 c. inches?

131. A **rectangular solid** has six faces, each of which is a rectangle.

A brick and an ordinary bar of soap are familiar examples.

132. Suppose that a solid is 4 feet long, 3 feet wide and 2 feet high; we can divide the edges respectively into 4, 3 and 2 equal parts, each of which parts will be 1 foot. If we draw planes through the points of division as in the diagram, the solid will be separated into a number of smaller blocks, each of which is a *cubic foot*; and since there are two layers, in each of which there are 3×4 blocks, we see that there are $2 \times 3 \times 4$ blocks altogether, and that the solid therefore contains $2 \times 3 \times 4$ cubic feet.



We can thus find the **volume**, or **cubical content**, of any rectangular solid by expressing the length, breadth and height in terms of a common unit of measurement, and multiplying the three numbers together. The product will give the volume of the solid in terms of the *corresponding* unit in cubic measure.

EXERCISE 164.

What is the volume of cubical blocks, with edges as given below?

1. 2 inches. 2. 4 inches. 3. 6 inches. 4. 8 inches.

What is the cubical content of rectangular boxes, the measurements of which are given below?

5. 4 in. by 2 in. by 1 in.
6. 5 in. by 3 in. by 2 in.
7. 7 in. by 4 in. by 3 in.
8. 6 in. by 5 in. by 4 in.
9. 8 in. by 9 in. by 6 in.
10. 13 in. by 7 in. by 2 in.
11. 10 in. by 12 in. by 3 in.
12. 14 in. by 10 in. by 6 in.

133. Using the words *volume*, *length*, *breadth* and *height*, or the letters V, l, b and h, to denote the *number of units* in each respectively, the relation between them may be stated thus:

$$\text{Volume} = \text{length} \times \text{breadth} \times \text{height}, \text{ or } V = l \times b \times h;$$

$$\therefore \text{length} = \frac{\text{volume}}{\text{breadth} \times \text{height}}, \qquad \text{or } l = \frac{V}{b \times h},$$

$$\text{breadth} = \frac{\text{volume}}{\text{length} \times \text{height}}, \qquad \text{or } b = \frac{V}{l \times h},$$

$$\text{and} \quad \text{height} = \frac{\text{volume}}{\text{length} \times \text{breadth}}, \qquad \text{or } h = \frac{V}{l \times b};$$

hence, if any three of the four are known, the fourth can be determined.

EXERCISE 165.

Find, in cubic feet, the capacity of a room whose dimensions are

	Length.	Breadth.	Height.		Length.	Breadth.	Height.
1.	20 ft.	12 ft.	10 ft.	2.	19 ft.	16 ft.	9 ft.
3.	24 ft.	18 ft.	15 ft.	4.	18 ft.	15 ft.	13 ft.
5.	13 ft.	13 ft.	13 ft.	6.	15 ft.	15 ft.	15 ft.

7. What is the height of a room which is 12 ft. long, 10 ft. wide and contains 1440 c. ft. of air?

8. A rectangular cistern 2 yds. 2 ft. long and 2 yds. wide contains 192 c. ft. of water; what is the depth of the water?

9. How many cubic yards of earth were removed in excavating the foundations of a house 72 ft. long, 45 ft. broad and 4 ft. deep?

10. How many persons can be accommodated in a hall whose height is 15 ft. and whose floor is 60 ft. by 40 ft., allowing 100 cubic feet of air space for each person?

11. A swimming bath is 40 yards long and 10 yards wide; how many cubic feet of water will be required to fill it to a depth of 4 ft.?

12. The floor of a room measures 25 ft. by 24 ft. and can accommodate 150 persons, allowing 80 cubic feet of air space to each person; what is the height of the room?

P. DECIMALS AND DECIMAL MEASURES.

P (1). Introduction.

134. In the METRIC SYSTEM the various weights and measures are so named that the relation which any denomination bears to another can easily be seen.

135. The unit measures of length, weight and capacity, are called respectively a **metre**, a **gram** and a **litre**.

136. The measures greater than the unit are respectively 10, 100, 1000 times the unit, and the words denoting these measures are formed by attaching the **GREEK** prefixes **deca-**, **hecto-**, **kilo-**, to the name of the unit.

Thus **deca-** denotes **10** times the unit,

hecto-..... **100** times .....,

and **kilo-** ..... **1000** times .....

137. The measures less than the unit are derived from the unit by dividing it into 10, 100, 1000 parts respectively.

The words denoting these sub-divisions are formed by attaching the **LATIN** prefixes **deci-**, **centi-**, **milli-** to the name of the unit.

Thus **10** times a **deci-** is equal to the unit,

100 times a **centi-**.....,

and **1000** times a **milli-**.....

Or **deci-** denotes a **tenth**-part; **centi-**, one **hundredth**-part; and **milli-**, one **thousandth**-part of the unit.

138. The initial *capital* letters are used as abbreviations for the Greek prefixes. Thus D stands for *deca-*, H for *hecto-* and K for *kilo-*. The initial *small* letters are used as abbreviations for the Latin prefixes. Thus *d* stands for *deci-*, *c* for *centi-* and *m* for *milli-*.

The initial *small* letters of the words *metre*, *gram*, and *litre*, namely, *m*, *g*, *l*, are used to denote these units respectively.

P (2). Decimal Measures of Length.

139. The metric unit of length is called a **metre**, and is a little more than 39 inches:

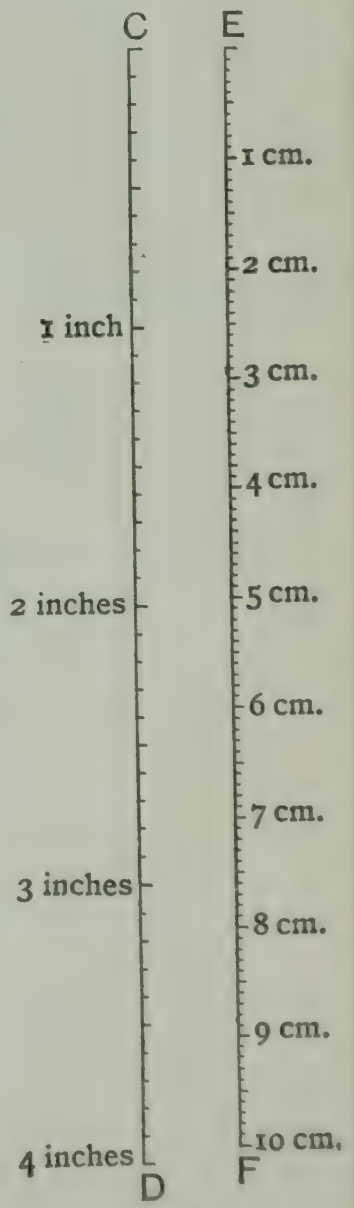
10 **millimetres** (mm.) = 1 **centimetre** (cm.).
10 **centimetres** = 1 *decimetre* (dm.).
10 *decimetres* = 1 **metre** (m.).
10 **metres** = 1 *decametre* (Dm.).
10 *decametres* = 1 *hectometre* (Hm.).
10 *hectometres* = 1 **kilometre** (Km.).

Long distances are usually expressed in kilometres and metres; ordinary lengths in metres and centimetres; very small lengths in centimetres and millimetres.

The *decimetre*, *decametre* and *hectometre* are little used.

ROUGH EQUIVALENTS:

5 cm. = 2 inches (nearly);
20 m. = 1 chain (or the length of a cricket pitch);
1 Km. = 5 furlongs.



140. The straight line CD in the margin is 4 **inches** in length, and each inch is divided into 10 equal parts.

The straight line EF is 10 **centimetres** long, and each centimetre is divided into 10 equal parts called **millimetres.**

141. If a line is measured by means of the scale EF in the margin, it may measure (say) 8 centimetres exactly. If it measures 8 centimetres and 5 millimetres over, its length may then be written in any of the forms

8 cm. 5 mm., or 85 mm., or 8·5 cm.

The last expression is read "*eight point five centimetres.*"

If a line measures 1 dm. 1 cm. 1 mm., its length may be written

1·11 dm., or 11·1 cm., or 111 mm.

The first expression is read "*one point one one decimetres,*" not "*one point eleven decimetres.*"

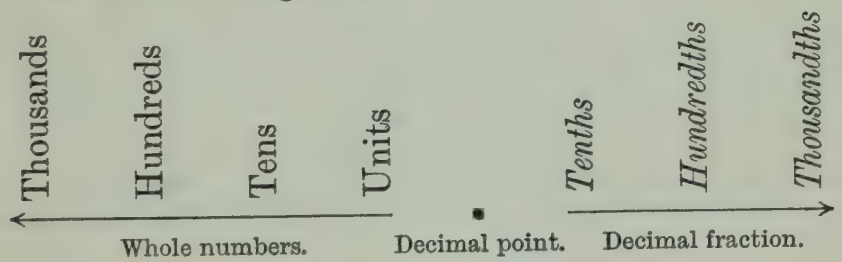
142. It is evident from these examples that the units' place must be distinguished in some way. This is done by placing a dot immediately after it. Thus 1·11 dm. means that the figure to the *left* of the dot denotes 1 decimetre, that the first figure to the *right* represents 1 *tenth* of a decimetre and that the next represents 1 *hundredth* of a decimetre. In 11·1 cm. the figures to the left of the point denote 11 centimetres and the first figure to the *right* represents 1 *tenth* of a centimetre.

143. An expression of the form ·11 is called a **DECIMAL**, and the point is called a **decimal point.**

Noughts may be placed at, or omitted from, the *right* of a decimal without changing its value. For example, 1·10 dm. is merely a longer way of writing 1·1 dm.; each denotes 1 dm. 1 cm.

It is quite otherwise however with 0's which occur immediately after the decimal point. For example, ·110 m. = 1 dm. 1 cm., but ·011 m. = 1 cm. 1 mm.

144. The positions and values of the figures may be understood from the following plan:



145. With this notation we can express a quantity by the metric system at once in terms of any denomination.

Thus,

$$4 \text{ m. } 7 \text{ dm. } 6 \text{ cm. } 8 \text{ mm.} = (4 + \tfrac{7}{10} + \tfrac{6}{100} + \tfrac{8}{1000}) \text{ metres}$$
$$= 4{\cdot}768 \text{ metres.}$$

Conversely,

$$4{\cdot}768 \text{ Km.} = 4 \text{ Km.} + \tfrac{7}{10} \text{ of a Km.} + \tfrac{6}{100} \text{ of a Km.} + \tfrac{8}{1000} \text{ of a Km.}$$
$$= 4 \text{ Km. } 7 \text{ Hm. } 6 \text{ Dm. } 8 \text{ m.}$$
$$= 4 \text{ Km. } 768 \text{ m.}$$

EXERCISE 166.

Express in centimetres:

1. 6 cm. 5 mm.
2. 13 cm. 6 mm.
3. 10 cm. 3 mm.
4. 1 dm. 5 cm. 2 mm.
5. 3 dm. 8 cm. 0 mm.
6. 7 dm. 0 cm. 8 mm.

Express in metres, decimetres, etc.:

7. 3·628 m.	8. 40·72 dm.	9. 354·6 cm.
10. 1862 cm.	11. 365 dm.	12. 4659 mm.

Express in Km. and decimals of a Kilometre:

13. 8015 m.	14. 5705 m.	15. 3450 m.
16. 15075 m.	17. 10507 m.	18. 16750 m.

Express in metres:

19. 4·765 Km.	20. 5·076 Km.	21. 6·708 Km.
22. 5·83 Km.	23. 9·8 Km.	24. 7·08 Km.

P (3). Decimal Measures of Weight.

146. The metric unit of weight is called a **gram.** It is equal to the weight of the water contained in a cubic centimetre, *i.e.* in a cubic vessel each of whose edges is one centimetre long. The names and the relations of the multiples and sub-divisions of the gram are similar to those of the metre.

Ordinary weights are usually expressed in kilograms (often termed *kilos*) and grams; very small weights in grams, centigrams and milligrams.

ROUGH EQUIVALENTS:

5 Kg. = 11 lb.; 1000 Kg. = 1 ton (nearly).

EXERCISE 167.

Express in centigrams:

1. 8 cg. 5 mg.	2. 16 cg. 8 mg.	3. 9 dg. 5 cg. 6 mg.

Express in grams, decigrams, etc.:

4. 7·306 g.	5. 371·4 cg.	6. 73·04 dg.

Express in Kg. and decimals of a Kilogram:

7. 4265 g.	8. 13056 g.	9. 20860 g.

Express in grams:

10. 5·875 Kg.	11. 4·73 Kg.	12. 7·4 Kg.

P (4). Decimal Measures of Capacity.

147. The metric unit of capacity is called a **litre.** It is equal to the volume of a cubic decimetre, *i.e.* of a rectangular vessel each of whose edges is 1 decimetre, or 10 cm., in length.

Since a litre is equal to a cubic decimetre, *i.e.* to 1000 cubic centimetres, it follows that

the weight of 1 litre of water = 1000 grams = 1 kilogram.

The centilitre, decilitre, litre and hectolitre are the measures in ordinary use.

ROUGH EQUIVALENT:

4 litres = 7 pints (nearly).

EXERCISE 168.

Express in Hl. and decimals of a Hectolitre:

1. 5 Hl. 17 l.
2. 8 Hl. 5 l.
3. 8 l. 9 dl. 7 cl.

Express in Hectolitres, litres, etc.:

4. 437 l.
5. 84597 dl.
6. 31725 cl.
7. 857·5 l.
8. 120·15 l.
9. 4·025 Hl.

P (5). Decimal Coinage.

148. In several countries the coinage has been arranged to suit the decimal notation.

In France, Belgium and Switzerland the principal coin is called a **franc** (f.), and this amount is divided into 100 equal parts, each of which is called a **centime** (c.).

In Germany the principal coin is called a **mark** (m.), and this amount is divided into 100 equal parts, each of which is called a **pfennig** (pf.).

In the United States the principal coin is called a **dollar** ($), and this amount is divided into 100 equal parts, each of which is called a **cent** (c.).

ROUGH EQUIVALENTS:

1 franc = $9\frac{1}{2}d.$ (about);
1 mark = 1*s.* (nearly);
1 dollar = 4*s.* 2*d.* (about).

EXERCISE 169.

Express in centimes:

1. 3 f. 13 c.
2. 14 f. 4 c.
3. 10 f. 10 c.
4. 25·5 f.

Express in francs and centimes:

5. 175 c.
6. 1005 c.
7. 2150 c.
8. 32·15 f.

Express in francs:

9. 25 f. 25 c.
10. 10 f. 10 c.
11. 20 f. 5 c.
12. 7 f. 3 c.

Express in pfennige:

13. 17 m. 15 pf. 14. 8 m. 2 pf. 15. 19 m. 80 pf. 16. 20·45 m.

Express in marks:

17. 115 pf. 18. 12 m. 12 pf. 19. 6 m. 1 pf. 20. 15 pf.

Express in cents:

21. 7 dol. 10 c. 22. 80 dol. 75 c. 23. 55 dol. 5 c. 24. $4·86.

Express in dollars and cents:

25. 967 c. 26. 806 c. 27. 5015 c. 28. 10065 c.

Express in dollars:

29. $8. 15 c. 30. $50. 10 c. 31. $65. 1 c. 32. $56. 5 c.

P (6). Decimal Notation: Multiplication and Division by 10.

EXERCISE 170.

Express in decimal notation:

1. 4 tenths. 2. 7 tenths. 3. 14 hundredths.
4. 75 hundredths. 5. 3 hundredths. 6. 795 thousandths.
7. 52 thousandths. 8. 8 thousandths. 9. 95 tenths.
10. 306 tenths. 11. 853 hundredths. 12. 2005 thousandths.

149. It follows from what has been said about place-values increasing as we move from the decimal point to the left, and decreasing as we move to the right, that in order *to* **multiply** *by* 10, 100, 1000, *etc., we move the decimal point* 1, 2, 3, *etc., places respectively to the* **right.**

EXAMPLE 1.
$$8·765 \times 10 = 87·65.$$
$$8·765 \times 100 = 876·5.$$
$$8·765 \times 1000 = 8765.$$

EXAMPLE 2. ·325 m. × 10 = 3·25 m.
= 3 m. 2 dm. 5 cm.

EXERCISE 171.

Multiply each of the following by 10:

1. 32·425. 2. ·025. 3. 58·02. 4. 5·002.
5. ·0002 Km. 6. ·45 g. 7. 4·5 l. 8. ·045 m.

Multiply each of the following by 100:

9. 4·245.	10. ·06275.	11. ·00025.	12. ·0525.
13. 10·1 Km.	14. ·001 g.	15. ·11 l.	16. 10·01 m.

Multiply each of the following by 1000:

17. 37·2751.	18. 5·2675.	19. 52·4278.	20. ·04725.
21. ·001 Km.	22. ·0075 g.	23. 15·625 l.	24. ·00005 m.

150. Similarly, *to* **divide** by 10, 100, 1000, *etc., we move the decimal point* 1, 2, 3, *etc., places respectively to the* **left.** *If it happens that there are no figures to the left of the decimal point, we imagine* 0*'s to be there.*

EXAMPLE 1.
$$54{\cdot}321 \div 10 = 5{\cdot}4321.$$
$$54{\cdot}321 \div 100 = {\cdot}54321.$$
$$54{\cdot}321 \div 1000 = {\cdot}054321.$$

EXAMPLE 2. 4·54 Kg. ÷ 10 = ·454 Kg.
= 4 Hg. 5 Dg. 4 g.

EXERCISE 172.

Divide each of the following by 10:

1. 16·375.	2. ·65.	3. ·352.	4. 13·5.
5. 125·025 Km.	6. ·01 g.	7. ·11 l.	8. ·011 m.

Divide each of the following by 100:

9. 47·75.	10. 324·005.	11. 365·2.	12. ·0465.
13. ·4 Km.	14. ·05 g.	15. ·006 l.	16. ·0005 m.

Divide each of the following by 1000:

17. 1002·65.	18. 365·275.	19. 25·56.	20. 5·825.
21. ·5 Km.	22. ·025 g.	23. ·0035 l.	24. ·04 m.

P (7). Addition and Subtraction.

151. In addition and subtraction, we place the decimals so that their decimal points are exactly one under another; we then add or subtract as if the decimals were whole numbers, and place the decimal point in the result under the other decimal points.

EXAMPLE. *Add together* 5·25 *m.*, ·025 *m.*, 3·05 *m.*, ·75 *m.*

m.		Verification. m. dm. cm. mm.
(i)	5·25	(ii) 5 . 2 . 5 . 0
	·025	2 . 5
	3·05	3 . 0 . 5 . 0
	·75	7 . 5 . 0
	9·075	9 . 0 . 7 . 5

It should be clearly understood that (i) and (ii) differ only in arrangement ; in (ii) we have expressed the quantities in terms of the separate successive denominations.

EXERCISE 173.

Verify **1–10** *by adding horizontally.*

Add together

1. 3·76, 82·54.
2. 2·05, 4·55.
3. 3·12, 4·56.
4. 17·01, 16·001.
5. 25·036, 36·123.
6. 48·135, 16·241, 8·005.
7. 75·413, 2·001, ·02.
8. 47·351, 176·37, 3·104.
9. ·001, ·05, ·41.
10. 56·37, 2·451, 28·56.

Verify **11–24** *by an alternative method.*

Express in metres, and add :

11. 19 m. 95 cm., 32 m. 8 cm., 4 m. 19 cm.
12. 285 cm., 4276 cm., 89 cm.
13. 4·236 m., 61·3 cm., 4·82 dm.
14. 17·6 cm., 361·45 dm., 729 mm.

Express in Kg., and add :

15. 170 Kg. 64 g., 3 Kg. 840 g., 25 Kg. 6 g., 964 g.

Express in Hl., and add :

16. 934 l., 1096 dl., 34675 cl., 4298 l.

Express as francs, marks or dollars, and add :

17. 75 f. 75 c., 145 f. 80 c., 19 f. 3 c.
18. 16 f. 48 c., 48 f. 10 c., 96 f. 2 c.
19. 3 c., 5 f. 48 c., 98 f. 56 c.
20. 3 f. 75 c., 5 f. 25 c., 4 f. 5 c.
21. 19 m. 95 pf., 10 m. 6 pf., 5 m. 65 pf.

22. 12 m. 8 pf., 120 m. 80 pf., 9 m. 76 pf.
23. $49. 16 c., $381. 29 c., $76. 5 c.
24. $16. 3 c., $19. 95 c., $17. 7 c.

EXAMPLE. *Subtract* ·032 *Kg. from* 6·2 *Kg.*

	Kg		Verification. Kg. Hg. Dg. g.
(i)	6·200	(ii)	6 . 2 . 0 . 0
	·032		3 . 2
	6·168		6 . 1 . 6 . 8

NOTE. The 0's at the end of the first decimal need not be written down.

EXERCISE 174.

Verify **1–9** *by Addition.*

Subtract

1. 4·75 from 6·82.
2. ·931 from 1·002.
3. ·001 from 1.
4. 3·01 from 4.
5. 1·98 from 8·471.
6. 1·875 from 11·2.
7. 8·45 from 19·336.
8. ·127 from 86·4.
9. 19·003 from 20·1.

Verify **10–25** *by an alternative method.*

Express as francs, dollars or marks, and subtract:

10. 15 f. 34 c. from 18 f.
11. 401 f. 3 c. from 500 f.
12. 4 c. from 3 f.
13. 115 c. from 3 f. 50 c.
14. $5. 14 c. from $8. 2 c.
15. 2 c. from $2.
16. 20 c. from $20.
17. 203 pf. from 5 m. 50 pf.
18. 23 pf. from 5 m. 5 pf.
19. 1 m. 12 pf. from 3 m.

Express as decimals of the measure of the highest denomination mentioned, and subtract:

20. 198 cm. from 5 m. 50 cm.
21. 89·4 cm. from 9·13 dm.
22. 38·7 cm. from 1 m.
23. 8 Kg. 95 g. from 12 Kg. 602 g.
24. 725 g. from 18 Kg.
25. 65 l. from 5 Hl. 5 l.

EXERCISE 175.

1. The height of A is 1·656 m., B is 1·25 cm. higher than A, and C is 3·5 mm. higher than B; find the height of C in metres.

2. A rod is 1 metre long; ·256 of it is coloured red, ·024 of it blue, ·127 of it yellow, ·085 of it green, and the rest is white; find, in centimetres, the total length of the coloured portion.

3. A weighs 67·5 Kg., B weighs ·851 Kg. more than A, C weighs 1·025 Kg. more than B, and D weighs 5·75 Kg. more than A; what is the total weight of A, B, C and D expressed in kilograms?

4. A man buys 14·85 m. of cloth for 32·74 f., 8·05 m. for 18·35 f., 25·70 m. for 56·15 f., and 10·08 m. for 23·05 f.; how many metres does he buy in all, and what money does he spend?

5. From a piece of cloth, 20 m. long, three pieces measuring 5·17 m., 6·075 m. and 4·95 m. are cut off; what length remains?

6. The length of a rod was measured and found to be 1·385 metres, but when 75 cm. had been cut off it was found that only 62 cm. remained; by how much was the first measurement wrong?

7. The distance from A to B is 25·384 Km. The distance of one inn from A is 12·625 Km., and the distance of another from B is 17·249 Km.; how far apart are the inns, if both of them lie between A and B?

8. Of \$500, A gets \$50·75, B \$25·15 more than A, and C \$20·50 more than B. After D has received his share, there is \$175·35 left; how much does D get?

P (8). Multiplication.

152. To multiply by a whole number.

We place the digit in the **units'** place of the multiplier under the **last** digit of the multiplicand, and we then proceed as in ordinary multiplication, placing the right-hand digit of each partial product **underneath that digit of the multiplier to the use of which it is due.** The decimal point in each partial product will then fall underneath the decimal point in the multiplicand.

EXAMPLE 1. *Multiply* 6·834 *by* 16.

Rough Approximation. The result must lie between 6×16 and 7×16, *i.e.* between 96 and 112.

We may in this particular case multiply either (i) by 16, or (ii) by the factors 4 and 4 in succession.

(i)	6·834
	16
	68·34
	41·004
	109·344

(ii)	6·834
	4
	27·336
	4
	109·344

EXAMPLE 2. *Multiply* 6·834 *by* 20.

$$6{\cdot}834 \times 10 \times 2 = 68{\cdot}34 \times 2$$
$$= \underline{136{\cdot}68.}$$

EXAMPLE 3. *Multiply* ·00025 *by* 26.

Rough Approximation. The result must lie between ·0002 × 26 and ·0003 × 26, *i.e.* between ·0052 and ·0078.

$$\begin{array}{r} {\cdot}00025 \\ 26 \\ \hline {\cdot}0050 \\ 150 \\ \hline \underline{{\cdot}0065} \end{array}$$

We omit the final 0 in the result.

EXERCISE 176.

Find the following products:

1. ·18 × 2.	**2.** ·05 × 7.	**3.** ·26 × 3.	**4.** 2·17 × 6.
5. 3·09 × 8.	**6.** 1·37 × 5.	**7.** ·145 × 9.	**8.** ·035 × 11.
9. ·007 × 12.	**10.** 10·5 × 4.	**11.** 9·04 × 5.	**12.** ·705 × 8.
13. ·37 × 50.	**14.** ·08 × 80.	**15.** ·45 × 70.	**16.** ·07 × 90.

Verify **15–24** *by using factors.*

17. 1·95 × 20.	**18.** 13·5 × 40.	**19.** ·826 × 18.	**20.** ·247 × 81.
21. ·076 × 27.	**22.** ·009 × 72.	**23.** ·625 × 36.	**24.** ·705 × 63.

Express the results **25–28** in metres. *Check each result.*

25. 2 m. 48 cm. × 13.	**26.** 61 m. 5 cm. × 71.
27. 57 cm. 4 mm. × 57.	**28.** 5 m. 25 cm. × 89.

Express the results **29–32** in Kg. *Check each result.*

29. 9 Kg. 614 g. × 85.	**30.** 3 Kg. 41 g. × 43.
31. 4 Kg. 9 g. × 38.	**32.** 27 Kg. 875 g. × 19.

153. To multiply by a decimal.

The multiplier must first be arranged in **standard form**, *i.e.* the form in which there is one digit to the left of the decimal point.

This is effected by moving the digits the required number of places to the left or right of the decimal point in the multiplier, and as many places in the *opposite direction* in the multiplicand; thus the value of the product remains unaltered.

We place the digit in the **units'** place of the multiplier under the **last** digit of the multiplicand, and we proceed in the manner explained in Art. 152.

EXAMPLE 1. *Multiply* 82·6 *by* 8·4.

Rough Approximation. The result must lie between 82×8 and 83×9, *i.e.* between 656 and 747.

The multiplier is already in standard form.

$$\begin{array}{r} 82{\cdot}6 \\ 8{\cdot}4 \\ \hline 660{\cdot}8 \\ 33{\cdot}04 \\ \hline \underline{693{\cdot}84} \end{array}$$

EXAMPLE 2. *Multiply* 43·2 *by* 560·8.

Rough Approximation. $40 \times 600 = 24000$.

The multiplier must first be arranged in standard form.

$$43{\cdot}2 \times 560{\cdot}8 = 4320 \times 5{\cdot}608.$$

$$\begin{array}{r} 4320 \\ 5{\cdot}608 \\ \hline 21600{\cdot} \\ 2592{\cdot}0 \\ 34{\cdot}560 \\ \hline \underline{24226{\cdot}56} \end{array}$$

EXAMPLE 3. *Multiply* 62·4 *by* ·025.

Arranging the multiplier in standard form,

$$62{\cdot}4 \times {\cdot}025 = {\cdot}624 \times 2{\cdot}5.$$

Rough Approximation. The result must lie between $\cdot624 \times 2$ and $\cdot624 \times 3$, *i.e.* between 1·248 and 1·872.

$$\begin{array}{r} {\cdot}624 \\ 2{\cdot}5 \\ \hline 1{\cdot}248 \\ {\cdot}3120 \\ \hline \underline{1{\cdot}56} \end{array}$$

We omit the 00 at the end of the result.

It is important first to find a rough approximation to a result, in order to detect any error in the position of the decimal point.

The number of decimal places in the product is always equal to the sum of the numbers of decimal places in the two factors.

154. The pupil should MEMORIZE, and be able to identify at once any of the following equivalent forms:

$\cdot 5 = \frac{1}{2}$, because $5 = \frac{1}{2}$ of 10;
$\cdot 25 = \frac{1}{4}$, ... $25 = \frac{1}{4}$ of 100;
$\cdot 75 = \frac{3}{4}$, ... $75 = \frac{3}{4}$ of 100;
$\cdot 125 = \frac{1}{8}$, ... $125 = \frac{1}{8}$ of 1000;
$\cdot 375 = \frac{3}{8}$, ... $375 = \frac{3}{8}$ of 1000;
$\cdot 625 = \frac{5}{8}$, ... $625 = \frac{5}{8}$ of 1000;
and $\cdot 875 = \frac{7}{8}$, ... $875 = \frac{7}{8}$ of 1000.

[Compare Art. 63.]

EXERCISE 177.

(1) *Make a preliminary rough approximation to each product; and* (2) *verify, either by interchanging multiplier and multiplicand, or by using another method.*

Find the following products:

1. $8\cdot4 \times 7\cdot2$.	2. $4\cdot9 \times 5\cdot6$.	3. $3\cdot6 \times 6\cdot4$.
4. $5\cdot4 \times \cdot27$.	5. $9\cdot6 \times \cdot45$.	6. $3\cdot5 \times \cdot48$.
7. $69\cdot2 \times 3\cdot05$.	8. $\cdot0125 \times 8\cdot32$.	9. $300\cdot75 \times 2\cdot908$.
10. $78\cdot15 \times 40\cdot96$.	11. $13\cdot13 \times 11\cdot17$.	12. $83\cdot75 \times 15\cdot45$.
13. $308\cdot5 \times \cdot63$.	14. $69\cdot2 \times \cdot305$.	15. $16\cdot72 \times \cdot014$.
16. $\cdot0076 \times \cdot017$.	17. $8\cdot691 \times \cdot0061$.	18. $654\cdot1 \times \cdot00167$.
19. $12\cdot375 \times 1\cdot25$.	20. $12\cdot375 \times \cdot125$.	21. $12\cdot375 \times \cdot0125$.
22. $12\cdot375 \times \cdot00125$.	23. $679\cdot85 \times \cdot00056$.	24. $396\cdot8 \times \cdot01609$.
25. $0\cdot7 \times \cdot008 \times 0\cdot09$.	26. $5\cdot6 \times 1\cdot01 \times 3\cdot03$.	
27. $\cdot1 \times 10\cdot2 \times \cdot3 \times \cdot5 \times \cdot2$.	28. $\cdot1 \times \cdot1 \times \cdot1 \times \cdot1 \times \cdot1 \times \cdot1$.	

155. The method of procedure in the following example should be noticed.

EXAMPLE. *Find the value of* 6 *m.* 75 *cm. of silk at* 8 *f.* 24 *c. per metre.*

Since 6 m. 75 cm. = 6·75 m.,
and 8 f. 24 c. = 8·24 f., } we have only to multiply 8·24 f. by 6·75.

Rough Approximation. The result must lie between 8 f. × 6 and 9 f. × 7, *i.e.* between 48 f. and 63 f.

	francs.
	8·24
	6·75
	49·44
	5·768
	·4120
	55·62

The required value = **55 f. 62 c.**

EXERCISE 178.

Find the value of

1. 40·125 m. at 8·40 f. per m.
2. 187·5 m. at 3·52 f. per m.
3. 173·34 Hl. at 15·50 f. per Hl.
4. 54·135 Kg. at 3·84 f. per Kg.
5. 125 Kg. 750 g. at 3 f. 40 c. per Kg.
6. 174 m. 50 cm. at 4 f. 8 c. per m.
7. 75 m. 25 cm. at 5 m. 64 pf. per m.
8. 148 Hl. 80 l. at 15 m. 20 pf. per Hl.

P (9). Division.

156. To divide by a whole number.

We divide the decimal as though it were a whole number, observing that the units' figure in the quotient is that which we get by using the units' figure in the dividend.

EXAMPLE 1. *Divide* 76·1275 *by* 5.

$$5 \mid 76{\cdot}1275$$
$$\underline{15{\cdot}2255}$$

EXAMPLE 2. *Divide* 56·787 *by* 12.

$$12 \mid 56{\cdot}787$$
$$\underline{4{\cdot}73225}$$

When we come to the end of the figures given in the dividend, we can affix noughts, and continue the operation until it terminates, or until we have obtained as many places as we require.

EXAMPLE 3. *Divide* 1·0076 *by* 44.

We divide by factors where possible.

$$11 \mid 1{\cdot}0076$$
$$4 \mid 0{\cdot}0916$$
$$\underline{0{\cdot}0229}$$

In the first line the first figure in the quotient, 9, must be put in the second place to the right of the decimal point, immediately below the last of the figures 1·00, used in obtaining it.

EXERCISE 179.

Verify each result.

Find the following quotients:

1. ·738 ÷ 2.	2. ·834 ÷ 3.	3. ·784 ÷ 4.
4. 11·75 ÷ 5.	5. 15·18 ÷ 6.	6. 24·15 ÷ 7.
7. 2·592 ÷ 8.	8. 5·625 ÷ 9.	9. 4·092 ÷ 11.
10. ·315 ÷ 9.	11. ·216 ÷ 8.	12. ·406 ÷ 7.
13. ·45 ÷ 6.	14. ·23 ÷ 5.	15. ·38 ÷ 4.
16. 9·96 ÷ 20.	17. 130·2 ÷ 60.	18. 18·6 ÷ 40.
19. 123·36 ÷ 24.	20. 4·144 ÷ 56.	21. 2·3112 ÷ 27.
22. 63·12 ÷ 16.	23. 6·705 ÷ 18.	24. ·1656 ÷ 45.

Express the results of **25** and **26** in metres.

25. 4 m. 48 cm. ÷ 80.
26. 122 m. 50 cm. ÷ 14.

Express the results of **27** and **28** in Kg.

27. 20 Kg. 125 g. ÷ 70.
28. 190 Kg. 980 g. ÷ 36.

Express the results of **29** and **30** in Hl.

29. 60 Hl. 75 l. ÷ 54.
30. 148 Hl. 68 l. ÷ 72.

157. If the divisor cannot easily be separated into small prime factors, the usual 'Long' Division arrangement must be adopted.

EXAMPLE. *Divide* 0·0021335 *by* 85.

The integral part of the dividend is 0; hence the integral part of the quotient is 0. Then comes the decimal point in the quotient. Then follow four noughts in the quotient, produced by the digits 0, 0, 2, 1 of the dividend, for we have to use *five* digits before we get a number 213, which is greater than 85. The fifth digit of the quotient is 2. The operation then goes on as in ordinary long division.

Quotient = **0·0000251**

```
85 | 0·0021335
     170
     ---
      433
      425
      ---
       85
       85
       --
```

To ensure the correct position of the point in the quotient it will be found convenient to **write each digit of the quotient immediately over that digit of the dividend which produces it.**

EXERCISE 180.

Find the following quotients:

1. $213{\cdot}35 \div 85$.	2. $21{\cdot}335 \div 85$.	3. $2{\cdot}1335 \div 85$.
4. $403{\cdot}593 \div 87$.	5. $85{\cdot}238 \div 17$.	6. $20{\cdot}09 \div 98$.
7. $3{\cdot}2153 \div 79$.	8. $\cdot03813 \div 31$.	9. $\cdot005635 \div 23$.
10. $\cdot004368 \div 52$.	11. $\cdot0031464 \div 46$.	12. $\cdot0005135 \div 65$.

In the following cases find the quotient as far as 4 decimal places:

13. $\cdot0638 \div 49$.	14. $6{\cdot}815 \div 84$.	15. $93{\cdot}15 \div 48$.
16. $91 \div 97$.	17. $\cdot83 \div 43$.	18. $146{\cdot}1 \div 29$.

158. The sign of division may be denoted by writing the dividend over the divisor with a line between them (Art. 35).

EXAMPLE 1. *Express* $\frac{59}{4}$ *in decimal form.*

We represent 59 by 59·0 ...

$$4 \mid 59{\cdot}00$$
$$\underline{14{\cdot}75}$$

EXAMPLE 2. *Express* $\frac{3}{32}$ *in decimal form.*

$$4 \mid 3{\cdot}00000$$
$$8 \mid 0{\cdot}75000$$
$$\underline{{\cdot}09375}$$

EXAMPLE 3. *Express* $\frac{359}{25}$ *in decimal form.*

$$\frac{359}{25} = \frac{359 \times 4}{100}$$ [Art. 63 (7).]
$$= 3{\cdot}59 \times 4$$
$$= \underline{14{\cdot}36.}$$

EXAMPLE 4. *Express* $\frac{359}{125}$ *in decimal form.*

$$\frac{359}{125} = \frac{359 \times 8}{1000}$$ [Art. 63 (7).]
$$= {\cdot}359 \times 8$$
$$= \underline{2{\cdot}872.}$$

159. A decimal in which a digit, or a set of digits, is repeated continually is called a **Recurring Decimal.**

Recurring decimals are written in a shortened form, the digits which are repeated being marked by dots placed over the first and last of them, thus: $\frac{8}{3}$ is written $2{\cdot}\dot{6}$ (pure recurring decimal) and $\frac{21}{22}$ is written $\cdot9\dot{5}\dot{4}$ (mixed recurring decimal).

EXAMPLE 1. *Express* $\frac{8}{3}$ *in decimal form.*

$$3 \mid 8{\cdot}00000 \ldots$$
$$2{\cdot}666666 \ldots$$

We can continue the decimal to as many places as we please, but it will never end.

The quotient $= \underline{2{\cdot}\dot{6}.}$

EXAMPLE 2. *Express* $\frac{21}{22}$ *in decimal form.*

$$2 \mid 21{\cdot}0$$
$$11 \mid 10{\cdot}5000000 \ldots$$
$${\cdot}9545454 \ldots$$

We can continue the operation, but the digits 5, 4, will be repeated.

The quotient $= \underline{\cdot9\dot{5}\dot{4}.}$

EXERCISE 181.

Express the quotients in decimal form:

1. $\frac{50}{4}$. 2. $\frac{50}{8}$. 3. $\frac{50}{16}$. 4. $\frac{50}{32}$. 5. $\frac{178}{64}$.

6. $\frac{72}{40}$. 7. $\frac{72}{45}$. 8. $\frac{72}{48}$. 9. $\frac{72}{50}$. 10. $\frac{81}{40}$.

11. $\frac{3745}{20}$. 12. $\frac{4932}{50}$. 13. $\frac{4383}{60}$. 14. $\frac{5042}{80}$. 15. $\frac{7455}{120}$.

Use the shortest method for 16–20.

16. $\frac{3945}{25}$. 17. $\frac{4765}{125}$. 18. $\frac{37912}{25}$. 19. $\frac{41926}{125}$. 20. $\frac{14226}{25}$.

Express in decimal form:

21. $\frac{5}{16}$. 22. $\frac{17}{40}$. 23. $\frac{1}{80}$. 24. $\frac{19}{32}$. 25. $\frac{5}{64}$.

Express as recurring decimals:

26. $\frac{1}{3}$. 27. $\frac{7}{9}$. 28. $\frac{5}{11}$. 29. $\frac{14}{33}$. 30. $\frac{4}{37}$.

31. $\frac{11}{15}$. 32. $\frac{13}{18}$. 33. $\frac{11}{12}$. 34. $\frac{1}{36}$. 35. $\frac{1}{44}$.

EXERCISE 182.

Using decimals, find the average of

1. 19·4, 148·04, 4·1. 2. 17·05, 15, 5·03, 12·36.

3. The daily readings of the barometer during a week were 30·21, 30·31, 30·34, 30·23, 29·95, 30·15, 29·79; find the average daily reading.

4. The rainfalls registered for the first six months of a year were 1·23 in., 1·38 in., 1·60 in., 1·73 in., 1·86 in., 2·10 in.; find the average monthly rainfall.

5. The average monthly rainfall for the last six months of a year was 2·23 in. The rainfalls for five months were respectively 0·26 in., 1·34 in., 1·34 in., 3·30 in., 3·60 in.; find the rainfall for the other month.

6. Find, to 2 decimal places, the average scores in completed innings of the following batsmen:

	Innings.	Times not out.	Runs.		Innings.	Times not out.	Runs.
A	25	1	1626.	B	40	0	2136.
C	27	2	1281.	D	47	5	2072.

160. To divide by a decimal.

The quotient obtained by dividing one number by another is the same as the quotient obtained by dividing any multiple of the first number by the same multiple of the second (Art. 184).

When the divisor is a decimal we may therefore arrange it in **standard form**, *i.e.* make the divisor a number between 1 and 10, by multiplying it, or dividing it, by the necessary power of 10, and then regulate the dividend by multiplying it, or dividing it, by the *same* power of 10.

EXAMPLE 1. *Divide* 52·143 *by* 9·1.

The divisor is already in standard form.

Rough Check. $\frac{52}{9}$ lies between 5 and 6, so that any error in the position of the decimal point can be easily detected.

Quotient = **5·73**

```
9·1 | 52·143
    | 45 5
      6 64
      6 37
        273
        273
```

Write the quotient over the dividend, the point in the quotient being placed over the point in the dividend.

EXAMPLE 2. *Divide* 3·93084 *by* ·716.

The divisor must be arranged in standard form.

Quotient = 39·3084 ÷ 7·16.

Rough Check. $\frac{39}{7}$ lies between 5 and 6, so that the decimal point ought not to be misplaced.

Quotient = **5·49**

```
7·16 | 39·3084
     | 35 80
       3 508
       2 864
         6444
         6444
```

EXAMPLE 3. *Divide* ·001778 *by* ·028.

It is sometimes convenient to make the divisor a whole number, and then divide by factors.

Quotient = 1·778 ÷ 28.

```
7 | 1·778
4 |  ·254
    ·0635
```

EXERCISE 183.

Check each result.

Find the following quotients:

1. $2{\cdot}4 \div {\cdot}3$.	2. $4{\cdot}35 \div {\cdot}5$.	3. ${\cdot}049 \div {\cdot}7$.
4. ${\cdot}036 \div {\cdot}04$.	5. ${\cdot}384 \div {\cdot}06$.	6. ${\cdot}0024 \div {\cdot}08$.
7. ${\cdot}156 \div {\cdot}24$.	8. ${\cdot}0063 \div {\cdot}45$.	9. ${\cdot}0042 \div {\cdot}084$.
10. ${\cdot}252 \div {\cdot}056$.	11. $5{\cdot}04 \div {\cdot}072$.	12. ${\cdot}21 \div {\cdot}035$.
13. $25{\cdot}5 \div 1{\cdot}7$.	14. ${\cdot}34371 \div 6{\cdot}7$.	15. ${\cdot}00918 \div {\cdot}017$.
16. $7{\cdot}4339 \div {\cdot}0079$.	17. $27{\cdot}2 \div {\cdot}085$.	18. $16{\cdot}9 \div {\cdot}013$.
19. $72{\cdot}5026 \div 7{\cdot}85$.	20. $7{\cdot}808 \div {\cdot}128$.	21. ${\cdot}086768 \div 13{\cdot}6$.
22. ${\cdot}16016 \div 1{\cdot}43$.	23. ${\cdot}16016 \div 14{\cdot}3$.	24. $384{\cdot}006 \div {\cdot}123$.
25. ${\cdot}000133 \div 8{\cdot}75$.	26. ${\cdot}282892 \div {\cdot}394$.	27. $3{\cdot}762505 \div {\cdot}0785$.
28. ${\cdot}065341 \div {\cdot}00475$.	29. $17{\cdot}091 \div {\cdot}0675$.	30. ${\cdot}0830304 \div 86{\cdot}4$.

In the following cases find the quotient as far as 3 decimal places:

31. $2{\cdot}934 \div 7{\cdot}9$.	32. $68{\cdot}36 \div {\cdot}69$.	33. $23{\cdot}745 \div {\cdot}0059$.
34. $12{\cdot}019 \div 54{\cdot}1$.	35. $347{\cdot}155 \div 10{\cdot}5$.	36. $92{\cdot}3785 \div {\cdot}623$.

EXERCISE 184.

Check each result.

Simplify the following, using the shortest method:

1. ${\cdot}125 \times {\cdot}25$.	2. ${\cdot}0265 \times 2{\cdot}5$.	3. $625{\cdot}2 \times {\cdot}025$.
4. $12{\cdot}375 \times {\cdot}125$.	5. $12{\cdot}375 \times 1{\cdot}25$.	6. $625{\cdot}2 \times {\cdot}0125$.
7. $783{\cdot}5 \div {\cdot}25$.	8. ${\cdot}0189 \div 2{\cdot}5$.	9. $12{\cdot}305 \div {\cdot}025$.
10. $783{\cdot}5 \div {\cdot}125$.	11. $1{\cdot}365 \div 1{\cdot}25$.	12. $7{\cdot}835 \div 12{\cdot}5$.

P (10). Problems.

161. The following are examples of problems, involving both multiplication and division, solved by the **Unitary Method.**

EXAMPLE. *If* 16 *Kg.* 1 *Hg. of sugar cost* 25 *f.* 30 *c., how much can be bought for* 14 *f.* 30 *c.?*

The weight which can be bought for 25·30 f. = 16·1 Kg.;

$\therefore$ ..... 1 f. $= \frac{16{\cdot}1}{25{\cdot}3}$ Kg;

$\therefore$ .....14·30 f. $= \frac{16{\cdot}1 \times 14{\cdot}3}{25{\cdot}3}$ Kg.

$= 9{\cdot}1$ Kg. $=$ **9 Kg. 1 Hg.**

EXERCISE 185.

1. If 57 m. of cloth cost 7 f. 60 c., how much would 15 m. 60 cm. cost?

2. What weight of copper is worth 723 f. 69 c., if 107 Kg. are worth 138 f. 3 c.?

3. If 184 Km. of wire cost 23 f. 80 c., how much should 82 Km. 8 Hm. cost?

4. If the cost of 30 Hl. 75 l. of wine is 9378 f. 75 c., what should be the cost of 2 Hl. 40 l.?

5. If 4 Hl. 55 l. of wine can be bought for 520 f., how much can be bought for 93 f. 60 c.?

6. If \$4. 80 c. are equivalent to 25 f. 20 c., what is the equivalent of \$50 in francs and centimes?

7. If 5 Kg. are equivalent to 11 lb., what is the equivalent of $2\frac{3}{4}$ cwt. in Kg.?

8. If a man walks 11 metres in 5 seconds, how many kilometres can he walk in an hour at the same rate?

9. If a train travels at the rate of 48 Km. per hour, how many metres does it travel in a minute?

10. If 127 f. are equivalent to £5, what is the value of 584 f. 20 c. in British money?

11. If \$179. 8 c. are equivalent to £37, what is the equivalent of \$121 in British money?

12. If 1 metre is equivalent to 39·37 inches, what is the equivalent of 196·85 yards in metres?

Q. APPROXIMATION.

162. A number, or a measure, is said to be **approximately** correct, when it is sufficiently near the true value for the purpose to be served.

For example, we will return to Ex. 1, Art. 159, and put it in this form: "A man sells cloth at the rate of 3 metres for 8 francs, and we buy exactly one metre of it; what must we pay?"

The exact value of 1 metre would be $2\frac{2}{3}$ francs, or 2·6666 ... francs, or 2 f. 66·66 c. But there are no coins by which we can pay exactly $\frac{2}{3}$ of a franc, or 66·666 ... centimes. The nearest we can get to it is 2 f. 67 c., which is therefore the *approximate* value of 1 metre of cloth. It is useless to find the value more accurately than to one-hundredth of a franc, *i.e.* to 2 decimal places.

163. Whatever may be the given number of decimal places to which the approximation is to be correct, the process is in all cases similar to that explained above; when therefore we omit all the figures after a certain number of places we determine the value of the last figure to be retained according to the following rule: If the first of the omitted figures be 5, 6, 7, 8, or 9, we **increase** the last figure retained by 1; but, if the first omitted figure be 0, 1, 2, 3, or 4, we **make no change** in the last figure retained. For example,

31·73 = 32, correct to units' figure;	7·3473 = 7·35, correct to 2 places;
31·49 = 31, correct to units' figure.	7·3439 = 7·34, correct to 2 places.

164. The phrase 'correct to 3 significant figures' means that the 3 figures of highest place value are correct in the sense explained above. For example,

33967 = 34000, correct to 3 significant figures,

·003047 = ·0030, correct to 2 significant figures,

and ·0003408 = ·000341, correct to 3 significant figures.

NOTE.—When the number is less than 1, *i.e.* when it is a pure decimal, the ciphers which may immediately follow the decimal point are not reckoned as significant figures.

EXERCISE 186.

State the values, correct to the units' figure, of

1. 37·34. 2. 54·56. 3. 78·49. 4. 69·8. 5. 50·67.

State the values, correct to 2 decimal places, of

6. 3·7841. 7. ·3684. 8. 9·396. 9. 12·354. 10. 5·098.

State the values, correct to the nearest centimetre, of

11. 3·567 m. 12. 14·84 cm. 13. 917 mm. 14. 3·8952 m.

State the values, correct to 3 significant figures, of

15. 37·84 Kg. 16. 15·87 inches. 17. 1456 metres.
18. 13·87 litres. 19. 2·206 lb. 20. 18456 yds.
21. 927564 miles. 22. ·003756 metres. 23. 39·3704 inches.

165. To find the approximate sum of two or more numbers, or the approximate difference of two numbers, we must first give to each number its approximate value correct to *two* places *more* than are required to be correct in the result. We then add or subtract the numbers in the ordinary way, and write down the result correct to the required number of places according to the rules explained above.

EXAMPLE. *Find, correct to two decimal places,* (i) *the sum of, and* (ii) *the difference of* 5·264216 *Kg. and* 3·376861 *Kg.*

In each case we express each number correct to 4 decimal places (2 more than we want), because we cannot be sure of having the 'correct' figure in the 2nd decimal place, unless we know the value of the figure in the 3rd decimal place, and the value of this figure will be affected by the carrying figure from the column of figures in the 4th decimal place.

5·26|42 Kg.
3·37|69 Kg.
Sum = 8·64|11 Kg.
Difference = 1·88|73 Kg.

Sum = **8·64** Kg. } correct to two
Difference = **1·89** Kg. } decimal places.

After a little practice the last two figures 11 (and 73) need not be written down.

EXERCISE 187.

Find, correct to the units' figure, the sum of

1. 75·3468, 10·0946, 8·7346, 60·7054.
2. 121·8456, 92·748, 13·735, 104·8176.
3. 87·876, 123·8975, 14·8984, 213·8.

Find, correct to the first decimal place, the difference between

4. 87·3698 and 75·9678.
5. 7·84039 and ·784689.
6. 9·74325 and ·9275.
7. 184·873499 and 97·09812.

Find, correct to two decimal places, the value of

8. 3·070209 + 31·732946 + 7·89123 + ·007896.
9. 10·84782 + 8·921 + 7·846972 + 14·8 + 17·896235.
10. ·9847892 − ·729674.
11. 29·75678 − 9·8472368.

Find, to the nearest centimetre, the value of

12. 3·0326 cm. + 7·324 m. + 75·3467 mm. + 8·4293 dm.
13. 8·356 m. + 13·76 mm. + 75·68 cm. + 8·5673 cm.
14. 12·7845 cm. − 111·845 mm. 15. 1 m. − 75·8472 cm.

166. To find the approximate product of two or more numbers, we must work throughout to *two* places *more* than the number of places required.

EXERCISE 188.

Find the following products, correct to two decimal places:

1. 713·245 × 9. 2. 71·3245 × 7. 3. 7·13245 × 5.
4. ·713245 × 6. 5. ·0713245 × 8. 6. 713·245 × 12.
7. 123·76592 × 4. 8. 123·76592 × 20. 9. 123·76592 × 300.
10. 123·76592 × ·7. 11. 123·76592 × ·08. 12. 123·76592 × ·006.

167. In the following examples the quotients are to be given correct to "so many significant figures."

EXAMPLE. *Divide* ·00017 *by* ·0349, *correct to* 3 *significant figures.*

We arrange the divisor in standard form.

Quotient = ·017 ÷ 3·49.

Rough Approximation. $\frac{\cdot 018}{3} = \cdot 006.$

Quotient = ·004871

3·49 | ·01700
1396
3040
2792
2480
2443
370
349
21

The required quotient = **·00487.**

EXERCISE 189.

Find the following quotients, correct to the number of significant figures given in brackets:

1. 7·45 ÷ ·32 (three). 2. ·00205 ÷ ·29 (four).
3. 1·7854 ÷ ·032 (five). 4. 14·327 ÷ 12·8 (three).
5. 15·125 ÷ 17·5 (three). 6. 37·5 ÷ 7·68 (four).
7. ·0002 ÷ ·0163 (four). 8. ·694 ÷ 17·96 (two).
9. ·482 ÷ 37·164 (two). 10. 281·5 ÷ 13·789 (five).

R. DECIMALS OF CONCRETE QUANTITIES.

168. To find what decimal one quantity (A) is of another (B) of the same kind.

When B is a simple quantity we proceed as follows:

EXAMPLE 1. *Express* £1. 17*s.* $3\frac{3}{4}$*d.* *as a decimal of* £5.

12 | 3·75 pence
20 | 17·3125 shillings
5 | 1·865625 pounds
·373125 of £5
The decimal = ·373125.

EXAMPLE 2. *Reduce* 5 *cwt.* 3 *qrs.* 7 *lb.* *to the decimal of* 5 *tons.*

4 | 3·25 qrs.
20 | 5·8125 cwt.
5 | 0·290625 tons
·058125 of 5 tons
The decimal = ·058125.

In Ex. 1, we first express $3\frac{3}{4}$*d.* as the decimal 3·75*d.* We then divide by 12 to reduce to shillings, and we get ·3125*s.* We add the 17*s.* from the given amount, and we get 17·3125*s.*; this, reduced to pounds, becomes £·865625. We then add the £1 from the given amount, and divide by 5.

In Ex. 2, the process is similar.

EXERCISE 190.

What decimal of 1*s.* is each of the following?

1. 6*d.* 2. 3*d.* 3. 9*d.* 4. $1\frac{1}{2}$*d.* 5. $4\frac{1}{2}$*d.* 6. $10\frac{1}{2}$*d.*

What decimal of £1 is each of the following?

7. 2*s.* 8. 10*s.* 9. 8*s.* 10. 14*s.*
11. 5*s.* 12. 3*s.* 13. 15*s.* 14. 19*s.*
15. 2*s.* 6*d.* 16. 17*s.* 6*d.* 17. 1*s.* 6*d.* 18. 7*s.* 6*d.*
19. 15*s.* 6*d.* 20. 18*s.* 6*d.* 21. 6*s.* 9*d.* 22. 13*s.* 9*d.*
23. 6*s.* 3*d.* 24. 17*s.* 3*d.* 25. 4*s.* $7\frac{1}{2}$*d.* 26. 10*s.* $10\frac{1}{2}$*d.*
27. 19*s.* $5\frac{1}{4}$*d.* 28. 12*s.* $6\frac{3}{4}$*d.* 29. 7*s.* $2\frac{1}{4}$*d.* 30. 15*s.* $9\frac{3}{4}$*d.*

EXERCISE 191.

In the following examples reduce the first of the two given quantities to the decimal of the second:

1. 1*s.* 6*d.*; 2*s.* 2. 1*s.* 3*d.*; 10*s.*
3. 3*s.* $4\frac{1}{2}$*d.*; 5*s.* 4. £1. 15*s.*; £2.

5. £1. 17*s.* 6*d.*; £3.
6. £3. 17*s.* 10½*d.*; £5.
7. 12*s.* 8¼*d.*; £10.
8. £5. 17*s.* 7½*d.*; £100.
9. £1. 11*s.* 3*d.*; £1000.
10. 12*s.* 6¾*d.*; 15*s.*
11. 7*s.* 10½*d.*; 1 guinea.
12. 3*s.* 1½*d.*; £1. 5*s.*

EXERCISE 192.

In the following examples reduce the first of the two given quantities to the decimal of the second:

1. 4 oz.; 1 lb.
2. 4 cwt.; 1 ton.
3. 5 cwt.; 1 ton.
4. 9 in.; 1 foot.
5. 220 yds.; 1 mile.
6. 1 pint; 1 gallon.
7. 1 cwt. 2 qrs.; 1 ton.
8. 1 qr. 14 lb.; 1 cwt.
9. 7 cwt. 3 qrs.; 1 ton.
10. 3 qrs. 14 lb.; 1 ton.
11. 1 qr. 10 lb. 8 oz.; 1 cwt.
12. 4 cwt. 2 qrs. 7 lb.; 1 ton.
13. 3 cwt. 2 qrs. 14 lb.; 5 tons.
14. 12 days 22 hrs. 3 min.; 30 days.

169. When B is a compound quantity we proceed thus:

EXAMPLE. *Express* 7*s.* 6*d.* *as a decimal of* 12*s.* 6*d.*

7*s.* 6*d.* = 15 *sixpences*;

12*s.* 6*d.* = 25 *sixpences.* [Compare Art. 106.]

The quotient of 15 divided by 25 = the required decimal.

$$\begin{array}{r|l} 5 & 15{\cdot}0 \\ \hline 5 & 3{\cdot}0 \\ \hline & {\cdot}6 \end{array}$$

The decimal = ·6.

EXERCISE 193.

In the following examples reduce the first of the two given quantities to the decimal of the second:

1. 15*s.* 9*d.*; 17*s.* 6*d.*
2. 14*s.* 6*d.*; 19*s.* 4*d.*
3. 13*s.* 10½*d.*; 18*s.* 6*d.*
4. 17*s.* 2½*d.*; 19*s.* 8*d.*
5. 6*s.* 6¾*d.*; 10*s.* 6*d.*
6. 3 qrs. 21 lb.; 1 cwt. 1 qr.
7. 3 min. 12 sec.; 5 min. 20 sec.
8. 14 lb. 8 oz.; 18 lb. 2 oz.
9. 5 tons 6 cwt. 2 qrs.; 17 tons 15 cwt.
10. 1 yd. 2 ft. 5 in.; 2 yds 2 ft. 8 in.

170. To find the value of a given decimal of a quantity.

EXAMPLE 1. *Find the value of* £3·6125.

£3 | ·6125
20
12 | ·25 shillings
12
3 | ·0 pence

Value = **£3. 12s. 3d.**

EXAMPLE 2. *Find the value of* ·6125 *of* £3.

·6125
3
£1 | ·8375
20
16 | ·75 shillings
12
9 | ·0 pence

Value = **£1. 16s. 9d.**

EXAMPLE 3. *Find the value of* 9·875 *of* 5*s.* 10*d.*

9·875 of 5*s.* 10*d.*
= 9·875 × 70 pence
= 691·25 pence
= 57*s.* 7¼*d.*
= **£2. 17s. 7¼d.**

EXAMPLE 4. *Find the value of* ·8125 *of* 2 *tons* 4 *cwt.*

·8125 of 2 tons 4 cwt.
= ·8125 × 44 cwt.
= 3·25 × 11 cwt.
= 35·75 cwt.
= **1 ton 15 cwt. 3 qrs.**

EXERCISE 194.

Find the value of

1. ·3625 of £1.
2. ·2875 of £1.
3. ·15625 of £1.
4. ·28125 of £1.
5. ·76875 of £1.
6. ·790625 of £1.
7. ·07625 of £5.
8. ·06875 of £4.
9. ·053375 of £100.
10. 9·68125 of £5.
11. 5·928125 of £1.
12. 4·3125 of 1 guinea.
13. ·78125 of 1 cwt.
14. 7·8125 of 1 cwt.
15. ·2375 of 1 week.
16. ·05875 of 5 tons.
17. ·430625 of 30 days.
18. ·003125 of 1760 yards.
19. ·6875 of 6*s.* 8*d.*
20. ·0375 of £3. 10*s.*
21. ·1875 of £4. 2*s.* 8*d.*
22. ·325 of 1 ton 10 cwt.
23. ·856 of 2 cwt. 26 lb.
24. ·625 of 365¼ days.
25. ·02625 of 3 cwt. 14 lb.

S. DECIMALISATION OF MONEY.

171. We know that

$2s. = \frac{1}{10}$ of £1 = £·1; dividing by 2, we find that $1s. =$ £·05;

and again dividing by 2, we find that $6d. =$ £·025.

By means of these results we can express as an exact decimal of £1 any amount which consists only of shillings and a sixpence. Thus $4s. =$ £·20; $7s. =$ £·35; $5s.\ 6d. =$ £·275; and $11s.\ 6d. =$ £·575. We will now show how sums of money less than $6d.$ may be dealt with.

We have found that $6d. =$ £·025; dividing by 6 we find that $1d. =$ £·$004\frac{1}{6}$; and again dividing by 4 we find that $\frac{1}{4}d. =$ £·$001\frac{1}{24}$. Thus we see that a farthing is nearly equal to £·001, and in order to express approximately as the decimal of £1 a sum of money *not greater* than $3d.$, we simply multiply £·001 by the number of farthings in it.

But if the sum of money is *greater* than $3d.$, and not greater than $6d.$, we multiply £·001 by 1 more than the number of farthings, in order to take into account, to some extent, the fraction $\frac{1}{24}$ in the relation $\frac{1}{4}d. =$ £·$001\frac{1}{24}$.

Thus $2\frac{1}{4}d. = 9f.$; $\therefore$ $2\frac{1}{4}d. =$ £·009 approximately;

but $4\frac{1}{2}d. = 18f.$; $\therefore$ $4\frac{1}{2}d. =$ £·019 approximately.

172. We can now express any sum of money as the decimal of £1, correct to 3 places. In practice we express the shillings and the sixpence (if there is one) as exact decimals of £1, and we add to the sum of these decimals the result of multiplying £·001 by the number of farthings (or that number increased by 1). Thus

£5. $7s.\ 4\frac{1}{2}d. =$ £$(5 + ·35 + ·019) =$ £5·369,

and £2. $8s.\ 7\frac{1}{2}d. =$ £$(2 + ·425 + ·006) =$ £2·431.

The operations here described should be performed mentally, the result only being written down.

EXERCISE 195.

Express the following sums of money as decimals of £1, correct to 3 places:

1. 2*s.* 6*d.*	2. 7*s.* 6*d.*	3. 15*s.* 6*d.*	4. 18*s.* 6*d.*
5. 5*s.* 1½*d.*	6. 13*s.* 2¼*d.*	7. 9*s.* 1¾*d.*	8. 17*s.* 2*d.*
9. 3*s.* 3*d.*	10. 11*s.* 4*d.*	11. 14*s.* 5½*d.*	12. 19*s.* 4¼*d.*
13. 10*s.* 6½*d.*	14. 1*s.* 7¾*d.*	15. 6*s.* 8*d.*	16. 12*s.* 8¼*d.*
17. 1*s.* 9½*d.*	18. 4*s.* 10*d.*	19. 13*s.* 10¾*d*	20. 18*s.* 11½*d.*

173. To convert the decimal of £1, correct to 3 places, into shillings and pence we reverse the process described in Art. 172. We translate the figures in the first 2 decimal places into shillings, and the part £·025 (if it occurs) into 6 pence; the remaining portion, if any, will lie between £·001 and £·023, and can be translated at once into farthings and then into pence. Thus

£·559 = £·550 + £·009 = 11*s.* + 9*f.* = 11*s.* 2¼*d.*;

£·665 = £·650 + £·015 = 13*s.* + 14*f.* = 13*s.* 3½*d.*;

£·379 = £·350 + £·025 + £·004 = 7*s.* + 6*d.* + 4*f.* = 7*s.* 7*d.*;

£·891 = £·850 + £·025 + £·016 = 17*s.* + 6*d.* + 15*f.* = 17*s.* 9¾*d.*

174. If the decimal of £1 is given to more than 3 places, its value correct to 3 places is obtained by omitting all the figures after the 3rd decimal place, and adding 1 to the figure in the 3rd place if the figure in the 4th place is 5 or greater than 5.

EXAMPLE. *Find, to the nearest farthing, the value of* £5·78456.

£5·78456 = £5·785, correct to 3 places, = **£5. 15s. 8½d.**

EXERCISE 196.

Express in £. *s. d.*, correct to the nearest farthing:

1. £·059.	2. £3·741.	3. £8·155.	4. £3·775.
5. £2·7163.	6. £·0696.	7. £8·7382.	8. £1·039.
9. £3·7645.	10. £5·2567.	11. £3·1459.	12. £·8481.

T. DECIMAL MEASURES OF AREA.

175. In the Metric System the unit of area is a square of which each side is a metre in length. It is therefore called a **square metre.**

If the pupil will draw on squared paper a square whose side contains 10 units of length, he can satisfy himself that this square contains 10×10 small squares, *i.e.* 100 units of area.

In the table of metric square measures each denomination is therefore 100 times the next smaller denomination:

10^2, or 100, sq. **millimetres** (sq. mm.) = 1 sq. **centimetre** (sq. cm.).
10^2, or 100, sq. **centimetres** = 1 sq. **decimetre** (sq. dm.).
10^2, or 100, sq. **decimetres** = 1 sq. **metre** (sq. m.).
10^2, or 100, sq. **metres** = 1 sq. **decametre** (sq. Dm.).
10^2, or 100, sq. **decametres** = 1 sq. **hectometre** (sq. Hm.).
10^2, or 100, sq. **hectometres** = 1 sq. **kilometre** (sq. Km.).

In the reduction of metric square measure we must remember always to have *two* figures to express the number of each denomination, and to supply the place of a missing denomination by *two* 0's.

EXAMPLE 1. *Express* 5 *sq. m.* 3 *sq. dm.* 51 *sq. cm. in sq. cm.*

5 sq. m. 3 sq. dm. 51 sq. cm. = | sq. m. | sq. dm. | sq. cm. |
| --- | --- | --- |
| 5 | 03 | 51 |

= **50351 sq. cm.**

EXAMPLE 2. *Express* 34567 *sq. cm. in sq. m., etc.*

3|45|67 sq. cm. = **3 sq. m. 45 sq. dm. 67 sq. cm.**

EXAMPLE 3. *Find in sq. metres the area of a rectangular pane of glass,* 1 *m.* 58 *cm. by* 1 *m.* 45 *cm.*

The length = 1·58 m., and the breadth = 1·45 m.;
∴ the area = $1{\cdot}58 \times 1{\cdot}45$ sq. m.
= **2·291 sq. m.**

1·58
1·45
1·58
·632
·0790
2·291

EXERCISE 197.

Express in square centimetres:

1. 39 sq. dm. 7 sq. cm.
2. 5 sq. m. 12 sq. cm.
3. 3 sq. m. 5 sq. dm. 8 sq. cm.

Express in square metres, etc.:

4. 63004 sq. cm.
5. 50921 sq. cm.
6. 5001509 sq. mm.

Verify **7–14** *by calculation, and give the results in sq. cm.*

On squared paper draw the squares having sides as given below, and find their area by counting squares:

7. 36 mm.
8. 73 mm.
9. 8·6 cm.
10. 6·8 cm.

On squared paper draw the rectangles, the measurements of which are given below, and find their area by counting squares:

11. 49 mm. by 38 mm.
12. 49 mm. by 1·8 cm.
13. 6·7 cm. by 39 mm.
14. 7·3 cm. by 8·6 cm.

Find, in sq. metres, the area of a floor whose length and breadth are respectively

15. 17 m. and 10 m. 5 dm.
16. 14 m. 25 cm. and 8 m. 40 cm.
17. 16·84 m. and 12·75 m.
18. 15·73 m. and 13·37 m.

Find the length of a room whose area and breadth are respectively

19. 1795·36 sq. m. and 45·8 m.
20. 190 sq. m. and 12 m. 50 cm.

21. A window, 1·616 sq. metres in area, is to be glazed with panes of glass each 25·25 cm. by 20 cm.; how many panes will be required, and what will be the cost at 75 centimes per pane?

22. Find the cost of paving a court, 27 m. 35 cm. by 19 m. 64 cm., at 5 f. 25 c. per square metre.

176. In measuring land, a square decametre is called an **are**. The terms derived from this are the **centiare** (= 1 *square metre*) and the **hectare** (= 1 *square hectometre*).

100 **centiares** (ca.) = 1 **are** (a.).
100 **ares** = 1 **hectare** (Ha.).
100 **hectares** = 1 sq. **kilometre.**

ROUGH EQUIVALENT:

2 Ha. = 5 acres.

EXAMPLE. *Find in ares the area of a field* 175 *m. by* 144 *m.*

Length = 17·5 Dm., and breadth = 14·4 Dm. ;

∴ the area = 17·5 × 14·4 sq. Dm.

= 252 ares

= **2 Ha. 52 a.**

NOTE. When the result is required in *ares* it is best to express the length and breadth in *decametres* ; the product will then be the number of sq. decametres, *i.e.*, *ares.*

EXERCISE 198.

Reduce

1. 68 sq. Km. 49 Ha. to ares.
2. 5 sq. Km. 7 Ha. 5 a. to ares.
3. 234587 centiares to Ha., etc.
4. 786845 ares to sq. Km., etc.

Find, in Hectares, ares and centiares, the area of a piece of land

5. 755 m. by 634 m.
6. 3·756 Km. by 786·5 m.

Find, in metres, the length of a piece of land whose area and breadth are respectively

7. 14 Ha. 67 a. 85 ca. and 775 m.
8. 585 Ha. 91 a. 35 ca. and 5·865 Km.
9. A piece of land is ·625 Km. long and ·38 Km. broad ; find its value at 730 f. per Ha.
10. A piece of land 2354 metres square is sold at 275 f. 25 c. per Ha. ; how many francs does the whole sell for ?

U. DECIMAL MEASURES OF VOLUME.

177. The metric measures of volume are derived from the metric measures of length in the same way as the British measures of volume are derived from the British measures of length.

10^3, or 1000, cubic **millimetres** (c.mm.) = 1 cubic **centimetre** (c.cm.).

10^3, or 1000, cubic **centimetres** = 1 cubic **decimetre** (c.dm.).

10^3, or 1000, cubic **decimetres** = 1 cubic **metre** (c.m.).

In the reduction of metric measures of volumes we must remember always to have *three* figures to express the number of each denomination, and to supply the place of a missing denomination by *three* 0's.

EXERCISE 199.

Express in cubic centimetres :

1. 72 c.dm. 571 c.cm. 2. 87 c.dm. 25 c.cm. 3. 3 c.m. 152 c.dm.

Express in cubic metres, etc. :

4. 37215 c.dm. 5. 1003401 c.cm. 6. 78064072 c.cm.

Find the volume of a cube whose edge measures

7. 3·4 m. 8. 5·9 m. 9. 9·6 m.

Find the cubic content of rectangular blocks of the following dimensions :

	Length.	Breadth.	Depth.
10.	5·7 cm.	3·1 cm.	1·7 cm.
11.	6·5 cm.	2·6 cm.	1·8 cm.
12.	18·7 cm.	8·4 cm.	4·5 cm.
13.	21·6 cm.	14·7 cm.	20 cm.
14.	1 m. 17 cm.	1 m. 17 cm.	1 m. 17 cm.
15.	8·5 m.	64 dm.	4 m. 24 cm.

V. FRACTIONS.

V (1). Introduction.

178. We will now treat more fully a matter which was mentioned briefly in Art. 37.

If a quantity be divided into any number of equal parts, one or more of these parts is called a **fraction** of the quantity.

If, for example, we have the amount of a shilling in penny pieces, we have 12 pennies altogether; and if we divide it into 3 equal parts we shall have 4 pennies in each part.

Each of these equal parts is called **one-third** of the whole shilling, and its relation to the shilling is expressed by the symbol $\frac{1}{3}$. Thus 4 pence $= \frac{1}{3}$ of 1 shilling. If we take 2 parts together there will be 8 pennies, and their relation to the whole shilling is described as **two-thirds**, and is expressed by the symbol $\frac{2}{3}$.

179. It will be seen that the number *below* the line gives a *name* to the parts; for this reason it is called the **denominator**.

Also, since the number *above* the line denotes the *number* of equal parts that are taken, it is called the **numerator**.

Thus in the symbol $\frac{2}{3}$, 3 is the *denominator* and 2 is the *numerator*.

180. Fractions represented in the form $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{8}$, etc., are called **VULGAR FRACTIONS**, that is, common fractions.

181. When both the numerator and the denominator are integers the fraction is called a **simple fraction**.

182. To find the value of a fraction of any given quantity, we divide the quantity by the denominator when the numerator is unity; but when the numerator is greater than one, it is generally more convenient to *multiply by the numerator first and then divide the product by the denominator.*

EXERCISE 200.

Find the value of

1. $\frac{1}{9}$ of a guinea.
2. $\frac{3}{5}$ of 10*s.* 5*d.*
3. $\frac{2}{3}$ of 4*s.* 6*d.*
4. $\frac{2}{7}$ of 14*s.* $3\frac{1}{2}$*d.*
5. $\frac{1}{6}$ of 1 yd. 6 in.
6. $\frac{2}{5}$ of 1 ft. 3 in.
7. $\frac{1}{9}$ of 2 gall. 1 qt.
8. $\frac{3}{4}$ of 2 gall. 2 qts.
9. $\frac{5}{6}$ of 16 lb. 8 oz.
10. $\frac{3}{10}$ of 17 tons 15 cwt.
11. $\frac{3}{4}$ of 1 hr. 21 min. 40 sec.
12. $\frac{11}{12}$ of 43 gall. 2 qts.
13. $\frac{9}{11}$ of 16 yds. 1 ft. 6 in.

183. We can pay a debt of 10*s.* by paying over either 1 half-sovereign, or 4 half-crowns, or 5 florins, or 10 shilling pieces, or 20 sixpenny pieces, or 40 threepenny pieces, and in many other ways too.

But 1 half-crown $= \frac{1}{8}$ of £1,

$\therefore$ 4 half-crowns $= \frac{4}{8}$ of £1.

Similarly 5 florins $= \frac{5}{10}$ of £1,

10 shillings $= \frac{10}{20}$ of £1,

and 20 sixpences $= \frac{20}{40}$ of £1.

Hence $\frac{4}{8}$ of £1, $\frac{5}{10}$ of £1, $\frac{10}{20}$ of £1, $\frac{20}{40}$ of £1, are each equal to 10*s.*, or $\frac{1}{2}$ of £1; and therefore the symbols $\frac{1}{2}$, $\frac{4}{8}$, $\frac{5}{10}$, $\frac{10}{20}$, $\frac{20}{40}$, are equivalent to one another. Thus the *same* fraction may be represented by many *different* symbols.

184. From this illustration we see that **the numerator and the denominator of a fraction may be divided, or multiplied, by the same number without altering the value of the fraction.**

185. When there are no factors common to numerator and denominator, the fraction is said to be in its **lowest terms.**

186. When there are factors common to numerator and denominator, a fraction may be **reduced to its lowest terms** by dividing both numerator and denominator by their H.C.F.

EXAMPLE 1. *Reduce* $\frac{63}{135}$ *to its lowest terms.*

$$\frac{63}{135} = \frac{7 \times \cancel{9}}{15 \times \cancel{9}} = \frac{7}{15}.$$

EXAMPLE 2. *Reduce* $\frac{357}{374}$ *to its lowest terms.*

$$\frac{357}{374} = \frac{3 \times 7 \times \cancel{17}}{2 \times 11 \times \cancel{17}} = \frac{21}{22}.$$

EXERCISE 201.

Reduce each of the following fractions to its lowest terms:

1. $\frac{12}{28}$.	2. $\frac{15}{27}$.	3. $\frac{55}{90}$.	4. $\frac{18}{27}$.	5. $\frac{18}{24}$.
6. $\frac{18}{32}$.	7. $\frac{32}{36}$.	8. $\frac{45}{48}$.	9. $\frac{36}{99}$.	10. $\frac{42}{60}$.
11. $\frac{27}{63}$.	12. $\frac{15}{65}$.	13. $\frac{48}{72}$.	14. $\frac{65}{91}$.	15. $\frac{75}{100}$.
16. $\frac{57}{243}$.	17. $\frac{115}{320}$.	18. $\frac{144}{156}$.	19. $\frac{117}{130}$.	20. $\frac{132}{143}$.
21. $\frac{112}{176}$.	22. $\frac{308}{392}$.	23. $\frac{324}{720}$.	24. $\frac{528}{608}$.	25. $\frac{675}{891}$.

V (2). Comparison of Fractions.

187. To compare one fraction with another we first substitute for them equivalent fractions in which the equal parts have the *same name*, that is, we determine equivalent fractions with equal *denominators.*

For example, in the case of $\frac{5}{6}$ the equal parts are *sixths*, and in the case of $\frac{7}{9}$ they are *ninths*. But

$$\frac{5}{6}=\frac{5\times3}{6\times3}=\frac{15}{18}, \quad \text{and} \quad \frac{7}{9}=\frac{7\times2}{9\times2}=\frac{14}{18}.$$

The parts are now 18*ths* in both cases, and we see at once that the first fraction is the greater.

188. The common denominator may be *any* common multiple of the original denominators, but we ought to take the *least*.

EXERCISE 202.

Determine which is the greater in each of the following pairs of fractions:

1. $\frac{1}{2}$ and $\frac{2}{3}$.
2. $\frac{2}{3}$ and $\frac{3}{4}$.
3. $\frac{3}{4}$ and $\frac{5}{6}$.
4. $\frac{5}{6}$ and $\frac{7}{8}$.
5. $\frac{6}{7}$ and $\frac{5}{8}$.
6. $\frac{3}{10}$ and $\frac{4}{9}$.
7. $\frac{2}{3}$ and $\frac{7}{12}$.
8. $\frac{4}{9}$ and $\frac{5}{12}$.

189. To compare, and arrange in order of magnitude, three or more fractions, we find the L.C.M. of the denominators, and replace each fractional symbol by one which has for its denominator this L.C.M.

EXAMPLE. *Reduce* $\frac{31}{63}$, $\frac{11}{14}$, $\frac{5}{21}$, $\frac{1}{42}$ *to equivalent fractions having a common denominator, and arrange them in order of magnitude.*

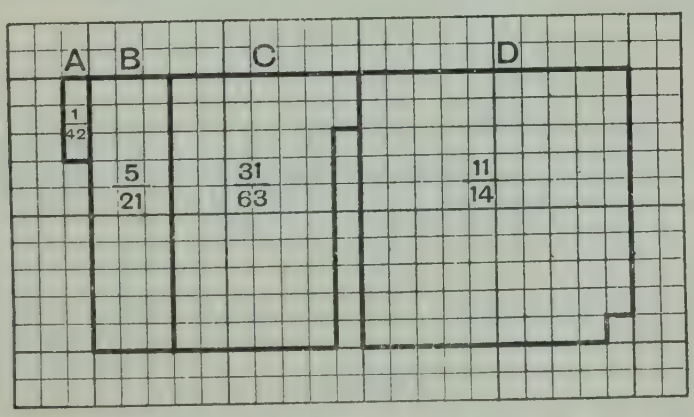
We find that the L.C.M. of the denominators = 126; and that

$$\frac{31}{63}=\frac{62}{126}, \quad \frac{11}{14}=\frac{99}{126}, \quad \frac{5}{21}=\frac{30}{126}, \quad \frac{1}{42}=\frac{3}{126};$$

hence their ascending order of magnitude is

$$\frac{3}{126}, \quad \frac{30}{126}, \quad \frac{62}{126}, \quad \frac{99}{126}, \quad \textit{i.e.}, \quad \frac{1}{42}, \quad \frac{5}{21}, \quad \frac{31}{63}, \quad \frac{11}{14}.$$

190. The four fractions in the preceding example may for comparison be represented graphically by the areas A, B, C, D



in the diagram. These areas contain 3, 30, 62 and 99 squares respectively. The unit in *each* case is made up of 126 small squares. It is only by using the same unit that comparison becomes possible.

EXERCISE 203.

Express by equivalent fractions having the least common denominator, and arrange in *ascending* order of magnitude:

1. $\frac{4}{5}$, $\frac{3}{4}$ and $\frac{7}{10}$.
2. $\frac{2}{7}$, $\frac{3}{8}$ and $\frac{5}{6}$.
3. $\frac{3}{5}$, $\frac{7}{10}$ and $\frac{5}{8}$.
4. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$ and $\frac{7}{12}$.
5. $\frac{5}{6}$, $\frac{5}{8}$, $\frac{2}{9}$ and $\frac{3}{10}$.
6. $\frac{5}{12}$, $\frac{2}{3}$, $\frac{4}{5}$ and $\frac{9}{10}$.

Express by equivalent fractions having the least common denominator, and arrange in *descending* order of magnitude:

7. $\frac{4}{5}$, $\frac{3}{10}$ and $\frac{7}{20}$.
8. $\frac{5}{6}$, $\frac{11}{12}$ and $\frac{11}{18}$.
9. $\frac{4}{9}$, $\frac{3}{5}$ and $\frac{8}{15}$.
10. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{17}{24}$ and $\frac{3}{4}$.
11. $\frac{1}{5}$, $\frac{3}{4}$, $\frac{7}{10}$ and $\frac{13}{25}$.
12. $\frac{2}{3}$, $\frac{7}{8}$, $\frac{11}{12}$ and $\frac{19}{20}$.

V (3). Improper Fractions and Mixed Numbers.

191. When the numerator of a fraction is *less than* the denominator, the fraction is called a **proper** fraction; when the numerator is *greater than* the denominator, the fraction is called an **improper** fraction. For example, $\frac{12}{13}$ is a *proper* fraction; and $\frac{14}{13}$ is an *improper* fraction.

192. A number made up of a whole number and a proper fraction is called a **mixed number**. For example, $3\frac{1}{4}$, which means $3+\frac{1}{4}$, is a mixed number.

193. In the following examples we will show how to express an *improper fraction* in the form of a *mixed number*.

EXAMPLE. *Express* $\frac{25}{3}$ *as a mixed number.*

$\frac{25}{3}$ = 25 *third-parts*
= 24 *third-parts* + 1 *third-part*
= 8 *units* + 1 *third-part*
= $8\frac{1}{3}$.

Hence, *if we divide the numerator of the improper fraction by the denominator, the quotient obtained will be the integral part of the mixed number; and the fractional part will have for its numerator the remainder after the division, and for its denominator the original denominator.*

194. If there is *no remainder* after the division, the improper fraction represents a whole number.

EXAMPLE. *Express* $\frac{60}{12}$ *as a whole number.*

The numerator can be divided by the denominator without remainder, and the quotient is 5.

Hence $\frac{60}{12}=5$.

EXERCISE 204.

Express each of the following improper fractions as a mixed number, or as a whole number:

1. $\frac{11}{6}$.	2. $\frac{42}{5}$.	3. $\frac{57}{8}$.	4. $\frac{63}{9}$.	5. $\frac{72}{7}$.
6. $\frac{83}{4}$.	7. $\frac{108}{12}$.	8. $\frac{125}{9}$.	9. $\frac{120}{7}$.	10. $\frac{129}{11}$.
11. $\frac{139}{8}$.	12. $\frac{7314}{7}$.	13. $\frac{1093}{9}$.	14. $\frac{5296}{11}$.	15. $\frac{1465}{12}$.

195. To express a *mixed number* in the form of an *improper fraction* is the converse of the preceding.

EXAMPLE. *Express* $3\frac{4}{5}$ *as an improper fraction.*

Each *unit* = 5 *fifth-parts*,

$\therefore$ 3 *units and* 4 *fifth-parts* = 19.....,

$\therefore 3\frac{4}{5} = \frac{19}{5}$.

Hence, *if we multiply the integral part of a mixed number by the denominator of the fractional part, and add to the product the numerator of the fractional part, the result will be the numerator of the improper fraction; and the original denominator will be its denominator.*

EXERCISE 205.

Express each of the following mixed numbers as an improper fraction:

1. $6\frac{7}{8}$.	2. $5\frac{8}{9}$.	3. $4\frac{6}{7}$.	4. $5\frac{11}{12}$.	5. $10\frac{4}{7}$.
6. $12\frac{7}{10}$.	7. $15\frac{2}{3}$.	8. $17\frac{3}{4}$.	9. $19\frac{5}{6}$.	10. $21\frac{2}{9}$.
11. $32\frac{3}{8}$.	12. $43\frac{4}{7}$.	13. $54\frac{3}{10}$.	14. $65\frac{7}{12}$.	15. $78\frac{9}{11}$.

Express the whole numbers

16. 3, 5, 7, 9 as fractions having 7 for their denominator.

17. 4, 6, 8, 10 as fractions having 11 for their denominator.

V (4). Addition and Subtraction.

196. We will now consider how two or more fractions may be added together, and the result expressed by a *single* fraction.

EXAMPLE. *Add together* $\frac{3}{5}$ *and* $\frac{2}{3}$.

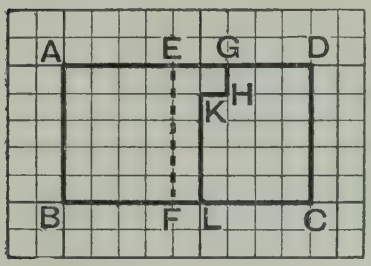
We must first express them as parts having the *same name*, *i.e.* as fractions with the same denominator. The L.C.M. of the denominators is 15, and we find that $\frac{3}{5} = \frac{9}{15}$, and $\frac{2}{3} = \frac{10}{15}$.

Hence the fractions to be added are equivalent to 9 *fifteenth*-parts and 10 *fifteenth*-parts, and the result of addition is clearly 19 *fifteenth*-parts. The process may be stated briefly as follows:

$$\frac{2}{3} + \frac{3}{5} = \frac{10}{15} + \frac{9}{15} = \frac{19}{15} = 1\frac{4}{15}.$$

In the same manner we add together three or more fractions.

197. The process may be illustrated graphically by the accompanying diagram. We will add together the fractions $\frac{4}{9}$ and $\frac{2}{15}$.



The figure ABCD represents the whole unit, and contains 45 squares.

The figure ABFE contains 20 squares, and therefore represents the fraction $\frac{20}{45}$, or $\frac{4}{9}$.

The figure EFLKHG contains 6 squares, and therefore represents the fraction $\frac{6}{45}$, or $\frac{2}{15}$.

Hence ABLKHG represents the sum of the two fractions. It contains, however, 26 squares, and it therefore represents also the fraction $\frac{26}{45}$. Hence the sum of $\frac{4}{9}$ and $\frac{2}{15}$ is $\frac{26}{45}$.

NOTE. The number of squares in the unit should be one which can be divided by each of the denominators (Art. 190). Generally the L.C.M. of the denominators will be the best number to take.

EXERCISE 206.

Draw diagrams to illustrate the following examples.

Add together the following fractions:

1. $\frac{2}{7}+\frac{3}{7}$.
2. $\frac{1}{6}+\frac{5}{6}$.
3. $\frac{1}{2}+\frac{1}{3}$.
4. $\frac{1}{2}+\frac{1}{5}$.
5. $\frac{1}{3}+\frac{1}{4}$.
6. $\frac{1}{3}+\frac{1}{5}$.
7. $\frac{1}{4}+\frac{1}{5}$.
8. $\frac{1}{5}+\frac{1}{6}$.
9. $\frac{1}{2}+\frac{1}{6}$.
10. $\frac{1}{4}+\frac{1}{6}$.
11. $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}$.
12. $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}$.
13. $\frac{5}{6}+\frac{7}{9}+\frac{3}{5}+\frac{3}{10}$.
14. $\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{7}{12}$.
15. $\frac{4}{5}+\frac{4}{9}+\frac{3}{10}+\frac{5}{6}$.
16. $\frac{3}{5}+\frac{2}{3}+\frac{7}{10}+\frac{3}{4}$.

198. If some of the fractions are *mixed numbers*, we add together the integral parts at once and treat the fractional parts as in the last section.

EXAMPLE. *Add together* $\frac{3}{5}+2\frac{1}{6}+3\frac{3}{4}+1\frac{7}{8}$.

$$\begin{aligned}\tfrac{3}{5}+2\tfrac{1}{6}+3\tfrac{3}{4}+1\tfrac{7}{8} &= 6+\tfrac{3}{5}+\tfrac{1}{6}+\tfrac{3}{4}+\tfrac{7}{8}\\ &= 6+\tfrac{72+20+90+105}{120}\\ &= 6+\tfrac{287}{120}\\ &= 6+2\tfrac{47}{120}\\ &= 8\tfrac{47}{120}.\end{aligned}$$

EXERCISE 207.

Add together the following fractions:

1. $1\frac{1}{3}+5+3\frac{2}{9}$.
2. $2\frac{1}{2}+1+5\frac{1}{4}$.
3. $\frac{4}{9}+7\frac{2}{3}+6+4\frac{5}{6}$.
4. $3\frac{1}{11}+2\frac{1}{4}+5\frac{3}{8}$.

5. $2\frac{1}{5}+\frac{7}{10}+4\frac{3}{8}$.
6. $7+\frac{1}{3}+3\frac{1}{2}+6\frac{2}{5}$.
7. $5\frac{2}{3}+8+2\frac{7}{8}+1\frac{2}{9}$.
8. $6\frac{1}{5}+\frac{7}{9}+2\frac{2}{3}+1\frac{1}{4}$.
9. $3\frac{7}{8}+4+7\frac{1}{2}+6\frac{1}{3}$.
10. $5\frac{1}{6}+3\frac{1}{2}+4\frac{1}{3}+1\frac{1}{4}$.
11. $\frac{5}{8}+4\frac{7}{10}+2\frac{1}{5}+10\frac{2}{3}$.
12. $\frac{3}{4}+2\frac{11}{12}+\frac{3}{8}+8\frac{1}{6}$.

199. In subtraction, as in addition, we express the given fractions as fractions having the same denominator; their difference will then be found by subtracting the smaller numerator from the greater.

EXAMPLE. *Find the difference between* $\frac{2}{3}$ *and* $\frac{3}{5}$.

$$\frac{2}{3}-\frac{3}{5}=\frac{10}{15}-\frac{9}{15}=\frac{10-9}{15}=\frac{1}{15}.$$

200. If the fractions are *mixed numbers*, we subtract the whole numbers and the fractional parts separately.

EXAMPLE. *Find the difference between* $6\frac{4}{9}$ *and* $2\frac{1}{6}$.

$$6\frac{4}{9}-2\frac{1}{6}=6-2+\frac{4}{9}-\frac{1}{6}=4+\frac{8-3}{18}=4\frac{5}{18}.$$

EXERCISE 208.

Find the value of each of the following expressions:

1. $\frac{5}{6}-\frac{1}{6}$.
2. $\frac{7}{9}-\frac{4}{9}$.
3. $\frac{1}{4}-\frac{1}{8}$.
4. $\frac{4}{5}-\frac{1}{2}$.
5. $\frac{2}{3}-\frac{1}{2}$.
6. $\frac{2}{3}-\frac{3}{5}$.
7. $\frac{3}{4}-\frac{2}{3}$.
8. $\frac{7}{9}-\frac{2}{3}$.
9. $6\frac{11}{12}-3\frac{3}{4}$.
10. $9\frac{1}{3}-3\frac{1}{6}$.
11. $5\frac{1}{2}-2\frac{1}{3}$.
12. $3\frac{4}{5}-2\frac{1}{10}$.
13. $5\frac{7}{8}-3\frac{1}{4}$.
14. $10\frac{7}{12}-3\frac{1}{2}$.
15. $5\frac{7}{9}-1\frac{3}{4}$.
16. $8\frac{2}{3}-3\frac{3}{7}$.

201. If the fractional part in the larger mixed number is less than in the smaller, we proceed as in the following example.

EXAMPLE. *Find the difference between* $4\frac{1}{3}$ *and* $2\frac{3}{4}$.

$$4\frac{1}{3}-2\frac{3}{4}=2+\frac{1}{3}-\frac{3}{4}=2+\frac{4-9}{12}=2-\frac{5}{12}=1\frac{7}{12}.$$

EXERCISE 209.

Find the value of each of the following expressions:

1. $15\frac{3}{8}-3\frac{1}{2}$.
2. $16\frac{1}{4}-7\frac{7}{12}$.
3. $10\frac{1}{9}-3\frac{1}{6}$.
4. $14\frac{2}{9}-9\frac{5}{8}$.
5. $8\frac{3}{7}-6\frac{4}{5}$.
6. $12\frac{2}{3}-7\frac{3}{5}$.
7. $13\frac{3}{4}-3\frac{4}{5}$.
8. $12\frac{3}{8}-7\frac{4}{5}$.
9. $15\frac{1}{8}-7\frac{3}{11}$.
10. $16\frac{3}{8}-6\frac{7}{12}$.
11. $18\frac{1}{10}-6\frac{5}{12}$.
12. $23\frac{1}{7}-10\frac{9}{10}$.

202. We will now consider some examples which involve both addition and subtraction.

EXAMPLE 1. *Express* $\frac{1}{2}+\frac{2}{3}-\frac{3}{4}+\frac{1}{6}-\frac{3}{8}$ *by a single fraction.*

We see, by inspection, that the L.C.M. of the denominators is 24.

Hence $\frac{1}{2}+\frac{2}{3}-\frac{3}{4}+\frac{1}{6}-\frac{3}{8}=\frac{12+16-18+4-9}{24}$.

The sum of the terms in the numerator which are preceded by + is 32, and the sum of those preceded by − is 27 ;

$\therefore$ the given expression $=\frac{32-27}{24}=\frac{5}{24}$.

EXAMPLE 2. *Simplify*

$2\frac{1}{2}+3\frac{1}{3}-4\frac{1}{5}-1\frac{2}{3}+6\frac{5}{6}$.

$$\begin{aligned}\text{Expression} &= 6+\tfrac{1}{2}+\tfrac{1}{3}-\tfrac{1}{5}-\tfrac{2}{3}+\tfrac{5}{6}\\ &= 6+\tfrac{15+10-6-20+25}{30}\\ &= 6+\tfrac{50-26}{30}\\ &= 6\tfrac{24}{30}\\ &= 6\tfrac{4}{5}.\end{aligned}$$

EXAMPLE 3. *Simplify*

$5\frac{1}{4}-\frac{1}{12}-1\frac{7}{8}-\frac{5}{6}$.

$$\begin{aligned}\text{Expression} &= 4+\tfrac{6-22-21-20}{24}\\ &= 4+\tfrac{6-63}{24}\\ &= 4-\tfrac{57}{24}\\ &= 4-1\tfrac{9}{8}\\ &= 4-2\tfrac{3}{8}\\ &= 1\tfrac{5}{8}.\end{aligned}$$

EXERCISE 210.

Simplify

1. $\frac{3}{4}+\frac{5}{8}-\frac{1}{2}$.
2. $\frac{2}{3}+\frac{5}{6}-\frac{2}{9}$.
3. $\frac{5}{8}-\frac{7}{12}+\frac{7}{10}$.
4. $15\frac{3}{4}+1\frac{1}{3}-3\frac{1}{9}$.
5. $3\frac{1}{3}+4\frac{1}{5}-1\frac{3}{10}$.
6. $13\frac{4}{5}-9\frac{3}{8}+2\frac{1}{4}$.
7. $10\frac{3}{5}-6\frac{7}{10}-4\frac{2}{3}+3\frac{1}{6}$.
8. $8\frac{3}{8}-2\frac{5}{6}-\frac{7}{12}-3\frac{4}{9}$.
9. $9\frac{5}{12}-\frac{7}{8}-2\frac{5}{6}-\frac{1}{2}$.
10. $8\frac{7}{8}-4\frac{4}{5}+7\frac{1}{6}-5\frac{3}{4}$.

V (5). Multiplication by a Whole Number.

203. Just as 4 *shillings* × 3 = 12 *shillings*,

so also 4 *fifth*-parts × 3 = 12 *fifth*-parts ;

that is, $\frac{4}{5}\times 3=\frac{12}{5}$.

Similarly $\frac{4}{15}\times 3=\frac{3\times 4}{15}=\frac{3\times 4}{3\times 5}=\frac{4}{5}$.

Hence, *to multiply a fraction by a whole number, we multiply the numerator, or* (when possible) *divide the denominator, by the number.*

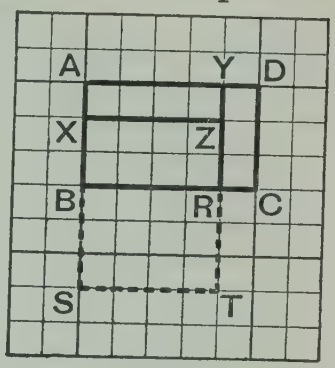
204. The resulting fraction must always be expressed in its lowest terms, and an improper fraction must be expressed as a mixed number.

EXAMPLE. *Multiply* $\frac{4}{15}$ *by* 6.

$$\tfrac{4}{15}\times 6=\frac{4\times 6}{15}=\frac{4\times 2\times 3}{15}=\frac{4\times 2}{5}=\frac{8}{5}=1\tfrac{3}{5}.$$

Hence, *if the multiplier consists of factors, some of which are also factors of the denominator and others not, the result may be obtained at once by removing common factors from the multiplier and the denominator, and multiplying the numerator by the factors of the multiplier that remain.* This process is called **cancelling** common factors.

205. The diagram in the margin illustrates the process graphically. The figure ABCD contains 15 squares, and the figure AXZY contains 4 squares. The figure AXZY represents therefore $\frac{4}{15}$ of the figure ABCD. Three such strips as AXZY make up the figure ABRY, which therefore represents the result of multiplying $\frac{4}{15}$ by 3. The figure ABRY contains 12 squares, and is therefore $\frac{12}{15}$ of the figure ABCD. Hence the result of multiplying $\frac{4}{15}$ by 3 is $\frac{12}{15}$, or $\frac{4}{5}$.



If we take 6 strips such as AXZY we make up the figure ASTY, which contains 24 squares and is therefore $\frac{24}{15}$ of the figure ABCD. Thus the result of multiplying $\frac{4}{15}$ by 6 is $\frac{24}{15}$, or $\frac{8}{5}$, or $1\frac{3}{5}$.

206. To multiply a mixed number, we multiply the whole number and the fractional part separately, and add the products.

EXAMPLE. *Multiply* $2\frac{1}{2}$ *by* 3.

$$2\tfrac{1}{2}\times 3 = 2\times 3 + \tfrac{1}{2}\times 3 = 6 + \tfrac{3}{2} = 6 + 1\tfrac{1}{2} = 7\tfrac{1}{2}.$$

EXERCISE 211.

Multiply

1. $\frac{2}{11}$ by 2 and 3.
2. $\frac{5}{28}$ by 5 and 9.
3. $\frac{5}{12}$ by 11 and 5.
4. $\frac{6}{11}$ by 9 and 12.
5. $\frac{8}{33}$ by 3 and 11.
6. $\frac{9}{49}$ by 7 and 14.
7. $2\frac{1}{2}$ by 4 and 5.
8. $4\frac{2}{3}$ by 7 and 11.
9. $6\frac{4}{5}$ by 3 and 12.
10. $7\frac{2}{7}$ by 4 and 15.
11. $8\frac{5}{14}$ by 13 and 19.
12. $3\frac{17}{22}$ by 2 and 11.

V (6). Division by a Whole Number.

207. Just as 12 *shillings* $\div 3 = 4$ *shillings*,

so also 12 *fifth*-parts $\div 3 = 4$ *fifth*-parts;

that is, $\frac{12}{5} \div 3 = \frac{4}{5}$.

Similarly $\frac{3}{14} \div 2 = 6$ *twenty-eighth* parts $\div 2$

$= 3$ *twenty-eighth* parts;

that is, $\frac{3}{14} \div 2 = \frac{3}{28}$.

Hence, *to divide a fraction by a whole number, we multiply the denominator by the number, or* (when possible) *divide the numerator by the number.*

EXAMPLE. *Divide* $2\frac{1}{6}$ *by* 7.

$$2\tfrac{1}{6} \div 7 = \frac{13}{6} \div 7 = \frac{13}{6 \times 7} = \frac{13}{42}.$$

Factors common to numerator and denominator should be removed first.

EXERCISE 212.

Divide

1. $\frac{3}{5}$ by 4, 5, 7. 2. $\frac{2}{9}$ by 3, 5, 7. 3. $\frac{12}{13}$ by 2, 3, 4.
4. $\frac{4}{19}$ by 2, 4, 8. 5. $\frac{15}{18}$ by 3, 5, 30. 6. $\frac{3}{20}$ by 6, 12, 24.
7. $2\frac{1}{2}$ by 2, 6, 14. 8. $3\frac{3}{4}$ by 4, 9, 11. 9. $16\frac{2}{3}$ by 5, 7, 15.
10. $3\frac{2}{5}$ by 17, 34, 85. 11. $4\frac{2}{9}$ by 19, 38, 95. 12. $3\frac{5}{6}$ by 5, 11, 46.

EXERCISE 213.

Find the average of the following numbers:

1. $2\frac{3}{4}$, $5\frac{1}{3}$, $4\frac{1}{6}$, $8\frac{1}{2}$. 2. $5\frac{1}{6}$, $3\frac{1}{2}$, $4\frac{1}{3}$, $1\frac{1}{4}$. 3. $3\frac{7}{8}$, 4, $7\frac{1}{2}$, $6\frac{1}{3}$.
4. $3\frac{4}{9}$, $1\frac{2}{3}$, 2, $1\frac{5}{6}$. 5. $4\frac{1}{5}$, $2\frac{7}{9}$, $2\frac{2}{3}$, $1\frac{1}{3}$. 6. $4\frac{1}{2}$, 15, $9\frac{1}{3}$, $2\frac{1}{5}$.

V (7). Multiplication by a Fraction.

208. To find $\frac{2}{3}$ of a quantity, the quantity must be divided into 3 equal parts, and 2 of these parts be taken. Hence, to find $\frac{2}{3}$ of $\frac{5}{7}$, we first divide $\frac{5}{7}$ by 3, and then multiply the result by 2.

Thus $$\frac{2}{3} \text{ of } \frac{5}{7} = 2 \times \frac{5}{3 \times 7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}.$$

Similarly $$\frac{3}{5} \text{ of } \frac{7}{11} = \frac{3 \times 7}{5 \times 11} = \frac{21}{55}.$$

Thus, *to multiply one fraction by another, we multiply the numerators together to obtain the numerator of the product, and we multiply the denominators to obtain the denominator of the product.*

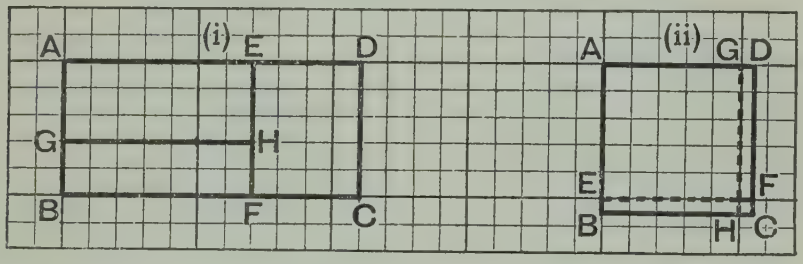
209. A fraction of a fraction is called a **compound fraction.**

210. The diagram (i) illustrates the multiplication of $\frac{3}{5}$ by $\frac{7}{11}$.

The figure ABCD contains 55 squares in 11 columns of 5 squares each, and the figure ABFE is made up of 7 of these columns. The figure ABFE is therefore $\frac{7}{11}$ of the figure ABCD.

Again the figure ABFE is made up of 5 rows of 7 squares each, and the figure AGHE is made up of 3 of these rows. The figure AGHE is therefore $\frac{3}{5}$ of the figure ABFE; but ABFE is $\frac{7}{11}$ of ABCD;

$$\therefore \text{ AGHE} = \tfrac{3}{5} \text{ of } \tfrac{7}{11} \text{ of ABCD.}$$



But AGHE $= \frac{21}{55}$ of ABCD, since AGHE contains 21 squares and ABCD contains 55. Hence we see that $\frac{3}{5}$ of $\frac{7}{11}$ of a thing is the same as $\frac{21}{55}$ of it.

The second diagram (ii) illustrates the multiplication of $5\frac{1}{2}$ by $5\frac{1}{2}$, the result being $30\frac{1}{4}$.

211. If there are any factors common to either numerator and either denominator, they should be removed before multiplication.

Thus $$\frac{8}{15} \times \frac{5}{12} = \frac{4 \times 2}{3 \times 5} \times \frac{5}{4 \times 3} = \frac{2 \times 1}{3 \times 3} = \frac{2}{9}.$$

212. It follows from Art. 208 that such an expression as

$$\frac{1}{2} \text{ of } \frac{3}{5} \text{ of } \frac{7}{11} = \frac{1}{2} \text{ of } \frac{3 \times 7}{5 \times 11}, \text{ and also that the latter} = \frac{1 \times 3 \times 7}{2 \times 5 \times 11}.$$

Thus *the product of any number of fractions is a fraction which has for its numerator the product of the original numerators, and for its denominator the product of the original denominators.*

213. If there should be any *mixed numbers* among the fractions to be multiplied, we substitute for them their equivalent improper fractions.

EXAMPLE. *Simplify* $3\frac{1}{5} \times 5\frac{5}{8} \times 3\frac{5}{9} \times \frac{3}{22} \times 2$.

$$\text{The product} = \frac{\overset{2}{\cancel{16}}}{\cancel{5}} \times \frac{\overset{5}{\cancel{45}}}{\cancel{8}} \times \frac{32}{\cancel{9}} \times \frac{3}{\underset{11}{\cancel{22}}} \times \frac{2}{1} = \frac{192}{11} = 17\tfrac{5}{11}.$$

EXERCISE 214.

Multiply

1. $\frac{2}{3}$ by $\frac{1}{2}$, $\frac{6}{11}$, $\frac{7}{12}$.
2. $\frac{5}{12}$ by $\frac{4}{5}$, $\frac{3}{10}$, $\frac{7}{15}$.
3. $\frac{14}{27}$ by $\frac{12}{35}$, $\frac{5}{28}$, $\frac{36}{35}$.
4. $\frac{6}{25}$ by $\frac{35}{54}$, $\frac{65}{66}$, $\frac{75}{92}$.
5. $2\frac{1}{2}$ by $\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$.
6. $3\frac{2}{3}$ by $\frac{2}{5}$, $\frac{1}{7}$, $\frac{5}{6}$.
7. $5\frac{3}{4}$ by $\frac{7}{11}$, $\frac{4}{16}$, $\frac{32}{46}$.
8. $3\frac{3}{4}$ by $2\frac{1}{2}$, $5\frac{1}{4}$, $7\frac{2}{3}$.
9. $7\frac{1}{2}$ by $1\frac{5}{6}$, $4\frac{1}{2}$, $2\frac{4}{5}$.
10. $5\frac{5}{12}$ by $3\frac{3}{5}$, $2\frac{7}{10}$, $5\frac{1}{4}$.

Find the value of

11. $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{5}$.
12. $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{1}{7}$.
13. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7}$.
14. $2\frac{1}{2} \times \frac{18}{35} \times \frac{10}{3}$.
15. $\frac{3}{7} \times 14 \times \frac{3}{2} \times 3\frac{5}{8}$.
16. $1\frac{1}{2} \times 1\frac{1}{3} \times 1\frac{1}{5} \times 1\frac{1}{6} \times 1\frac{1}{7}$.
17. $\frac{4}{7}$ of $8\frac{3}{10} \times \frac{5}{7} \times 9\frac{4}{5}$.
18. $5\frac{3}{7}$ of $\frac{3}{19} \times 2\frac{1}{22}$ of $3\frac{8}{45}$.

V (8). Division by a Fraction.

214. It is essential to the common definition of division that the divisor should be a whole number, but if we extend the definition and consider *dividend*, *divisor*, and *quotient*, merely as quantities, such that

$$Dividend = Divisor \times Quotient,$$

we may allow the divisor to be a fraction or a mixed number, and we may consider the quotient as merely answering the question—"*By what must the divisor be multiplied in order that the result may be the dividend?*"

Thus the quotient of 7 divided by 8 is $\frac{7}{8}$; that of $3\frac{1}{2}$ divided by $\frac{1}{2}$ is 7; that of 7 by $3\frac{1}{2}$ is 2; and that of 28 by 3 is $9\frac{1}{3}$.

We may thus consider a fraction to represent such a quantity, that *when it is multiplied by the denominator the result is the numerator.*

215. Again to divide $\frac{9}{16}$ by $\frac{5}{7}$, we have to find a quantity such that if we multiply it by $\frac{5}{7}$, the product will be $\frac{9}{16}$;

$$\therefore \text{ the quantity} \times \tfrac{5}{7} = \tfrac{9}{16};$$

$$\therefore \text{ the quantity} \times \tfrac{5}{7} \times 7 = \tfrac{9}{16} \times 7;$$

$$\therefore \text{ the quantity} \times 5 = \tfrac{9}{16} \times 7;$$

$$\therefore \text{ the quantity} = \tfrac{9}{16} \times \tfrac{7}{5} = \tfrac{63}{80}.$$

Hence, *to divide one fraction by another, we invert the divisor and multiply.*

EXAMPLE 1. *Divide* $\frac{4}{9}$ *by* $\frac{16}{21}$.

$$\frac{4}{9} \div \frac{16}{21} = \frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{7}{\cancel{21}}}{\underset{4}{\cancel{16}}} = \frac{7}{12}.$$

We cancel factors common to numerators and denominators *after* the divisor has been inverted.

EX. 2. *Divide* $3\frac{5}{12}$ *by* $4\frac{11}{28}$.

$$3\frac{5}{12} \div 4\frac{11}{28} = \frac{\overset{1}{\cancel{41}}}{\underset{3}{\cancel{12}}} \times \frac{\overset{7}{\cancel{28}}}{\underset{3}{\cancel{41}}} = \frac{7}{9}.$$

Mixed numbers must be first expressed as improper fractions.

EXERCISE 215.

Divide

1. $\frac{9}{16}$ by $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{7}{9}$.
2. $\frac{5}{10}$ by $\frac{5}{100}$, $\frac{8}{7}$, $\frac{3}{10}$, $\frac{4}{5}$.
3. $\frac{8}{15}$ by $\frac{9}{20}$, $\frac{8}{9}$, $\frac{4}{15}$, $\frac{6}{13}$.
4. $\frac{7}{8}$ by $\frac{14}{19}$, $\frac{3}{8}$, $\frac{4}{7}$, $\frac{5}{16}$.
5. 29 by $\frac{3}{4}$, $\frac{2}{3}$, $\frac{4}{7}$, $\frac{12}{13}$.
6. $\frac{3}{8}$ by $5\frac{1}{2}$, $2\frac{1}{3}$, $4\frac{1}{5}$, $6\frac{2}{3}$.
7. $\frac{9}{19}$ by $1\frac{7}{38}$, $5\frac{2}{5}$, $4\frac{1}{2}$, $1\frac{8}{19}$.
8. $4\frac{3}{4}$ by $5\frac{1}{2}$, $3\frac{4}{5}$, $9\frac{1}{4}$, $4\frac{1}{4}$.

Simplify the expressions:

9. $3\frac{11}{17} \div 6\frac{4}{51}$.
10. $1\frac{14}{15} \div 2\frac{53}{75}$.
11. $18\frac{21}{52} \div 4\frac{59}{65}$.
12. $3\frac{11}{15} \div \frac{28}{85}$.
13. $\frac{21}{32} \div 6\frac{9}{16}$.
14. $2\frac{13}{25} \div 1\frac{4}{45}$.
15. $5\frac{27}{47} \div 1\frac{37}{94}$.
16. $8\frac{41}{50} \div 2\frac{13}{25}$.
17. $3\frac{11}{12} \div 6\frac{4}{15}$.

V (9). Brackets.

216. We have already illustrated brackets with simple quantities (Art. 19, 29, 30 and 39).

From those illustrations we learnt that *numbers enclosed in brackets must be reduced to their simplest form, before any of the operations indicated by signs outside the bracket can be performed.*

EXAMPLE. *Simplify* $(5\frac{1}{6} - 2\frac{1}{12}) \div (9\frac{3}{8} + 4\frac{1}{2})$.

$$(5\tfrac{1}{6} - 2\tfrac{1}{12}) \div (9\tfrac{3}{8} + 4\tfrac{1}{2}) = 3\tfrac{1}{12} \div 13\tfrac{7}{8}$$
$$= \frac{37}{12} \times \frac{8}{111}$$
$$= \frac{\overset{1}{\cancel{37}}}{\underset{3}{\cancel{12}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{111}}}$$
$$= \frac{2}{9}.$$

NOTE. As an introduction to Exercise **216** pupils should revise Exercises **15** and **20**.

EXERCISE 216.

Simplify the expressions:

1. $15\frac{1}{4} - (7\frac{3}{4} + 6\frac{2}{3})$.
2. $15\frac{1}{4} - (7\frac{3}{4} - 6\frac{2}{3})$.
3. $4\frac{2}{11} \div (5\frac{7}{8} + 1\frac{5}{16})$.
4. $1\frac{29}{45} \div (\frac{2}{3} - \frac{4}{15})$.
5. $(15\frac{1}{4} - \frac{2}{3}) \div 7\frac{3}{4}$.
6. $(10\frac{1}{4} + \frac{5}{6}) \times 4\frac{1}{6}$.
7. $(15\frac{1}{4} + \frac{2}{3})(7\frac{3}{4} - 6\frac{2}{3})$.
8. $(8\frac{5}{6} - 4\frac{2}{3}) \div (3\frac{3}{4} + 7\frac{5}{12})$.

217. The difference in meaning between

$$3\tfrac{1}{3} \div 2\tfrac{1}{2} \times 7\tfrac{1}{4} \text{ and } 3\tfrac{1}{3} \div 2\tfrac{1}{2} \text{ of } 7\tfrac{1}{4}$$

is that, in the former, the sign ÷ extends only to the next quantity $2\frac{1}{2}$, and no farther; but, in the latter, $2\frac{1}{2}$ of $7\frac{1}{4}$ is a single quantity.

EXERCISE 217.

Simplify the expressions:

1. $2\frac{1}{4} \times 1\frac{1}{2} \div 1\frac{1}{8}$ of $2\frac{2}{5}$.
2. $2\frac{1}{4}$ of $1\frac{1}{2} \div 1\frac{1}{8} \times 2\frac{2}{5}$.
3. $2\frac{1}{4}$ of $1\frac{1}{2} \div 1\frac{1}{8}$ of $2\frac{2}{5}$.
4. $\frac{7}{55}$ of $\frac{13}{20} \div \frac{52}{33}$ of $\frac{21}{20}$.
5. $2\frac{1}{4}$ of $\frac{7}{66} \div \frac{1}{2}$ of $\frac{6}{11}$.
6. $\frac{2}{3}$ of $4\frac{1}{2} \div \frac{4}{13}$ of $3\frac{1}{4}$.

218. The following examples involve addition and subtraction as well as multiplication and division.

EXAMPLE. *Simplify* $\frac{1}{5}$ *of* $2\frac{5}{7} - 10\frac{5}{6} \div 1\frac{12}{13} + \frac{3}{7}$ *of* $3\frac{3}{4}$ *of* $3\frac{4}{5}$.

$$\begin{aligned}
\text{The expression} &= \tfrac{1}{5} \times \tfrac{19}{7} - \tfrac{65}{6} \times \tfrac{13}{25} + \tfrac{3}{7} \times \tfrac{15}{4} \times \tfrac{19}{5} \\
&= \tfrac{19}{35} - \tfrac{169}{30} + \tfrac{171}{28} \\
&= \tfrac{19}{35} - 5\tfrac{19}{30} + 6\tfrac{3}{28} \\
&= 1 + \tfrac{228-266+45}{420} \\
&= 1\tfrac{7}{420} \\
&= 1\tfrac{1}{60}.
\end{aligned}$$

NOTE. As an introduction to Exercise 218 pupils should revise Exercise 28.

EXERCISE 218.

Find the value of

1. $15\frac{1}{4} + \frac{2}{3}$ of $7\frac{3}{4}$.
2. $\frac{3}{7} \div 1\frac{4}{11}$ of $1\frac{1}{21} - \frac{1}{5}$.
3. $\frac{5}{13}$ of $5\frac{4}{7} - \frac{3}{5}$ of $\frac{5}{4}$ of $2\frac{1}{3}$.
4. $3\frac{1}{5} + 4\frac{1}{3} - 5\frac{1}{4}$ of $\frac{6}{7}$.
5. $8\frac{2}{3} - 3\frac{5}{8}$ of $1\frac{7}{9} + 2\frac{3}{5}$ of $2\frac{3}{26}$.
6. $3\frac{1}{2} - 4\frac{1}{3}$ of $\frac{7}{26} + 4\frac{1}{5}$ of $\frac{3}{7}$.
7. $3\frac{7}{8} \div \frac{9}{10} - 1\frac{3}{16}$.
8. $5\frac{2}{5} + 3\frac{1}{5} \div 1\frac{1}{3} - 4 \div 1\frac{9}{11}$.
9. $\frac{11}{17}$ of $21\frac{6}{7} - \frac{11}{19}$ of $10\frac{5}{9} + \frac{11}{21} \div \frac{33}{61}$.
10. $\frac{7}{11}$ of $5\frac{1}{2} \times 1\frac{4}{7}$ of $\frac{4}{11} - \frac{3}{8} \div 1\frac{1}{4}$.

V (10). Complex Fractions.

219. A fraction which has a fractional numerator or a fractional denominator, or both, is called a **complex** fraction.

Complex fractions are simplified by making use of the principle stated in Art. 184, namely, that the value of a fraction is

unaltered if the numerator and the denominator are multiplied by the same number. How this multiplier should be chosen will be seen from the following examples.

EXAMPLE 1. *Simplify*

$$\frac{3\frac{1}{2}-2\frac{1}{3}}{3\frac{1}{4}+\frac{5}{6}}.$$

If we here multiply the numerator and the denominator by 12, the L.C.M. of denominators 2, 3, 4, and 6, both the numerator and the denominator become integral and the fraction is no longer complex.

$$\text{Thus the fraction} = \frac{42-28}{39+10} = \frac{14}{49} = \frac{2}{7}.$$

EXAMPLE 2. *Simplify*

$$\frac{\frac{2}{3}-\frac{1}{7} \text{ of } 3\frac{1}{2}}{\frac{3}{5} \text{ of } 3\frac{1}{3}-\frac{1}{4}}.$$

$$\text{The expression} = \frac{\frac{2}{3}-\frac{1}{7}\times\frac{7}{2}}{\frac{3}{5}\times\frac{10}{3}-\frac{1}{4}} = \frac{\frac{2}{3}-\frac{1}{2}}{2-\frac{1}{4}} = \frac{8-6}{24-3} = \frac{2}{21}.$$

EXERCISE 219.

Simplify the following expressions:

1. $\frac{\frac{7}{8}}{\frac{3}{11}}$.
2. $\frac{7\frac{1}{2}}{\frac{5}{17}}$.
3. $\frac{28\frac{1}{2}}{7\frac{1}{8}}$.
4. $\frac{4\frac{1}{4}}{3\frac{5}{6}}$.
5. $\frac{\frac{3}{4}}{\frac{6}{5}\times\frac{5}{7}}$.
6. $\frac{\frac{5}{7}\times1\frac{6}{13}}{2\frac{5}{7}\times\frac{4}{13}}$.
7. $\frac{\frac{1}{9} \text{ of } 1\frac{1}{4} \text{ of } 4\frac{1}{2}}{\frac{5}{56}\times1\frac{1}{3}\times3\frac{1}{2}}$.
8. $\frac{1\frac{1}{3}\times7\frac{1}{2}}{3\frac{7}{8}\div6\frac{1}{5}}$.
9. $\frac{3\frac{2}{3}\div1\frac{3}{8}}{6\frac{2}{5}\div5\frac{1}{3}}$.
10. $\frac{\frac{7}{8}\times3\frac{2}{3}\div\frac{2}{25}}{\frac{1}{3}\times8\frac{3}{4}\div7\frac{1}{2}}$.
11. $\frac{2\frac{1}{2}+3\frac{1}{4}}{1\frac{1}{3}-\frac{3}{8}}$.
12. $\frac{6\frac{3}{4}-1\frac{5}{14}}{2\frac{1}{6}+1\frac{3}{7}}$.
13. $\frac{\frac{1}{2} \text{ of } 1\frac{2}{5}}{3\frac{3}{8}+1\frac{14}{16}}$.
14. $\frac{\frac{2}{9} \text{ of } 1\frac{2}{3} \text{ of } 2\frac{2}{5}}{\frac{7}{9}+\frac{5}{12}-\frac{3}{4}}$.
15. $\frac{3\frac{1}{12}-8\frac{2}{9}+5\frac{1}{6}}{4\frac{1}{16} \text{ of } 2\frac{6}{13}}$.
16. $\frac{2\frac{1}{4}-\frac{2}{3} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \text{ of } 3\frac{1}{3}+\frac{13}{36}}$.

V (11). Fractions of Concrete Quantities.

220. To find what fraction one quantity (A) is of another (B) of the same kind is the same as to find how many times the former contains the latter.

If we express the quantities in terms of a common unit, *the required fraction has for its numerator the number of these units contained in* A, *and for its denominator the number contained in* B.

EXAMPLE 1. *What fraction of £5 is 4s. 6d.?*

We take *sixpence* for the common unit.

$$\text{Fraction} = \frac{4s.\ 6d.}{£5} = \frac{9 \text{ sixpences}}{200 \text{ sixpences}} = \frac{9}{200}.$$

EXAMPLE 2. *Reduce 2 qrs. 8 lb. to the fraction of 2 cwt.*

$$\text{Fraction} = \frac{2 \text{ qrs. } 8 \text{ lb.}}{2 \text{ cwt.}} = \frac{64 \text{ lb.}}{224 \text{ lb.}} = \frac{2 \times 32}{7 \times 32} = \frac{2}{7}.$$

EXERCISE 220.

What part of 1*s.* is each of the following?

1. 6*d.* 2. 3*d.* 3. 4*d.* 4. 2*d.* 5. 1*d.* 6. $1\frac{1}{2}d.$

What part of £1 is each of the following?

7. 10*s.* 8. 5*s.* 9. 4*s.* 10. 2*s.*
11. 1*s.* 12. 6*d.* 13. 2*s.* 6*d.* 14. 6*s.* 8*d.*
15. 3*s.* 4*d.* 16. 1*s.* 8*d.* 17. 1*s.* 3*d.* 18. 1*s.* 4*d.*

What fraction of £1 is each of the following?

19. 15*s.* 20. 12*s.* 21. 14*s.* 22. 16*s.*
23. 18*s.* 24. 7*s.* 6*d.* 25. 12*s.* 6*d.* 26. 17*s.* 6*d.*
27. 13*s.* 4*d.* 28. 16*s.* 8*d.* 29. 8*s.* 4*d.* 30. 18*s.* 4*d.*

EXERCISE 221.

What fraction of

1. 1 yard is 2 ft. 3 in.?
2. 1 yard is 1 ft. 9 in.?
3. 1 yard is 1 ft. $10\frac{1}{2}$ in.?
4. 1 gallon is 3 pints?
5. 1 lb. is 12 oz.?
6. 1 lb. is 10 oz.?
7. 1 lb. is 14 oz.?
8. 1 ton is 15 cwt.?
9. 1 ton is 2 cwt.?
10. 1 ton is 6 cwt.?
11. 1 cwt. is 1 stone?
12. 1 cwt. is 84 lb.?

EXERCISE 222.

What fraction of

1. 6*s.* 3*d.* is 2*s.* 6*d.*?
2. 17*s.* 6*d.* is 15*s.* 9*d.*?
3. 14*s.* 8*d.* is 9*s.* 2*d.*?
4. 19*s.* 4*d.* is 14*s.* 6*d.*?
5. 19*s.* $5\frac{3}{4}d.$ is 11*s.* $8\frac{1}{4}d.$?
6. 19*s.* 8*d.* is 17*s.* $2\frac{1}{2}d.$?
7. 1 cwt. is 2 qrs. 14 lb.?
8. 16 lb. 8 oz. is 13 lb. 12 oz.?
9. 1 cwt. 1 qr. 3 lb. is 1 qr. 5 lb.?
10. 17 yds. 1 ft. 3 in. is 3 yds. 6 in.?

W. PRACTICE.

221. When one quantity (A) is contained in another quantity (B) an exact number of times, A is said to be an **aliquot part** of B. For example, 3*d.* is an *aliquot* part, namely $\frac{1}{4}$, of 1*s.*

222. **Practice** is a method of calculating *by addition of aliquot parts* the value of a simple or of a compound quantity, when the value of a unit of one denomination is given.

W (1). Simple Practice.

223. The method of **Simple Practice** will be understood from the following example.

EXAMPLE. *Find the value of* 243 *things at* £4. 17*s.* 6*d. each.*

Method 1. The value at £1 each is clearly £243. The value at £4 each is found by multiplying the value at £1 by 4.

Again, 10*s.* is an aliquot part, namely $\frac{1}{2}$, of £1 ; hence the value at 10*s.* each is found by dividing the value at £1 by 2.

Similarly, we find the value at 5*s.* each by dividing the value at 10*s.* by 2, and the value at 2*s.* 6*d.* each by dividing the value at 5*s.* by 2. The sum of these results will be the value at £4. 17*s.* 6*d.* each.

	£	*s.*	*d.*			
	243	0	0	= value at £1 each ;		
			4			
	972	0	0	=	£4	..... ;
10*s.* = $\frac{1}{2}$ of £1.	121	10	0	=	10*s.*	each ;
5*s.* = $\frac{1}{2}$ of 10*s.*	60	15	0	=	5*s.*	..... ;
2*s.* 6*d.* = $\frac{1}{2}$ of 5*s.*	30	7	6	=	2*s.* 6*d.*	..... ;
	£1184	**12**	**6**	= value at £4. 17*s.* 6*d.* each.		

Method 2. It is often easier to find the value at a "round" sum *higher* than the given price, and then to subtract from the amount thus found the value at the difference between this "round" sum and the given price. Thus :

	£	*s.*	*d.*			
	243	0	0	= value at £1 each ;		
			5			
	1215	0	0	=	£5	..... ;
2*s.* 6*d.* = $\frac{1}{8}$ of £1, ∴	30	7	6	=	2*s.* 6*d.*	each ;
By subtraction,	**£1184**	**12**	**6**	= value at £4. 17*s.* 6*d.* each.		

Method 3 (using decimals).

£4. 17*s.* = £4·85 ;

	£	
243		= value at £1 each ;
	4·85	
972		= ..... £4 ..... ;
194·4		= ..... 17*s*..... ;
12·15		= ..... 6*d*. ..... ;
6*d*. = $\frac{1}{40}$ of £1, ∴	6·075	
Required value =	£1184·625	
=	**£1184. 12s. 6d.**	

Verification (using fractions).

Since £4. 17*s*. 6*d*. = £4$\frac{7}{8}$;

∴ the value = £243 × 4 + £243 × $\frac{7}{8}$

= £972 + £212. 12*s*. 6*d*.

= **£1184. 12s. 6d.**

Otherwise, multiply £4. 17*s*. 6*d*. by 243.

EXERCISE 223.

Check each result by using fractions.

Using **two** aliquot parts, find the value of the following numbers of things, the given price being the price of *one* article :

1. 225 at 15*s*.	**2.** 294 at 12*s*.	**3.** 195 at 12*s*. 6*d*.
4. 213 at 17*s*. 6*d*.	**5.** 265 at 13*s*. 4*d*.	**6.** 268 at 11*s*. 8*d*.
7. 371 at 7*s*. 6*d*.	**8.** 483 at 6*s*.	**9.** 285 at 8*s*. 4*d*.
10. 368 at 18*s*. 4*d*.	**11.** 228 at 14*s*.	**12.** 235 at 16*s*. 8*d*.

EXERCISE 224.

(1) *State suitable aliquot parts for each of the sums of money mentioned below ; and* (2) *check each result by using another method.*

Find the value of the following numbers of things, the given price being the price of *one* article :

1. 530 at £1. 15*s*.	**2.** 778 at £2. 16*s*.
3. 579 at £3. 18*s*.	**4.** 870 at £2. 17*s*. 6*d*.
5. 713 at £7. 13*s*. 4*d*.	**6.** 563 at £8. 16*s*. 8*d*.
7. 538 at £9. 18*s*. 4*d*.	**8.** 247 at £4. 19*s*.
9. 109 at £5. 19*s*. 6*d*.	**10.** 138 at £6. 19*s*. 9*d*.

W (2). Compound Practice.

224. In **Compound Practice** we find the value of a compound quantity, when the value of a unit of *one* denomination is given.

EXAMPLE. *Find the value of* 11 *tons* 6 *cwt. at* £3. 13*s.* 4*d. per ton.*

The value of 11 tons is obtained by multiplying £3. 13*s.* 4*d.* by 11. To the product we add the value of 6 cwt., obtained by taking aliquot parts. Thus

	£	*s.*	*d.*		
	3	13	4	= value of 1 ton	
			11		
	40	6	8	=	11 tons
4 cwt. = $\frac{1}{5}$ of 1 ton		14	8	=	4 cwt.
2 cwt. = $\frac{1}{2}$ of 4 cwt.		7	4	=	2 cwt.
	£41	8	8	= value of 11 tons 6 cwt.	

Verification (using fractions).

Since 11 tons 6 cwt. = $11\frac{3}{10}$ of 1 ton,

$\therefore$ the value = £3. 13*s.* 4*d.* $\times 11\frac{3}{10}$ = £40. 6*s.* 8*d.* + £1. 2*s.*

= **£41. 8s. 8d.**

Otherwise, multiply £3. 13*s.* 4*d.* by 113 and divide the product by 10.

EXERCISE 225.

Verify each result by using fractions.

Using aliquot parts of the price of the given unit, find the value of

1. 12 tons 15 cwt. at £1. 5*s.* 6*d.* per ton.
2. 14 cwt. 3 qrs. at 18*s.* 4*d.* per cwt.
3. 8 lb. 12 oz. at 3*s.* 9*d.* per lb.
4. 24 lb. 10 oz. at 1*s.* 4*d.* per lb.
5. 25 lb. 6 oz. at 4*s.* 8*d.* per lb.
6. 14 tons 12 cwt. at £2. 5*s.* per ton.
7. 11 cwt. 14 lb. at £2. 14*s.* per cwt.
8. 13 gall. 3 qts. at 1*s.* 4*d.* per gallon.
9. 10 gall. 1 qt. 1 pt. at 15*s.* 4*d.* per gallon.
10. 12 yds. 2 ft. 3 in. at 9*s.* 6*d.* per yard.
11. 72 yds. 1 ft. 9 in. at 12*s.* 6*d.* per yard.
12. 11 months 21 days at £25. 12*s.* 6*d.* per month (28 days).

X. PROBLEMS ON FRACTIONS, AREAS, AND VOLUMES.

225. The following are examples of problems involving vulgar fractions.

EXAMPLE 1. *A commercial traveller rode by rail $\frac{1}{3}$ of a journey, drove $\frac{2}{5}$ of it, and walked the rest; what part of it did he walk?*

Fraction of whole journey which he did *not* walk $= \frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15} = \frac{11}{15}$;

$\therefore$ fraction of whole journey which he *did* walk $= 1 - \frac{11}{15} = \frac{4}{15}$.

EXAMPLE 2. *A boy spends $\frac{2}{3}$ of his money, then $\frac{2}{3}$ of the remainder, and then has 3s. 6d.; what had he at first?*

Let x = the number of shillings which the boy had at first;

then $\frac{2}{3}x$ = the number of shillings he spends first,

and $\frac{1}{3}x$ = ... has left;

$\therefore \frac{2}{3} \times \frac{1}{3}x$ = ... spends the second time,

and $\frac{1}{3} \times \frac{1}{3}x$ = ... has left finally;

$\therefore \frac{1}{3} \times \frac{1}{3}x = 3\frac{1}{2}$; $\therefore \frac{1}{9}x = 3\frac{1}{2}$;

$\therefore x = 3\frac{1}{2} \times 9 = 31\frac{1}{2}$;

$\therefore$ the boy's original money $= 31\frac{1}{2}s. =$ **£1. 11s. 6d.**

EXAMPLE 3. A *working alone can do a piece of work in 2 days,* B *in 4 days, and* C *in 6 days; in what time could they do it if they all worked together?*

A can do the whole in 2 days, B in 4 days, and C in 6 days;

$\therefore$ A can do $\frac{1}{2}$ of the work in 1 day,

B ..... $\frac{1}{4}$ ..... 1 ...,

and C ..... $\frac{1}{6}$ ..... 1 ...;

$\therefore$ A, B and C together can do $(\frac{1}{2} + \frac{1}{4} + \frac{1}{6})$ of it in one day,

$\therefore$ ..... $\frac{6+3+2}{12}$ .....,

$\therefore$ ..... $\frac{11}{12}$ .....,

$\therefore$ .... the whole in $1 \div \frac{11}{12}$ days,

i.e. in $\frac{12}{11}$ days;

$\therefore$ time required $= 1\frac{1}{11}$ **days.**

EXERCISE 226.

1. A man paid $\frac{2}{3}$ of an account in notes, $\frac{1}{4}$ in gold, and the rest in silver; what fraction of the account was paid in silver?

2. A man travelled $\frac{5}{16}$ of his journey by coach, $\frac{7}{20}$ by rail, and walked the remaining distance; what fraction of his journey did he walk?

3. One quarter of an estate was left to the eldest daughter, $\frac{1}{3}$ to the second daughter, and $\frac{2}{3}$ of the remainder to a son; how much was over?

4. If I pay away $\frac{2}{3}$ of my money, and then $\frac{3}{7}$ of what remains, how much of the whole is left?

5. A boy, after spending $\frac{1}{4}$ of his pocket-money at one shop and $\frac{7}{8}$ of the remainder at another, has 9*d.* left; how much had he at first?

6. A boy spent $\frac{3}{5}$ of his money, and then found that $\frac{3}{5}$ of the remainder was half-a-crown; how much had he at first?

7. A working alone can do a piece of work in 5 hours, and B in 8 hours; what part of the work will they do in one hour, working together?

8. One pipe fills a cistern in 8 minutes, another fills it in 7 minutes and another in 5 minutes; how much of the cistern will be filled in a minute, if all three pipes be running?

9. A can do a piece of work in 6 days, B in 8 days and C in 9 days; in how many days would all three working together finish it?

10. If a cistern can be filled by 1 pipe in 3 hours, by a second in 5 hours and by a third in 7 hours, in what time will it be filled by all three in action together?

11. A cistern can be filled by 2 pipes, A and B, in 20 and 24 minutes respectively, and can be emptied by a pipe C in 30 minutes; if all be turned on when the cistern is empty, in what time will it be filled?

12. Pipes A and B can fill a cistern in 2 hours and 3 hours respectively, and C can empty it in 5 hours; in what time will the cistern be filled when A, B and C are all turned on?

226. The following are further examples of problems which can be solved by the **Unitary Method.**

EXAMPLE 1. *If $\frac{3}{5}$ of a house is worth £210, what is the value of the whole house?*

The value of $\frac{3}{5}$ of the house = £210;
∴ ..... $\frac{1}{5}$ ..... = £70;
∴ .....the whole house = £70 × 5 = **£350.**

EXAMPLE 2. *If $\frac{2}{3}$ of a yard of silk cost 4s., what will $\frac{3}{4}$ of a yard cost?*

The cost of $\frac{2}{3}$ yd. of silk = 4*s.*;
∴ ..... $\frac{1}{3}$ yd. ..... = 2*s.*;
∴ ..... 1 yd. ..... = 6*s.*;
∴ ..... $\frac{3}{4}$ yd. ..... = 6*s.* × $\frac{3}{4}$ = **4s. 6d.**

EXERCISE 227.

1. If $\frac{3}{5}$ lb. of tea costs 1*s.* 3*d.*, what is the price per lb.?
2. What will 1 lb. of butter cost, if $\frac{5}{8}$ lb. costs 10*d.*?
3. If $3\frac{1}{3}$ tons of coal cost £4. 10*s.*, what is the cost of a ton?
4. What is the value of a sack of flour, if $2\frac{1}{5}$ sacks cost £3. 6*s.*?
5. If $\frac{3}{5}$ of a property be worth £882, what is the value of $\frac{1}{7}$ of it?
6. If $2\frac{1}{3}$ yards of silk cost £1. 15*s.*, what will $\frac{1}{4}$ yard cost?
7. If $\frac{3}{4}$ of a share is worth £36. 15*s.*, what are $7\frac{1}{2}$ shares worth?
8. If $3\frac{3}{4}$ shares are worth £27. 10*s.*, what are $4\frac{5}{8}$ shares worth?
9. If $10\frac{3}{8}$ yards of ribbon cost 8*s.* $7\frac{3}{4}$*d.*, what will 12 yards cost?
10. If $10\frac{3}{4}$ lb. of raisins cost 7*s.* 2*d.*, what will $12\frac{1}{2}$ lb. cost?

227. The following are further examples in areas and volumes, involving vulgar fractions.

EXAMPLE 1. *Find the area of the floor of a room 12 ft. 8 in. long and 10 ft. 9 in. wide.*

Length of room = $12\frac{2}{3}$ ft. = $\frac{38}{3}$ ft.,
and width ..... = $10\frac{3}{4}$ ft. = $\frac{43}{4}$ ft.;
∴ area ..... = $\frac{38}{3} \times \frac{43}{4}$ sq. ft. = $\frac{817}{6}$ sq. ft.
= **136 sq. ft. 24 sq. in.**

EXAMPLE 2. *Find the cubic content of a rectangular block of wood* 16 *ft.* 6 *in. long,* 9 *in. wide and* 4 *in. thick.*

$$\text{Length} = 16\tfrac{1}{2} \text{ ft.}, \quad \text{width} = \tfrac{3}{4} \text{ ft.}, \quad \text{thickness} = \tfrac{1}{3} \text{ ft.};$$

$$\therefore \text{ cubic content} = \tfrac{33}{2} \times \tfrac{3}{4} \times \tfrac{1}{3} \text{ c. ft.} = \tfrac{33}{8} \text{ c. ft.}$$

$$= 4\tfrac{1}{8} \text{ c. ft.}$$

$$= \underline{\textbf{4 c. ft. 216 c. in.}}$$

EXERCISE 228.

1. Find the area of a floor, 14 ft. 6 in. by 13 ft. 2 in., and that of an adjoining room, 18 ft. 10 in. by 15 ft. 7 in.

2. Find the length of a floor whose area is 540 sq. feet and whose breadth is 13 ft. 6 in.

3. How many panes of glass, each 2 ft. by 1 ft. 5 in., will be required to glaze a window 8 ft. by 5 ft. 8 in.?

4. How many turfs, each 1 yard long and 27 in. wide, will be wanted for a lawn 27 ft. by 17 ft. 6 in.?

5. Find the area of the walls of a room, 49 ft. 6 in. long, 33 ft. 9 in. wide and 10 ft. high.

6. Find the area of the walls of a room, 25 ft. 7 in. long, 16 ft. 9 in. wide and 13 ft. 6 in. high; and the cost of painting the same at 2*s.* per square yard.

7. Find the cubic content of a rectangular block 10 ft. 4 in. by 5 ft. 3 in. by 4 ft.

8. Find the length of a block of stone whose volume is $12\frac{1}{2}$ cubic feet, width $2\frac{1}{2}$ ft. and thickness $1\frac{1}{4}$ ft.

9. How many bricks, each 9 in. by $4\frac{1}{2}$ in. by 3 in., will be needed for a wall 25 yds. long, 15 ft. high and 1 ft. $10\frac{1}{2}$ in. thick?

10. If a pint of water weighs $1\frac{1}{4}$ lb. and a cubic foot weighs 1000 ounces, find how many gallons a cistern, 3 ft. 4 in. long, 2 ft. 8 in. broad and 2 ft. 3 in. deep, will hold.

A′. RATIO.

228. We have learnt how to find the number of times one quantity is contained in another (Art. 106); also how to express one quantity as the fraction of another (Art. 220).

229. The relation which one quantity bears to another of the same kind with respect to *magnitude*, expressed by the number of times the first quantity contains the second, is called the **ratio** of the one to the other.

The ratio, for example, of 3 inches to 2 inches may be expressed either in the form 3 : 2, or in the form $\frac{3}{2}$.

In the ratio 3 : 2, 3 is called the **antecedent** or **first term**, 2 the **consequent** or **second term.**

230. It is important to note that ratio can exist only between quantities of the *same kind.* Thus, $\frac{3}{2}$ does not express the ratio of 3 inches to 2 pence.

231. To find the ratio between two compound quantities, we reduce them to simple quantities of the *same name.*

EXAMPLE. *Find the ratio which* 1 *yd.* 2 *ft.* 6 *in. bears to* 2 *yds.* 2 *ft.* 3 *in.*

$$\text{Ratio} = \frac{1 \text{ yd. } 2 \text{ ft. } 6 \text{ in.}}{2 \text{ yds. } 2 \text{ ft. } 3 \text{ in.}} = \frac{66 \text{ in.}}{99 \text{ in.}} = \frac{66}{99} = \frac{2}{3} \text{ or } \underline{2 : 3.}$$

$$\text{Or, Ratio} = \frac{5\frac{1}{2} \text{ ft.}}{8\frac{1}{4} \text{ ft.}} = \frac{22}{33} = \frac{2}{3} \text{ or } \underline{2 : 3.}$$

EXERCISE 229.

Write down the results by inspection whenever possible.

Express in its simplest fractional form the ratio of

1. 8 to 2.
2. 2 to 8.
3. 4 to 12.
4. 16 to 14.
5. 1 : $\frac{1}{5}$.
6. 2 : $\frac{1}{2}$.
7. $\frac{1}{2} : \frac{1}{4}$.
8. $\frac{3}{4} : \frac{1}{2}$.
9. 3 oz. to 1 lb.
10. 3 min. to 1 hour.
11. 3 cwt. to 1 ton.
12. 12*s.* 6*d.* : £1.
13. £1 : 15*s.*
14. 1*s.* 3*d.* : 3*s.* 4*d.*
15. 5 to ·75.
16. 2·25 to ·75.
17. 39 to 110·5.
18. 56 cm. : 4 m. 48 cm.
19. 122 Km. 500 m. : 8·75 Km.
20. 13·5 Kg. : 540 Kg.

B′. PROPORTION.

B′ (1). Introduction.

232. When two ratios are equal, the four quantities are said to be in **proportion.**

Thus the ratio of 4 to 6 is equal to the ratio of 2 to 3; for we may divide 4 and 6 by the same number 2 without affecting the ratio. This is expressed by saying that

$$4 \text{ is to } 6 \text{ as } 2 \text{ is to } 3;$$

and is written $$4 : 6 : : 2 : 3,$$

or $$4 : 6 = 2 : 3 \quad \text{or} \quad \frac{4}{6} = \frac{2}{3}.$$

The fractional form is the best for working purposes.

Each of these expressions indicates a *proportion.* The terms 4, 6, 2, 3 are called *proportionals,* and are distinguished as the 1st, 2nd, 3rd and 4th proportional respectively. The first and fourth proportionals, 4 and 3, are called the **extremes.** The second and third proportionals, 6 and 2, are called the **means.**

233. In any proportion the **product of the extremes is equal to the product of the means.**

Thus the proportion $4 : 6 = 2 : 3$, written in the fractional form, becomes $\frac{4}{6} = \frac{2}{3}$; and if we express the two fractions with the same denominator (6×3), we get

$$\frac{4 \times 3}{6 \times 3} = \frac{2 \times 6}{3 \times 6}.$$

Hence, if we know three of the quantities, we can always find the fourth.

EXAMPLE. *Find the unknown term x in the proportion*

$$6 : 4 = 3 : x.$$

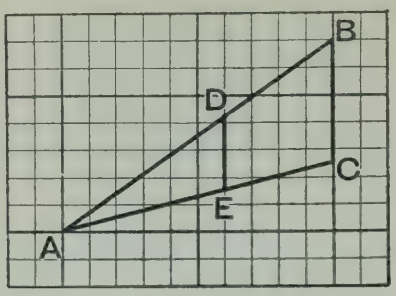
In fractional form we have

$$\frac{6}{4} = \frac{3}{x};$$

$$\therefore \; 6x = 12;$$

$$\therefore \; x = 2.$$

234. The meaning of proportion can be illustrated graphically by using squared paper in the following manner :



If through a point A a straight line AB is drawn in any direction as in the figure, it will be seen to be divided into 10 equal parts at the points where it cuts the vertical lines on the paper. We see also that AD contains 6, and that BD contains 4, of these equal parts ;

$$\therefore \frac{AD}{DB}=\frac{6}{4}=\frac{3}{2}.$$

If a second line AC be drawn from A in another direction so as to cut the same number of vertical lines, we find in the same way that

$$\frac{AE}{EC}=\frac{6}{4}=\frac{3}{2}.$$

Hence it follows that $\frac{AD}{DB}=\frac{AE}{EC}$.

It is clear that the argument is independent of the number of the vertical lines which are made use of, and we therefore conclude that, generally, a straight line drawn parallel to the base of a triangle divides the sides into parts which are proportionals.

EXERCISE 230.

Verify each result.

Complete the following proportions by finding the value of x :

1. $x : 3 :: 54 : 6$.
2. $\frac{15}{x}=\frac{5}{7}$.
3. 12 yds. : 4 yds. = £9 : £x.
4. $\frac{5}{16} : \frac{1}{4} :: 15 : x$.
5. $\frac{8}{6}=\frac{x}{3}$.
6. 20 tons : 25 tons = £x : £10.
7. $5\frac{1}{2} : x :: 16 : 32$.
8. $\frac{x}{44}=\frac{16}{64}$.

9. x hrs. : 9 hrs. = 10 men : 3 men.

10. $1{\cdot}5 : 7{\cdot}5 :: x : 3$.

11. $\frac{8}{7} = \frac{1}{x}$.

12. 12 days : x days = £18 : £12.

235. Problems involving Proportion, which we have solved by the Unitary Method (Arts. 54, 98, 107, 118, 161 and 226), can also be solved by the Ratio Method.

B′ (2). Simple Proportion.

EXAMPLE 1. *If* 9 *copies of a book cost* £1. 17*s.* 6*d.*, *what must be paid for* 11 *copies?*

(Solution by the **Unitary Method**, page 91.)

The cost of the books is clearly proportional to the number bought; thus, if x pounds be the cost of 11 copies, it follows that x pounds must bear the same ratio to £1. 17*s.* 6*d.*, or £$1\frac{7}{8}$, that 11 copies bear to 9 copies.

Hence $$\frac{£x}{£1\frac{7}{8}} = \frac{11 \text{ books}}{9 \text{ books}}; \quad \therefore \frac{x}{1\frac{7}{8}} = \frac{11}{9};$$

$$\therefore x = \frac{11}{9} \times 1\tfrac{7}{8} = \frac{11}{9} \times \frac{15}{8} = \frac{55}{24} = 2\tfrac{7}{24};$$

$\therefore$ Cost of 11 books = **£2. 5s. 10d.**

Alternative Method. The problem might also have been stated:

9 books : 11 books = £$1\frac{7}{8}$: £x.

EXAMPLE 2. *If* 6 *copies of a book cost* £3. 16*s.* 6*d.*, *how many copies can be bought for* £8. 5*s.* 9*d.*?

(Solution by the **Unitary Method**, page 104.)

The number of books bought is clearly proportional to the money spent; thus if x be the number bought for £8. 5*s.* 9*d.*, or $165\frac{3}{4}$*s.*, it follows that x books must bear the same ratio to 6 books that $165\frac{3}{4}$*s.* bears to $76\frac{1}{2}$*s.*

Hence $$\frac{x}{6} = \frac{165\frac{3}{4}}{76\frac{1}{2}} = \frac{663}{306} = \frac{13}{6};$$

$$\therefore x = 13;$$

$\therefore$ No. of books for £8. 5*s.* 9*d.* = **13.**

Alternative Method. The problem might also have been stated:

$76\frac{1}{2}$*s.* : $165\frac{3}{4}$*s.* = 6 books : x books.

EXERCISE 231.

Verify by an alternative method.

Solve by the Ratio Method :

1. If the railway fare for 48 miles be 7*s.*, what will a ticket cost for a journey of 120 miles ?

2. If for 17*s.* 6*d.* I can travel 120 miles, how far can I travel for 9*s.* 4*d.* ?

3. If a train travels 48 miles in $1\frac{1}{2}$ hours, how long will it take to travel 120 miles at the same rate of speed ?

4. If a train travels 64 miles in 2 hrs., how far will it travel at this rate in 3 hrs. 45 min. ?

5. If 5 pennies placed end to end extend 6 inches, how many pennies will extend a mile ?

6. If 3 pennies weigh 1 oz., how many pennies will there be in a bag the contents of which weigh $1\frac{1}{4}$ cwt. ?

SUPPLEMENTARY EXAMPLES.

Re-work by the Ratio Method: Exercises **68**, 1-12 ; **129**, 5-12 ; **144**, 1-5 ; **185**, 1-12 ; **227**, 5-10.

B′ (3). Inverse Proportion.

EXAMPLE. *If* 19 *men can complete certain work in* 437 *days, in how many days should* 23 *men be able to complete it ?*

(Solution by the **Unitary Method**, page 46.)

The time taken to do the work *diminishes* at the same rate as the number of men employed *increases*. Thus, if x days be the time required by 23 men, we have

$$\frac{x}{437} = \frac{19}{23} ;$$

$$\therefore\ x = \frac{19 \times 437}{23} = 361 ;$$

$\therefore$ Time required = **361 days.**

Alternative Method. The problem might also have been stated :

23 men : 19 men = 437 days : x days.

EXERCISE 232.

Verify by an alternative method.

Solve by the Ratio Method :

1. A train travelling at 32 miles an hour can complete its journey in $3\frac{3}{4}$ hours ; how many hours would it take if the rate were 48 miles an hour ?

2. A train travelling at 48 miles an hour can do a certain journey in $1\frac{3}{4}$ hours; in what time would the journey be completed if the rate were 30 miles an hour?

3. Food is provided to last 525 soldiers for 20 days; how long will it suffice if the camp is increased by 475 soldiers?

4. A farmer has corn enough for 21 horses for 18 days; if he buys 6 more horses, how many days earlier will the corn be consumed?

5. By travelling at 18 knots a steamer completes a voyage in 45 days; how many more days would she take if the speed were reduced by 3 knots?

6. A piece of work can be done by 16 men in 3 days; how many extra men must be engaged to complete the work in 2 days?

SUPPLEMENTARY EXAMPLES.

Re-work by the Ratio Method: Exercises 68, 13-20; 144, 6-10.

B′ (4). Compound Proportion.

236. When the fractions which represent two or more simple ratios are multiplied together the result will represent the ratio **compounded** of the simple ratios.

Thus the ratio compounded of $\frac{8}{6}$ and $\frac{5}{8}$ is $\frac{8}{6} \times \frac{5}{8}$ or $\frac{5}{6}$.

237. In the preceding problems we have found the change in *one* quantity corresponding to the change in *one* other. In many cases, however, a change in one quantity depends upon changes in two others. Such problems may be solved either by the Unitary Method or by the Ratio Method. They are generally spoken of as problems in **Compound Proportion.**

EXAMPLE. *If* 6 *horses can plough* 480 *acres in* 8 *days, how many acres can* 8 *horses plough in* 5 *days?*

Unitary Method.

In 8 days 6 horses can plough 480 acres;
∴ ... 1 day 6 horses ..... 60 acres;
∴ ... 5 days 6 horses ..... 300 acres.
∴ ... 5 days 1 horse ..... 50 acres;
∴ ... 5 days 8 horses ..... 400 acres.

Ratio Method. The required number of acres depends on two different ratios: (i) so far as concerns the number of **horses** employed, the number of acres is *increased* in the ratio $\frac{8}{6}$; and (ii) so far as concerns the number of **days** to be taken, the number of acres is *decreased* in the ratio $\frac{5}{8}$. Thus, if x acres be the number required, we have

$$x = 480 \times \frac{8}{6} \times \frac{5}{8} = 400;$$

$\therefore$ No. of acres required $= \underline{400}$.

EXERCISE 233.

Verify by the Unitary Method.

Solve by the Ratio Method:

1. If 8 men can do a piece of work in 2 days, in how many days will 6 men do three times the quantity?

2. If 6 men working 10 hours a day do a piece of work in 16 days, how many men will be required to complete it in 24 days, working 8 hours a day?

3. If 4 men working 10 hours a day do a piece of work in 9 days, in how many days, working 8 hours a day, could 3 men do the same piece of work?

4. If 12 men do a piece of work in 24 days, working 8 hours a day, how many hours a day must 16 men work to do the same piece of work in 18 days?

5. If 5 fires burn 3 tons of coal in 6 months, how many tons will 4 fires burn in 10 months? Assume that each fire burns the same amount of coal per month.

6. If the wages of 4 men for 12 days be £30, what would be the wages of 6 men for 10 days at the same rate of pay?

7. If the wages of 5 men amount to £57 in 36 days, how many men would receive £76 in wages in 60 days at the same rate of pay?

8. If it costs £28 to cover with carpet a room 12 feet long and $10\frac{1}{2}$ feet wide, how much will it cost to carpet a room 18 feet long and $13\frac{1}{2}$ feet wide, with material of the same quality?

9. If a pair of blankets, each measuring 6 ft. 9 in. by 5 ft. 3 in., costs £1. 11*s*. 6*d*., what would be the cost of a pair of the same quality, each measuring 9 ft. 9 in. by 8 ft. 3 in.?

10. If a train moving at the rate of 36 miles per hour covers the distance between two towns in $2\frac{1}{2}$ hours, how long will it take a train moving at the rate of 45 miles per hour to travel two-thirds of the distance ?

11. If a man walks 14 miles in 4 hours, how many minutes will he take to walk half a mile, his speed being increased in the ratio of 8 to 7 ?

12. If a man can walk 660 yards in 5 minutes, how many miles can he walk in 1 hour at two-thirds of the former rate ?

B′ (5). Proportionate Division.

238. When we know the ratio between the parts of a given quantity, however many the parts may be, we can easily determine the parts themselves.

EXAMPLE. *Divide* £1. 4*s*. 6*d*. *between Smith and Brown, so that Smith may have* 3 *shares and Brown* 4 *shares.*

(Alternative Solution, page 105.)

If we divide the whole into 7 equal parts, Smith will get 3 of these parts and Brown will get 4;

$\therefore$ Smith's share $= \frac{3}{7}$ of 24*s*. 6*d*. $= \frac{3}{7} \times 4\frac{9}{2}s. = \frac{21}{2}s. =$ 10s. 6d.,

and Brown's share $= \frac{4}{7}$ of 24*s*. 6*d*. $= \frac{4}{7} \times 4\frac{9}{2}s. =$ 14s.

Verification. *The sum of the parts must be equal to the whole;*

10*s*. 6*d*. + 14*s*. = £1. 4*s*. 6*d*.

EXERCISE 234.

Verify each result.

Divide

1. £10 into two parts in the ratio of 3 : 5.
2. 27 lb. 8 oz. into two parts in the ratio of 3 : 5.
3. 2 yds. 1 ft. 1 in. into two parts proportional to 2 and 3.
4. £12 into three parts in the ratio of 4 : 5 : 6.
5. 36·5 inches into three parts proportional to 2 : 3 : 5.
6. 100 tons into three parts proportional to 8, 7 and 1.
7. $1\frac{1}{2}$ cwt. into three parts proportional to 3 : 5 : 6.
8. A train has passengers in the three classes proportional in number to 2 : 5 : 9. If the total number of passengers is 848, what is the number in each class ?

9. At an examination A, B and C obtain altogether 351 marks in the proportion of 6 : 4 : 3 ; how many marks does each get ?

10. At the same examination P, Q and R also obtain altogether 351 marks in the proportion of 2 : 3 : 4 ; how many marks does each get ?

SUPPLEMENTARY EXAMPLES.

Work again Exercise **145.**

C′. MEASUREMENTS TO SCALE.

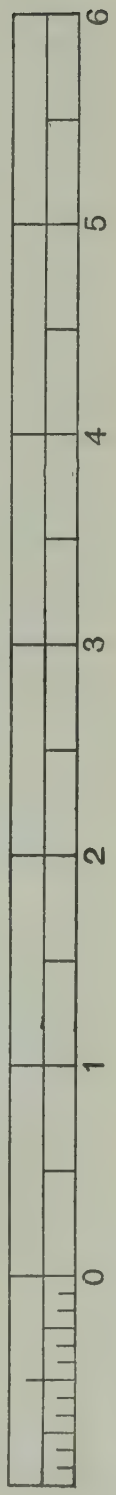
239. It is often desirable to make a drawing of an object showing exactly the relationship of one part to another, and yet it may not be possible to make the lines in the drawing the same length as they are in the object.

For example, supposing we want to make a drawing or diagram of a cupboard door, which is 5 ft. high and 2 ft. 6 in. wide, on a sheet of paper no longer than the page of this book. It is necessary to make each line in the drawing a certain fraction of the corresponding line in the door; thus, if we make a drawing in which 1 inch stands for 1 foot, it is clear that 5 feet will be represented by a line 5 inches long and 2 feet 6 inches by a line $2\frac{1}{2}$ inches long.

This operation is known as **drawing to scale.**

NOTE. Architects, engineers, etc., use a single dash (′) to express feet and two dashes (″) to express inches.

240. The *scale* must be indicated on the diagram in one of three ways: (i) Scale, 1″ to 1′; or (ii) scale $\frac{1}{12}$; or (iii) by a drawing of the scale on the diagram. Thus the figure in the margin shows a drawing of $\frac{3}{4}$″ to 1′, showing inches, and reading to 7′.



EXERCISE 235.

1. Examine the maps in your atlas, and find out how the scale is expressed.

2. On your map of England find the distance, in a straight line, between London and Bristol.

NOTE. Other pairs of towns can also be taken: London-Manchester, London-Nottingham, Liverpool-Southampton.

3 A drawing is made in which $\frac{3}{4}$″ stands for 1 foot; what will be the real length of a line which measures 3″ in the drawing?

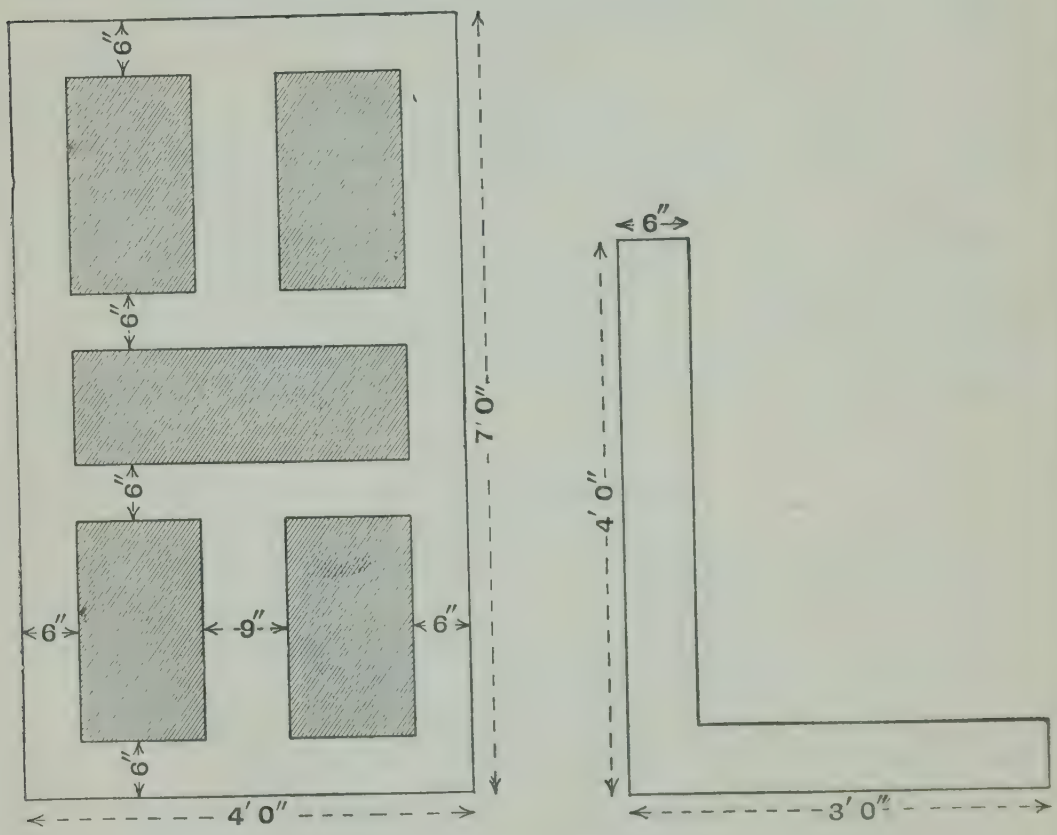
4. A plan is drawn to a scale of 6″ to a mile; what is the length of a line on the plan representing a road 620 yards long? Give the answer, in inches, correct to 3 significant figures.

5. A gunner takes his range from a map, drawn to a scale of 2″ to a mile. On the map the distance is represented by 2·43 inches; find the range in yards to the nearest 50.

6. Measure in inches a cupboard door 5 ft. by 2 ft. 6 in. On squared paper (tenth of an inch) make a drawing of the door to a scale of $\frac{1}{10}$. Calculate its area.

7. A door is 6 ft. 6 in. high and 4 ft. wide. Make a drawing of it (i) on squared paper to a scale of 0·4 inch to 1 foot, (ii) on plain paper to a scale of $\frac{3}{4}$″ to 1′. Calculate the area.

8. Taking as a model the letter L in the figured sketch below, on squared paper draw a letter L with the following dimensions: height 2 in., base $1\frac{1}{2}$ in., width of letter $\frac{3}{10}$ in. Find its area in square inches.



EXERCISE 236.

1. Find the area of the letter L from the figured sketch.

2. Measure the drawing of L in the figured sketch; hence state the scale.

3. In the same style and to the same scale as L draw figured sketches of the letters E, F, H, T; and calculate the area of each letter.

4. Find the area of the panels shaded in the figured sketch of a door.

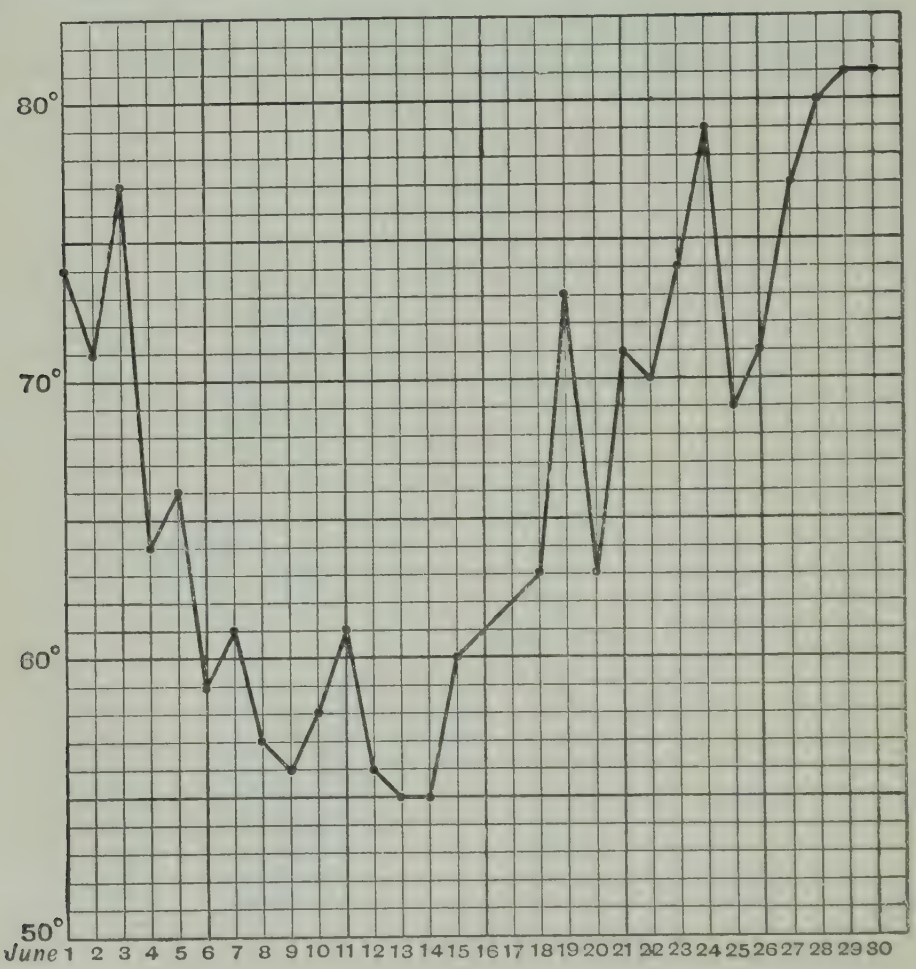
5. A slate is $12\frac{1}{2}$ in. by $8\frac{1}{2}$ in., and the frame round it is $\frac{1}{2}$ in. wide. Make a figured sketch of the frame, and find its area in square inches.

6. A picture frame 10 in. long and 7 in. broad is made of wood $\frac{1}{2}$ in. wide. Make a figured sketch of the frame, and find how many square inches of wood it contains.

D′. SIMPLE GRAPHS.

241. It is sometimes convenient to record observations of temperature, the height of the barometer, and various statistics, graphically, by means of lines drawn upon squared paper.

For example, the following diagram shows the highest temperature for each day of June in a certain year:



It will be seen that each space in a horizontal line represents a single day, while each space in a vertical line represents 1 degree of temperature. The graph is obtained by joining the points which show the highest temperatures for consecutive days by straight lines, the maximum for June 1 being connected with that for June 2, that for June 2 with that for June 3, and so on.

EXERCISE 237.

SQUARED PAPER EXERCISE.

1. The temperatures in a schoolroom at 9 a.m. for ten successive days are 48° F., 52° F., 60° F., 58° F., 62° F., 54° F., 57° F., 61° F., 63° F., 66° F. Show by means of a graph the changes in temperature which take place.

2. The attendances at a day school during one week are: 196, 206, 198, 208, 200, 210, 204, 209, 199, 201. Draw a graph showing these variations clearly.

3. At intervals of 5 years the average price of a certain article was respectively: 16*s.* 6*d.*, 17*s.*, 18*s.*, 18*s.* 6*d.*, 19*s.* 6*d.*, 20*s.* 6*d.*, 21*s.*, 22*s.* 6*d.*, and 25*s.* Show the variation by means of a graph.

4. Show by means of a graph the variation in the expectation of life presented by the following occupations, the period at a certain age being given in each case: Lawyer, 30 years; doctor, 10; clergyman, 25; collier, 15; sailor, 17; policeman, 19; cotton-spinner, 21; farm labourer, 28; tram-driver, 16; shopkeeper, 23.

5. During an attack of fever a patient's temperature varies as follows: 99° F., 100° F., 101·5° F., 103° F., 104° F., 103° F., 104° F., 103·5° F., 102·5° F., 101° F., 100° F., 101° F., 99° F. Show the variation graphically.

6. During ten successive Sundays the collections in a church were: £5. 10*s.*, £6. 5*s.*, £10. 10*s.*, £5. 15*s.*, £7. 10*s.*, £8. 15*s.*, £5. 15*s.*, £9. 10*s.*, £10, and £6. 15*s.* Draw a graph showing the variations.

E′. PERCENTAGE.

242. It is often desirable to compare quantities by reference to a common standard. To do this we express the fraction which one quantity is of another by a symbol which has 100 for its denominator.

The first quantity is thus estimated in *hundredths*, or as a **percentage**, of the other.

E′ (1). To Find the Rate Per Cent.

243. To find how much per cent. one quantity (A) is of another (B) of the same kind, we express A as a vulgar fraction of B and we then convert this fraction into a fraction having 100 for its denominator.

EXAMPLE 1. *Compare as percentages:* 13 *marks out of a possible* 20 *and* 16 *marks out of* 25.

(i) 13 marks out of $20 = \frac{13}{20}$ of maximum $= \frac{65}{100}$ of maximum;

i.e. 13 marks = **65 per cent. of the maximum.**

(ii) 16 marks out of $25 = \frac{16}{25}$ of maximum $= \frac{64}{100}$ of maximum;

i.e. 16 marks = **64 per cent. of the maximum.**

The words *per cent.* are expressed by the symbol %.

EXERCISE 238.

What percentage is equivalent to each of the following?

1. 3 of 6. 2. 3 of 4. 3. 3 of 5. 4. 3 of 8.

Express the following ratios as percentages:

5. 14 : 56. 6. 28 : 35. 7. 16 : 25. 8. 21 : 40.

What rate per cent. is equivalent to each of the following?

9. $\frac{2}{3}$. 10. $\frac{1}{6}$. 11. $\frac{7}{40}$. 12. $\frac{9}{80}$.

Find, correct to 1 decimal place, what percentage

13. 50 is of 64. 14. 34 is of 95. 15. 402 is of 560.

Find, correct to 3 significant figures, what percentage

16. 9 is of 14. 17. 52 is of 85. 18. 97 is of 130.

EXAMPLE 2. *What percentage of* £5 *is* 4*s.* 6*d.* ?

(Compare Example 1, page 178.)

$$\text{The ratio} = \frac{4s.\ 6d.}{£5} = \frac{9}{200};$$

$$\therefore \text{ the percentage} = \frac{9}{200} \text{ of } 100 = \underline{4\tfrac{1}{2}\%}.$$

EXERCISE 239.

Find what percentage

1. 3*d.* is of 1*s.*
2. 6*d.* is of £1.
3. 2*d.* is of 1*s.*
4. 2*s.* 6*d.* is of £1.
5. 2 oz. is of 1 lb.
6. 9 in. is of 1 yard.
7. 1 pint is of 1 gallon.
8. 220 yds. is of 1 mile.
9. 1 cwt. is of 1 ton.
10. 45 min. is of 1 hour.

EXERCISE 240.

1. At an examination a boy obtained 135 marks out of a total of 250; what percentage of the total is this ?

2. Forty boys enter for an examination and 37 pass satisfactorily; what is the percentage of failure ?

3. Last term a school had 280 pupils, and this term there are 294 pupils; what is the increase per cent. ?

4. The population of a village ten years ago was 4250 and it is now 3740; what is the decrease per cent. on the former number ?

5. The correct weight of a sack of coals should be 224 lb. A dealer sells a sack of coals weighing 196 lb.; of what percentage of the correct weight is a customer cheated ?

6. Out of a mixture of 3 cwt. 21½ lb. of coffee and 17 lb. 8 oz. of chicory, what percentage of the whole is coffee ?

7. In a form of 32 boys, 25 are present every time, that is 10 times in a week, six, 9 times, and one, 8 times; what is the percentage of attendance ? Give the answer in decimal form.

8. At an examination a boy obtains 419 marks out of a possible 560; what percentage of the maximum is this? Give the answer correct to 1 decimal place.

9. At an examination there are 1883 candidates, and of this number 538 are successful. (i) What percentage of the candidates are successful, (ii) what percentage fail? Give the answer correct to 3 significant figures.

10. At an examination, in which the maximum mark for each subject is 50, a boy receives 48, 32, 18, 39 and 20 respectively for five subjects; what percentage of the total possible marks does he obtain? Give the answer correct to 3 significant figures.

EXERCISE 241.

The following table shows the result of the county cricket championship during a certain year. Show in the last column the number of points obtained, expressed as a percentage as far as 2 decimal places of the number of points possible.

	No. of Points Possible.	No. of Points Obtained.	Percentage.
Middlesex - -	100	77	
Lancashire - -	130	97	
Surrey - - -	115	79	
Kent - - -	120	82	
Yorkshire - -	120	81	
Sussex - - -	140	90	
Nottinghamshire -	85	52	
Gloucestershire -	85	40	

SUPPLEMENTARY EXAMPLES.

Re-work Exercises 220-222, expressing all the results as percentages. Re-number the Exercises, 241 A, B, C.

E′ (2). To Find a Percentage.

244. To find a percentage of a number or quantity, we first express the rate per cent. as a vulgar or decimal fraction.

EXAMPLE 1. *In an examination, out of a possible* 60 *marks a boy gets* 65 *per cent.; how many marks does he get?*

$$\text{No. of marks} = \tfrac{65}{100} \text{ of } 60 = 60 \times \tfrac{13}{20} = 3 \times 13 = \underline{39}.$$

EXERCISE 242.

What fraction is equivalent to each of the following ?

1. 50%.	2. 25%.	3. 20%.	4. 15%.
5. 10%.	6. $33\frac{1}{3}$%.	7. $12\frac{1}{2}$%.	8. $7\frac{1}{2}$%.
9. $3\frac{3}{4}$%.	10. $2\frac{1}{2}$%.	11. $1\frac{1}{4}$%.	12. 40%.
13. 80%.	14. 30%.	15. $66\frac{2}{3}$%.	16. $37\frac{1}{2}$%.
17. $62\frac{1}{2}$%.	18. $87\frac{1}{2}$%.	19. $6\frac{1}{4}$%.	20. $18\frac{3}{4}$%.

EXERCISE 243.

1. A school has 350 names on the roll. If 4 per cent. of the pupils are absent, what is the number of absentees ?

2. At the beginning of the year 560 boys attended a certain school. By the end of the year the number had increased by $6\frac{1}{4}$ per cent. ; how many boys were in the school at the end of the year ?

3. At an examination there were 40 candidates, $92\frac{1}{2}$ per cent. of whom passed. The rest failed ; how many candidates failed ?

4. A sack of coals should contain 224 lb. There is a deficiency of $12\frac{1}{2}$ per cent. in the weight given by a dealer ; how many pounds of coal did his sack contain ?

5. The population of a village is now 3740. Five years ago the population was 15 per cent. more than this number ; what was the population five years ago ?

6. In a town of 30000 inhabitants the death-rate for a certain year was 10·3 per 1000. What percentage was this ? And how many deaths were there ?

EXAMPLE 2. *Find the value of* 4% *of* £2. 10*s.*

Value $= \frac{4}{100}$ of £2. 10*s.* $= \frac{1}{25}$ of 50*s.* $=$ **2s.**

EXERCISE 244.

Find the value of

1. 50% of £5. 14*s.*	2. $33\frac{1}{3}$% of £5.	3. 25% of £6. 6*s.*
4. 20% of £7. 15*s.*	5. 15% of £150.	6. $12\frac{1}{2}$% of £3. 16*s.*
7. 10% of £9. 5*s.*	8. $7\frac{1}{2}$% of £26.	9. 5% of £1. 15*s.*
10. $3\frac{3}{4}$% of £17.	11. $2\frac{1}{2}$% of £42.	12. $1\frac{1}{4}$% of £90.
13. 40% of £1. 5*s.*	14. 80% of £1. 16*s.* 8*d.*	

Find the value of

15. 30% of £1. 15*s*. 10*d*.
16. $66\frac{2}{3}$% of £1. 15*s*. 6*d*.
17. 75% of £1. 10*s*. 8*d*.
18. 90% of £2. 12*s*. 6*d*.
19. $37\frac{1}{2}$% of £2. 6*s*. 8*d*.
20. $62\frac{1}{2}$% of £2. 8*s*. 8*d*.

EXERCISE 245.

Find the value of

1. $12\frac{1}{2}$% of 14 tons 6 cwt.
2. $33\frac{1}{3}$% of 23 lb. 4 oz.
3. 2% of 3 days 7 hrs. 10 min.
4. 75% of 3 yds. 2 ft. 8 in.
5. 20% of 7 miles 6 fur. 5 chains.
6. 50% of 1 gall. 3 qts.
7. $8\frac{1}{3}$% of 8 sq. yds. 4 sq. ft. 72 sq. in.
8. 10% of 126 c. yds. 18 c. ft.

EXERCISE 246.

1. In a school of 250 pupils 24 per cent. are candidates at an examination. Of this number 5 per cent. fail ; how many candidates pass ?

2. At an examination 325 candidates are presented, of whom 28 per cent. fail in Modern Languages, 16 per cent. in Classics and 12 per cent. in Mathematics ; how many candidates pass in each group of subjects ?

3. A man had £375. He first spent 10 per cent. of his money and then 12 per cent. of the remainder ; how much has he left ?

4. A mixture of coffee and chicory weighing 35 lb. contains $6\frac{1}{4}$ per cent. of chicory. The remainder is coffee ; how much coffee and how much chicory does the mixture contain ?

5. If 75 per cent. of gunpowder is nitre, 10 per cent. sulphur and 15 per cent. charcoal, what weight of each will be required in making 15 cwt. of gunpowder ?

6. The proportion of nitrogen in the air is 79 per cent. and of oxygen 20 per cent. What quantity of nitrogen and oxygen respectively will there be in a room containing 181700 cubic feet of air ?

E′ (3). Calculations correct to the nearest penny.

245. The usual method of calculation is to reckon $\frac{1}{2}d$. and fractions greater than $\frac{1}{2}d$. as 1*d*., and to ignore fractions less than $\frac{1}{2}d$.

Thus $6\frac{3}{8}d$. is reckoned as 6*d*. and $6\frac{5}{8}d$. as 7*d*.

EXERCISE 247.

Find the value of the following correct to the nearest penny :

1. $12\frac{1}{2}\%$ of 11*s.* 7*d.* 2. $37\frac{1}{2}\%$ of 4*s.* 3*d.* 3. $62\frac{1}{2}\%$ of 5*s.* 9*d.*
4. $87\frac{1}{2}\%$ of 7*s.* 11*d.* 5. $6\frac{1}{4}\%$ of 4*s.* 2*d.* 6. $18\frac{3}{4}\%$ of 14*s.* 5*d.*
7. $31\frac{1}{4}\%$ of 15*s.* 7*d.* 8. $43\frac{3}{4}\%$ of 17*s.* 5*d.*

E′ (4). Business Applications of Percentage.

246. When a person employs an agent to buy or sell property of any kind he pays the agent for his trouble by giving him a *percentage* on the value of the property bought or sold. The money thus paid to the agent is called his **commission**. The agent is sometimes called a **broker**, and his commission is sometimes called **brokerage.**

247. In the case of an *Insurance Company*, which, in return for certain payments, undertakes to make good a loss from fire or shipwreck, or to pay a certain sum of money after a man's death to his representatives, the person who insures his property or his life pays annually to the company a certain percentage of the sum insured. The percentage in this case is called a **premium**. The written agreement is called a **policy.**

248. Tradesmen sometimes encourage customers to pay promptly by allowing a reduction on the amount of an account. This reduction is usually a percentage on the amount of the account, and is called **cash discount.**

EXERCISE 248.

Complete the following table :

DISCOUNT TABLE.

50 % =	in the £.	10 % =	in the £.
$33\frac{1}{3}\%$ =	.....	$7\frac{1}{2}\%$ =	.....
25 % =	.....	5 % =	.....
20 % =	.....	$3\frac{3}{4}\%$ =	.....
15 % =	.....	$2\frac{1}{2}\%$ =	.....
$12\frac{1}{2}\%$ =	.....	$1\frac{1}{4}\%$ =	.....

249. The method employed in business for the calculation of discount will be understood from the following example.

EXAMPLE. *A tradesman's account for* £61. 12*s.* 9*d. is subject to a discount of* $2\frac{1}{2}\%$ *for cash. Find how much a customer should pay in ready money, correct to the nearest penny.*

Discount $= \frac{1}{40}$ of £61. 12*s.* 9*d.* = £1. 10*s.* $9\frac{3}{40}$*d.*

= £1. 10*s.* 10*d.*, correct to the nearest penny;

∴ Amount payable = £61. 12*s.* 9*d.* − £1. 10*s.* 10*d.*

= **£60. 1s. 11d.**

Or, using **Decimals**: £61. 12*s.* 9*d.* = £61·638. [Art. 172.]

$$\text{Discount} = \frac{£61{\cdot}638}{40} = £1{\cdot}541 \qquad \text{[Art. 173.]}$$

= £1. 10*s.* 10*d.*, correct to the nearest penny.

EXERCISE 249.

Calculations to be made correct to the nearest penny.

1. What is an auctioneer's commission at 10% for selling furniture worth £875?

2. A broker buys £2000 worth of cotton; what is his commission at $\frac{1}{4}\%$?

3. A house-agent collects my rents and charges me 5% commission; how much do I receive if my rents amount to £145?

4. A farmer sends 10 cows to market to be sold. They fetch 45 guineas each. The auctioneer deducts his commission of $2\frac{1}{2}\%$ and pays over the balance; how much does the farmer receive?

5. A land-agent sells property to the value of £5692. 15*s.*; what is his commission at $1\frac{1}{2}\%$?

6. A man insures his life in an insurance company for £1500 at the rate of £2. 2*s.* 6*d.* per cent. per annum; what does the annual premium on the policy of assurance amount to?

7. The annual premium on a fire insurance policy being $\frac{1}{8}\%$, what is the amount of premium to be paid for insuring property for 1000 guineas?

8. A bookseller used to take 2*d.* in the shilling off the published price of books. (i) What was the cash price of a book published at 10*s.* 6*d.*? (ii) Express the discount price as a percentage of the published price.

9. For cash payment a shopkeeper allows $2\frac{1}{2}\%$ discount. What is the amount to be deducted from an account for £17. 10*s.* ?

10. What is the amount actually paid in settlement of an account for £15. 16*s.* 6*d.*, less a cash discount of $2\frac{1}{2}\%$?

E′ (5). Inverse Questions on Percentage.

250. The method of working such questions will be understood from the following example.

EXAMPLE. *In an examination a boy obtains* 39 *marks or* 65 *per cent. of the highest possible number ; what is the maximum ?*

Method 1. Let x = the number ; then

$$\frac{65}{100} \text{ of } x = 39 ;$$

$$\therefore\ x = \frac{39 \times 100}{65} = 60 ;$$

$\therefore$ required maximum = $\underline{60.}$

Method 2.

Maximum from which 65 marks are obtained = 100 ;

$\therefore$ ..... 39 marks ..... $= \frac{39 \times 100}{65} = \underline{60.}$

EXERCISE 250.

1. How long is an arithmetic lesson if 9 minutes is 20% of the time ?

2. What are the highest possible marks for a test paper in arithmetic if 45 marks are 75% of the total ?

3. In a form $87\frac{1}{2}\%$ of the pupils on the roll are present ; if the actual number present is 21, what is the number on the roll ?

4. From a length of cord 10·08 feet are cut off ; if this quantity is 16% of the original length, what was the original length ?

5. Of the oranges in a case $7\frac{1}{2}\%$ are bad and 370 are good ; how many oranges does the case contain ?

6. At an examination 36 boys passed. The percentage of failure was 20 ; how many boys entered for the examination ?

7. The population of a certain town is 6923, and is 40% more than it was ten years ago ; what was the population then ?

8. An auctioneer's commission was $7\frac{1}{2}\%$ of the value of the property which he sold. He received £3. 18*s.* ; what did the property realize ?

9. An agent charges me $2\frac{1}{2}\%$ for collecting rents ; how much money does he collect if, after deducting his commission, he pays me £195 ?

10. A house is insured for three-fourths of its value at $1\frac{1}{2}\%$, and the premium is £18 ; what is the value of the house ?

F′. PROFIT AND LOSS.

251. The price at which an article is bought is called the **cost price**; the price at which it is sold is called the **selling price.** When an article is sold for more than it cost, it is said to be sold at a **profit**; when it is sold for less, it is said to be sold at a **loss.**

Supposing A sells for £6 an article which cost him £5, and that B sells for £5 an article which cost him £4, we see that in each case the profit is £1; but the *rate of profit* is not the same: A makes a profit of £1 on an outlay of £5 and B makes the same profit on an outlay of £4.

To compare the two rates of profit, we must take a **standard outlay** (or **cost price**), say £100, thus:

A's Rate of Profit.	B's Rate of Profit.
Profit on £5 = £1;	Profit on £4 = £1;
∴ profit on £1 = £0·2;	∴ profit on £1 = £0·25;
∴ profit on £100 = £20.	∴ profit on £100 = £25.

252. Profit or loss may be reckoned in general terms as **a percentage of the money originally spent** (or the **cost price**).

The standard cost price is represented by £100.

A selling price giving 20% profit is represented by £120.

A selling price giving 20% loss is represented by £80.

253. To find the profit or loss per cent., given the cost price and the selling price.

EXAMPLE. *An article bought for* £3 *is sold for* £3. 15*s.*; *what is the rate of profit per cent.?*

Profit on 60*s.* = 15*s.*;

$$\therefore \text{ profit on } 100s. = \frac{100}{60} \times 15s. = 25s.;$$

∴ rate of profit per cent. = 25.

EXERCISE 251.

Find the rate of profit or loss per cent. in the following cases:

	Cost Price.	Selling Price.		Cost Price.	Selling Price.
1.	3*s.*	3*s.* 9*d.*	2.	3*s.*	2*s.* 3*d.*
3.	1*s.* 8*d.*	1*s.* 9*d.*	4.	4*s.* 2*d.*	3*s.* 11*d.*

Find the rate of profit or loss per cent. in the following cases:

	Cost Price.	Selling Price.		Cost Price.	Selling Price.
5.	6*s.* 8*d.*	7*s.* 2*d.*	6.	5*s.*	4*s.* 3*d.*
7.	4*s.* 7*d.*	7*s.* 4*d.*	8.	3*s.* 9*d.*	2*s.* 3*d.*
9.	4*s.* 6*d.*	6*s.*	10.	1*s.* 4*d.*	1*s.* 2*d.*

EXERCISE 252.

1. A man buys a house for £800 and sells it for £1000; what percentage profit does he make on his outlay?

2. A profit of 4*s.* 8*d.* is made by selling an article for £2. 11*s.* 4*d.*; what is the profit per cent. on the cost price?

3. A dealer buys a gross of articles for £9 and sells them at 1*s.* 4*d.* each; what is the gain per cent. on his outlay?

4. A provision dealer buys 1 cwt. of butter for £14 and sells it at 2*s.* 9*d.* per lb.; what does he gain or lose, and what does he gain or lose per cent. on his outlay?

5. A shopkeeper buys tea at £10 per cwt. and sells it at the rate of $2\frac{1}{4}d.$ per oz.; what is the profit per cent. on his outlay?

6. A profit of 2*s.* 3*d.* in the pound is made on an outlay of £262. 5*s.*; what is the total profit (to the nearest penny), and what is the profit per cent.?

7. An ironmonger buys screws at 10*d.* per gross and sells them at the rate of 10 for a penny; on an outlay of £1 what is the profit (to the nearest penny), and what is the profit per cent.?

8. A grocer buys sugar at £5. 2*s.* 8*d.* per cwt. and sells it at 1*s.* per lb.; what does he gain on 100 lb., and what does he gain per cent. on his outlay?

254. To find the selling price, given the cost price and the profit or loss per cent.

EXAMPLE. *An article bought for* £3 *is sold at a profit of* 25 *per cent.; what is the selling price?*

Method 1.

Cost price = 60*s.*,

profit = $\frac{25}{100}$ of 60*s.* = 15*s.*;

$\therefore$ selling price = 75*s.* = **£3. 15s.**

Method 2.

Ratio of selling price to cost price = $\frac{125}{100} = \frac{5}{4}$;

$\therefore$ selling price = $\frac{5}{4}$ of £3 = **£3. 15s.**

EXERCISE 253.

Find the selling price in the following cases :

Cost Price.	Profit.	Cost Price.	Loss.
1. 3*s.*	25 %.	2. 3*s.*	25 %.
3. 1*s.* 8*d.*	5 %.	4. 5*s.*	5 %.
5. 4*s.* 6*d.*	$33\frac{1}{3}$%.	6. 1*s.* 4*d.*	$12\frac{1}{2}$%.
7. 6*s.* 8*d.*	$7\frac{1}{2}$%.	8. 4*s.* 2*d.*	6 %.
9. 9*s.* 2*d.*	60 %.	10. 3*s.* 9*d.*	40 %.

EXERCISE 254.

1. A shopkeeper buys 3 dozen cricket balls for £16. 16*s.* and sells each ball at a profit of $12\frac{1}{2}$% on his outlay ; what is the selling price of each ball, and what is his total profit ?

2. A draper buys 23 yards of silk for £30 and sells it so as to gain 15% on the cost price ; what is the selling price per yard ?

3. A man buys two pictures, one for £40, the other for £60 ; he sells the first at a profit of 30% and the second at a profit of 15%. At what price does he sell each picture ; and what is his percentage profit on his whole outlay ?

4. A grocer buys 1 cwt. of sugar for 5 guineas and sells it by the pound at a profit of $6\frac{2}{3}$% on his outlay ; how much per lb. does he charge ?

5. A provision dealer buys a cheese, weighing 53 lb., for £4. 8*s.* 4*d.* ; at what price per lb. must he sell it so as to gain 15% on his outlay ?

6. A shopkeeper buys £260 worth of goods. He sells three-fourths of them at a gain of 15% and the rest at a loss of 5% on his outlay ; what is the total amount received from the sale ?

7. A grocer buys butter at £14 per cwt. and he sells half at 3*s.* per lb. ; at what price per lb. must he sell the rest so as to gain 40% on his outlay ?

8. A Parisian shopkeeper buys a certain commodity at the rate of 1500 francs per kilogram ; what must be his selling price per gram in order to gain 8% on his outlay ?

255. To find the cost price, given the selling price and the profit or loss per cent.

EXAMPLE 1. *An article is sold for* £3. 15*s.* *at a profit of* 25 *per cent. on the cost price ; what is the cost price ?*

$$\begin{aligned}\text{Selling price} &= \tfrac{125}{100} \text{ of cost price}\\ &= \tfrac{5}{4} \text{ of cost price};\\ \therefore \text{ cost price} &= \tfrac{4}{5} \text{ of selling price}\\ &= \tfrac{4}{5} \text{ of } 75s.\\ &= \underline{\text{£3.}}\end{aligned}$$

EXAMPLE 2. *An article is sold for* £2. 5*s.* *at a loss of* 25 *per cent. on the cost price ; what is the cost price ?*

$$\begin{aligned}\text{£2. 5}s. &= \tfrac{75}{100} \text{ of cost price}\\ &= \tfrac{3}{4} \text{ of cost price};\\ \therefore \text{ cost price} &= \tfrac{4}{3} \text{ of £2. 5}s.\\ &= \underline{\text{£3.}}\end{aligned}$$

EXERCISE 255.

Find the cost price in the following cases :

Selling Price.	Profit.	Selling Price.	Loss.
1. 3*s.* 9*d.*	25 %.	2. 2*s.* 3*d.*	25 %.
3. 1*s.* 9*d.*	5 %.	4. 4*s.* 9*d.*	5 %.
5. 1*s.* 6*d.*	$12\frac{1}{2}$%.	6. 1*s.* 2*d.*	$12\frac{1}{2}$%.
7. 7*s.* 2*d.*	$7\frac{1}{2}$%.	8. 3*s.* 11*d.*	6 %.
9. 7*s.* 4*d.*	60 %.	10. 4*s.* 3*d.*	15 %.

EXERCISE 256.

1. A piano was sold for £84 at a profit of $16\frac{2}{3}$% on the cost price ; what was the cost price ?

2. A picture was sold for £55. 16*s.* at a loss of 7% on the cost price ; what was the cost price ?

3. A number of articles are sold at 4*s.* 7*d.* each at a profit of 10% on the cost price. The total profit is £12 ; how many articles are sold ?

4. A shopkeeper sells walnuts for 6*s.* per 100 and gains 20% on his outlay ; at what price per score did he buy walnuts ?

5. A grocer sells butter at 2*s.* 3*d.* per lb. at a profit of $12\frac{1}{2}\%$ on his outlay ; what is the cost price per cwt. ?

6. A dealer by selling coal at 2*s.* $7\frac{1}{2}$*d.* per cwt. gains 40% on his outlay ; what does the coal cost him per ton ?

7. A shopkeeper sells potatoes at 3*s.* per stone (14 lb.), and gains 60% on his outlay ; at what price per ton did he buy potatoes ?

8. A Parisian shopkeeper sells wire at 188 francs per kilometre at a loss of 6% on his outlay ; how many centimes per metre did the wire cost him ?

256. To compare (i) selling prices, and (ii) profits or losses per cent.

EXAMPLE 1. *An article is sold for* £2. 5*s.* *at a loss of* 25 *per cent. ; what selling price would have given a profit of* 25 *per cent. ?*

£2. 5*s.* $= \frac{75}{100}$ of cost ;

$\therefore$ cost $= \frac{100}{75}$ of £2. 5*s.*

$= \frac{4}{3}$ of £2. 5*s.* ;

reqd. selling price $= \frac{125}{100}$ of cost

$= \frac{5}{4} \times \frac{4}{3}$ of £2. 5*s.*

$= \frac{5}{3}$ of £2. 5*s.*

$=$ £3. 15s.

Or thus: Reqd. selling price $= \frac{125}{75}$ of £2. 5*s.* = etc.

EXAMPLE 2. *An article sold for* £3. 15*s.* *gives a profit of* 25 *per cent. ; what would be the gain or loss per cent., if it had been sold for* £2. 5*s.* *?*

£3. 15*s.* $= \frac{125}{100}$ of cost ;

$\therefore$ £2. 5*s.* $= \frac{\text{£2. 5s.}}{\text{£3. 15s.}}$ of $\frac{125}{100}$ of cost

$= \frac{45}{75} \times \frac{5}{4}$ of cost

$= \frac{3}{4}$ of cost

$= \frac{75}{100}$ of cost ;

$\therefore$ rate of loss $= 25\ \%$.

EXERCISE 257.

1. By selling an article for 16*s.* 6*d.* a shopkeeper loses 12% on his outlay ; for how much must he sell it to gain 12% ?

2. By selling a camera for £2. 2*s.* a boy gained 5% on his outlay ; at what price should he have sold it in order to gain 15% ?

3. If 25% is gained by selling an article for 11*s.* 8*d.*, at what price should it be sold so as to gain only $12\frac{1}{2}$% ?

4. If by selling sugar at £4. 16*s.* per cwt. a grocer would lose 4%, at what price per lb. must he sell it to gain 12% ?

5. A shopkeeper sells goods for £30 at a loss of 10% on his outlay. He ought to have sold them at a profit of 30% ; how much under their proper price were they sold ?

6. By selling an article for 1*s.* 7*d.* a shopkeeper loses 5% on his outlay ; what would he have gained or lost per cent. by selling it for 1*s.* 9*d.* ?

7. A shopkeeper selling an article for 12*s.* $10\frac{1}{2}$*d.* gains 3% on his outlay ; what would have been the gain per cent. if he had sold it for 13*s.* 6*d.* ?

8. A shopkeeper selling an article for £2. 12*s.* 6*d.* gains 5% on his outlay ; what would he have gained or lost per cent. by selling it for £2. 7*s.* 6*d.* ?

9. By selling goods for £6. 18*s.* 9*d.* a shopkeeper gains $12\frac{1}{2}$% on his outlay ; what would have been his gain or loss per cent. if he had sold them for £6. 15*s.* 8*d.* ?

10. If 6 kilograms of a commodity are sold for 37 francs 80 centimes at a profit of 5% on the cost price, what will be the gain per cent. if the selling price is increased by 20 centimes per kilogram ?

G′. INTEREST.

257. **Interest** is the money paid for the use of money lent.
The interest is reckoned at so much for every £100 lent for each year, *i.e.* at a certain **Rate per cent. per annum.**
The sum of money lent is called the **Principal.**

258. When it is agreed between the lender and the borrower that the interest shall be paid over periodically, as soon as it becomes due, the Principal is said to be lent at **Simple Interest.**

Thus, if the rate be 1 per cent. per annum, the interest for 1 year is £1 on every £100. At 7 per cent. per annum it follows that the interest for 1 year is £7 on £100; for 2 years £14; for half a year £3. 10*s.*, and so on.

G′ (1). Simple Interest.

EXAMPLE. *Find the Simple Interest on* £325 *for* 3 *years at* 7 *per cent. per annum.*

Method 1.

Interest on £325 for 1 year at $1\% = £\frac{325}{100} = £3{\cdot}25$.

..... 1 year .... $7\% = £3{\cdot}25 \times 7 = £22{\cdot}75$.

..... 3 years ... $7\% = £22{\cdot}75 \times 3 = £68{\cdot}25$

$= \underline{£68.\ 5s.}$

Method 2.

Interest on £100 for 3 years $= £7 \times 3 = £21$;

$\therefore$ ..... £325 for 3 years $= £325 \times \frac{1}{100} \times 21 = £\frac{13}{4} \times 21$

$= £68\frac{1}{4}$

$= \underline{£68.\ 5s.}$

259. **To find the Simple Interest**: *Multiply the Principal by the rate per cent. and by the number of years, and divide the result by* 100.

260. When we add the Interest to the Principal we get the **Amount,** *i.e.* the sum to which the Principal will amount at the end of a given time.

Thus, £325 lent at 7 per cent. per annum will amount to £393. 5*s.* (£325 + £68. 5*s.*) at the end of 3 years.

EXERCISE 258.

Write down the answers at sight.

Find 1 per cent. of the following sums of money :

1. £150.	2. £420.	3. £560.	4. £290.
5. £380.	6. £570.	7. £740.	8. £830.
9. £625.	10. £475.	11. £815.	12. £545.
13. £385.	14. £295.	15. £735.	16. £265.

EXERCISE 259.

Write down the results at sight.

Find (i) the Simple Interest for 1 year on, and (ii) the Amount of

1. £100 at 5%.	2. £200 at 5%.	3. £500 at 6%.
4. £150 ... 8%.	5. £420 ... 7%.	6. £560 ... 6%.
7. £290 ... 5%.	8. £625 ... 5%.	9. £475 ... 6%.
10. £385 ... 4%.	11. £545 ... 7%.	12. £815 ... 8%.

EXERCISE 260.

Check each result.

Find (i) the Simple Interest on, and (ii) the Amount of

1. £100 for 2 years at 5%.	2. £200 for 3 years at 5%.
3. £150 .... 2 ..... 8%.	4. £420 .... 3 ..... 7%.
5. £560 .... 2 ..... 6%.	6. £380 .... 3 ..... 6%.
7. £625 .... 2 ..... 4%.	8. £815 .... 3 ..... 8%.
9. £545 .... 2 ..... 7%.	10. £295 .... 3 ..... 4%.

EXERCISE 261.

Check each result.

Find (i) the Simple Interest on, and (ii) the Amount of

1. £200 for 6 months at 10%.	2. £500 for 2 months at 6%.
3. £250 .... 6 ..... 4%.	4. £250 .... 3 ..... 8%.
5. £570 .... 6 ..... 8%.	6. £265 .... 3 ..... 4%.
7. £475 .... 6 ..... 6%.	8. £645 .... 4 ..... 6%.

261. If the date on which a sum of money is lent and the date on which it is repaid with interest are given, *one only* of the days named is included in calculating the interest.

Thus the period from March 7th to May 19th

$= (24 + 30 + 19)$ days $= 73$ days $= \frac{73}{365}$ of a year $= \frac{1}{5}$ year.

EXERCISE 262.

Check each result.

Find (i) the Interest, and (ii) the Amount in the following cases:

	Principal.	Time.	Rate per annum.
1.	£740.	March 7th–May 19th.	5%.
2.	£525.	May 19th–October 12th.	5%.
3.	£390.	April 15th–November 20th.	5%.
4.	£385.	March 14th–December 31st.	5%.
5.	£650.	August 7th–December 31st.	10%.
6.	£260.	March 5th–October 10th.	10%.

262. Bankers disregard fractions of a penny in calculations. In the following example we find the interest to the *nearest* penny.

EXAMPLE. *Find correct to the nearest penny the Simple Interest on* £1715. 1*s.* 8*d.* *for* 2 *years* 6 *months at* 6 *per cent. per annum.*

$$\text{Interest} = \tfrac{6}{100} \times 2\tfrac{1}{2} \text{ of £1715. 1}s\text{. 8}d.$$
$$= \tfrac{3}{20} \text{ of £1715. 1}s\text{. 8}d.$$
$$= \text{£5145. 5}s\text{. 0}d. \div 20$$
$$= \underline{\text{£257. 5s. 3d.}}$$

EXERCISE 263.

Check each result.

Find the Simple Interest on

1. £473. 15*s.* for 2 years at 5%.
2. £250. 17*s.* 6*d.* for 2 years at 10%.
3. £227. 10*s.* for 2 years at 8%.
4. £390. 12*s.* 6*d.* for 2 years at 6%.
5. £162. 10*s.* for 3 years at 6%.

Find the Simple Interest on

6. £869. 7*s*. 6*d*. for 2 years at 7%.
7. £551. 15*s*. for 10 months at 6%.
8. £139. 7*s*. 6*d*. for 6 months at 4%.

EXERCISE 264.

Check each result.

Find, to the nearest penny, the Simple Interest on

1. £554. 10*s*. for 3 months at 4%.
2. £2160. 12*s*. 6*d*. for 1 year at 6%.
3. £365. 10*s*. for 6 months at 6%.
4. £5982. 7*s*. 6*d*. for $2\frac{1}{2}$ years at 6%.
5. £652. 7*s*. 6*d*. for $4\frac{1}{2}$ years at 8%.
6. £1261. 17*s*. 6*d*. for $1\frac{1}{4}$ years at 5%.
7. £286. 15*s*. 6*d*. for 3 years at 4%.
8. £3265. 12*s*. 6*d*. for $5\frac{1}{2}$ years at 4%.

G′ (2). Simple Interest and Decimal Monetary Systems.

263. The following examples require a knowledge of certain foreign monetary systems (Art. 148).

FRANCE:

100 centimes (c.) = 1 **franc** (f.).

UNITED STATES OF AMERICA

100 cents (c.) = 1 **dollar** ($).

EXERCISE 265.

Give your results correct to the nearest centime (*or cent*).

Find the Simple Interest on

1. 942 francs for 1 year at 7%.
2. 670 f. 75 c. for 8 months at 5%.
3. 215 dollars for 1 year at $6\frac{1}{4}$%.
4. 1052 dollars 80 c. for $2\frac{1}{2}$ years at 7%.
5. 216 francs for $2\frac{1}{2}$ years at $3\frac{1}{4}$%.
6. 422 f. 50 c. for 6 years at 9%.
7. 9216 dollars for $1\frac{3}{4}$ years at $5\frac{1}{2}$%.
8. 4968 dollars 75 c. for 3 years at $5\frac{1}{4}$%.

EXERCISE 266.

1. If £365 is borrowed on 17th May at $7\frac{1}{2}$ per cent. per annum, what sum will repay the debt on 9th November ?

2. A man borrows £20, interest being at the rate of $7\frac{1}{2}$ per cent. per annum, but he pays the interest monthly ; how much does the monthly interest amount to ?

3. I buy a house for £860 ; what rent must I charge in order to obtain $5\frac{3}{4}$ per cent. per annum on my purchase money if I have to pay an average of £5. 11*s.* per annum for repairs and insurance ?

4. Show that the Simple Interest on £225 lent at 5 per cent. will in 4 years be the same as the Simple Interest on £250 lent for 3 years at 6 per cent.

5. Find the difference in the Simple Interest on £525 lent for 2 years at $6\frac{1}{2}$ and 7 per cent. respectively.

6. What is the ratio of the Simple Interest on £60 for 6 months at 5 per cent. to that on £90 for 3 months at 10 per cent.

G′ (3). Inverse Simple Interest.

264. Of the four quantities, *Principal*, *Rate per cent.*, *Time* and *Interest*, any one can be found when the other three are given.

EXAMPLE 1. *What Principal will yield* £68. 5*s.* *interest in* 3 *years at* 7% *?*

The Interest on £100 = £7 × 3 = £21 ;

∴ Principal which yields £21 = £100 ;

∴ ..... £68·25 $= \frac{£100 \times 68{\cdot}25}{21} = £\frac{6825}{21}$

$= \underline{£325.}$

EXAMPLE 2. *In how many years will the interest on* £325 *become* £68. 5*s.* *at* 7% *?*

Interest on £325 at 7% for one year = £22. 15*s.*

and ..... in reqd. time = £68. 5*s.* ;

∴ required number of years $= \frac{£68.\ 5s.}{£22.\ 15s.}$

$= \frac{273}{91}$

$= \underline{3.}$

EXAMPLE 3. *At what rate per cent. will the interest on* £325 *become* £68. 5*s*. *in* 3 *years?*

Interest on £325 for 3 years at **one per cent.** = £9. 15*s*.

and ..... reqd. rate % = £68. 5*s*.;

$$\therefore \text{ required rate per cent.} = \frac{\text{£68. 5}s.}{\text{£9. 15}s.} = \frac{273}{39} = \underline{7}.$$

Check. The worked example (Art. 258) verifies the three results (Art. 264).

EXERCISE 267.

Check each result.

Find the Principal when the Interest in 1 year is

1. £4. 10*s*. at 10%. 2. £3. 18*s*. at 5%. 3. £5. 5*s*. at $6\frac{1}{4}$%.

Find in what time the Simple Interest on

4. £65 will become £3. 5*s*. at 5% per annum.
5. £85 will become £25. 10*s*. at 10% per annum.
6. £60 will become £2. 16*s*. at 7% per annum.

Find the rate per cent. per annum when the Simple Interest on

7. £60 in 1 year is £3. 8. £90 in 3 months is £2. 5*s*.

EXERCISE 268.

Check each result.

Find the Principal when the Simple Interest is

1. £28. 10*s*. for 1 year at 6%.
2. £29. 5*s*. for 2 years at 8%.
3. £2. 15*s*. 9*d*. for 6 months at 4%.
4. 65 francs 94 centimes for 1 year at 7%.

Find in what time the Simple Interest on

5. £560 will become £33. 12*s*. at 6% per annum.
6. £227. 10*s*. will become £36. 8*s*. at 8% per annum.
7. $40 will become $6.50 c. at 5% per annum.

Find the rate per cent. per annum when the Simple Interest on

8. £420 in 1 year is £29. 8*s*.
9. £162. 10*s*. in 3 years is £29. 5*s*.
10. $200 in 6 years is $90.

265. The Amount is equal to the sum of the Principal and Interest (Art. 260); and therefore when any two of these quantities are given, the third can be readily found.

EXERCISE 269.

Check each result.

Find in what time

1. £545 will amount to £621. 6*s*. at 7%.
2. £162. 10*s*. will amount to £191. 15*s*. at 6%.
3. £237. 10*s*. will amount to £245. 8*s*. 4*d*. at 5%.

Find at what rate per cent. per annum

4. £560 will amount to £627. 4*s*. in 2 years.
5. £139. 7*s*. 6*d*. will amount to £142. 3*s*. 3*d*. in 6 months.
6. £337. 10*s*. will amount to £421. 17*s*. 6*d*. in 4 years.

EXAMPLE 4. *What Principal will amount to* £393. 5*s*. *in* 3 *years at* 7% ?

£100 amounts to £121 under the given conditions.

Principal which amounts to £121 = £100;

$\therefore$ ..... $£393\frac{1}{4} = £100 \times \frac{393\frac{1}{4}}{121}$

$= £100 \times \frac{1573}{4} \times \frac{1}{121}$

$= £25 \times 13$

$= \underline{£325.}$

EXERCISE 270.

Check each result.

Find the Principal which will amount to

1. £675 in 2 years at 4%.
2. £489. 5*s*. in 6 months at 6%.
3. £263. 18*s*. in 2 years at 8%.

Find the Principal which will amount to

4. £259. 7*s.* in 2 years at 7%.
5. £421. 17*s.* 6*d.* in 4 years at $6\frac{1}{4}$%.
6. £347. 12*s.* 6*d.* in 8 months at $4\frac{1}{2}$%.

EXERCISE 271.

1. If £100 is lent at interest for £9. 2*s.* 6*d.* per cent. per annum, what is the interest per day ? (365 days = 1 year.)

2. A man borrows £1, agreeing to pay threepence per week as interest ; at what rate per cent. per annum is he paying ?

3. A moneylender charges £1 for a loan of £10 for a month ; how much per cent. per annum does he charge ?

4. A sum of £325 is lent on January 10th, 1921, and £334. 2*s.* is received on June 5th, as payment of principal and interest ; what rate of interest per annum is charged ?

5. The yearly rent of a house being £130, what should be the cost of the house that the purchaser may get $6\frac{1}{2}$% on his investment ?

6. A sum of £350 is lent on June 15th and is repaid as soon as the interest amounts to £3. 10*s.* ; at what date is the principal repaid ? (Rate of interest : 5% per annum.)

G′ (4). Compound Interest.

266. In **Compound Interest** the interest at the end of a stated period is added to the Principal : the amount so formed gives the Principal on which interest is reckoned for the next period ; and so on.

For example, suppose £100 be lent for 3 years at 5 %, the interest being payable yearly. At the end of the first year the interest due is £5. This is retained by the borrower and the principal becomes £105. For the second year the interest on £105 is £5. 5*s.* This again is retained, and the principal becomes £110. 5*s.* The interest for the third year is £5. 10*s.* 3*d.* ; therefore at the end of the 3 years the total amount due is £115. 15*s.* 3*d.*, and the increase is £15. 15*s.* 3*d.*

267. The following method should be adopted : we work throughout in decimals, writing results correct to 5 places, since the result is required to the nearest penny ; *i.e.* correct to 3 decimal places.

EXAMPLE. *Find to the nearest penny the amount, and the increase of,* £325 *in* 3 *years at* 7% *Compound Interest.*

	£325	= Principal for first year.
7% = $\frac{7}{100}$; ∴	22·75	= Interest for first year.
	347·75	= Principal for second year.
7% = $\frac{7}{100}$; ∴	24·3425	= Interest for second year.
	372·0925	= Principal for third year.
7% = $\frac{7}{100}$; ∴	26·04648	= Interest for third year.

Amount at end of 3 years = £398·139 = **£398. 2s. 9d.**;

∴ Increase = **£73. 2s. 9d.**

NOTE.—Compare the Compound and the Simple Interest on the same Principal for 3 years at 7 % (Art. 258).

EXERCISE 272.

Find the amount and the increase of the following sums, at Compound Interest, *to the nearest penny.*

1. £500 at 10% in 2 years.
2. £200 at 5% in 2 years.
3. £250 at 4% in 2 years.
4. £480 at 5% in 2 years.
5. £561. 2*s.* 6*d.* at 5% in 2 years.
6. £1760. 10*s.* at 4% in 2 years.
7. £4000 at 5% in 3 years.
8. £1750. 12*s.* 6*d.* at 4% in 3 years.
9. £923. 17*s.* 6*d.* at $2\frac{1}{2}$% in 3 years.
10. £4000 at $7\frac{1}{2}$% in 3 years.
11. £5000 at $4\frac{1}{2}$% in 3 years.
12. £1034. 3*s.* at $5\frac{1}{2}$% in 3 years.

In the next examples, add the interest half-yearly.

13. £500 at 5% in 2 years.
14. £1000 at 8% in 2 years.

H′. SQUARE ROOT.

H′ (1). General Method: Introduction.

268. We have learnt how to find the square root of a number which can be easily separated into prime factors (Arts. 68-71).

269. The general method of extracting the square root of a number is based upon the formula

$$(a+b)^2 = a^2 + 2\times a\times b + b^2 = a^2 + (2\times a + b)\times b. \quad \text{[Art. 52.]}$$

EXAMPLE. *Find the square of* 38.

Let $a = 30$ and $b = 8$;

$$\begin{aligned}\therefore\ 38\times 38 = (30+8)^2 &= (30)^2 + 2\times 30\times 8 + 8^2\\ &= (30)^2 + (2\times 30 + 8)\times 8\\ &= 900 + (60+8)\times 8\\ &= 900 + 544\\ &= \underline{1444.}\end{aligned}$$

The converse of this process will give the square root of 1444.

EXERCISE 273.

Using the above Method, and setting down all the working, find the value of

1. 13^2.	2. 17^2.	3. 19^2.	4. 23^2.	5. 31^2.
6. 37^2.	7. 47^2.	8. 53^2.	9. 73^2.	10. 79^2.
11. 89^2.	12. 97^2.	13. 44^2.	14. 75^2.	15. 84^2.

H′ (2). Square Root of a Whole Number.

270. Since $\sqrt{100} = 10$, and $\sqrt{10000} = 100$, and $\sqrt{1000000} = 1000$, etc., it follows that the square roots of all numbers between 100 and 10000 lie between 10 and 100, and consist of *two* digits; that the square roots of all numbers between 10000 and 1000000 lie between 100 and 1000, and consist of *three* digits, and so on.

Hence we see that *two* additional digits in a number lead to *one* additional digit in the square root.

If therefore we mark off the digits of a number in periods of *two*, beginning at the right hand, the number of these periods will be the same as the number of digits in the square root.

The left-hand period will sometimes contain only *one* digit.

For example, by marking off the digits of the numbers 169, 1444 and 56644 as follows 1'69, 14'44 and 5'66'44, we see that the square roots of these numbers have respectively, 2, 2 and 3 digits.

271. The general method of extracting the square root of a number will be seen from the following examples :

EXAMPLE 1. *Find the square root of* 1444.

This number is obtained by multiplying 38 by 38 (Art. 269). Hence the square root is 38.

We can find the number of *tens* in the square root by ascertaining what multiple of 10 has its square *next less than* 1444 : this is clearly 30, for $40^2 = 1600$, and is too great.

Having found the *tens*, we next find the *units*. Subtracting 30^2 or 900 from 1444, the remainder is 544 ; and (Art. 269)

$$544 = (2 \times 30 + 8) \times 8.$$

If we could divide 544 by $2 \times 30 + 8$, we should get the number of units 8 at once. But since the divisor itself involves this unascertained number 8, the plan adopted is to use the part 2×30 as a **trial** divisor, to find by means of it a **trial** quotient, and then to see whether

$$(2 \times 30 + \text{trial quotient}) \times \text{trial quotient} = 544.$$

The first trial quotient may not prove correct ; if it is too great, we try the number next less, and so on.

In this particular case, if we divide 544 by 2×30, *i.e.* by 60, we get a trial quotient 9. But $(60 + 9) \times 9 = 621$, and is too great ; we therefore try 8, and we find that $(60 + 8) \times 8 = 544$.

The digits of the square root 38 are thus found in succession.

The above operations may be stated concisely thus :

$$\begin{array}{r|l} 14,44 & 30+8, \\ 900 & \\ \hline 60+8 \mid 544 & \\ 544 & \\ \hline \end{array}$$

or thus

$$\begin{array}{r|l} & 3\ \ 8 \\ \hline 14,44 & \\ 9 & \\ \hline 68 \mid 544 & \\ 544 & \\ \hline \end{array}$$

The latter is the practical form, and the process is briefly as follows :

Step i. We mark off the digits in twos, beginning at the right hand.

Step ii. The greatest square in 14 is 9, and its square root is 3. We place 3 in the root place. We multiply 3 by 3 and subtract the product from 14. The remainder is 5.

Step iii. To this remainder we annex the period 44, and our dividend becomes 544. **Twice** the root digit 3 is 6, so we put 6 in the divisor.

Instead of 60 we take 6 as a *trial divisor*, and we take 54 as a *trial dividend*. Proceeding as explained above, we find that 8 is the true quotient. We annex 8 to the 3 in the root place, and also to the divisor. We multiply 68 by 8 and, subtracting the product from 544, we get no remainder. The square root is thus 38.

EXERCISE 274.

Find the square root of

1. 169.	2. 289.	3. 361.	4. 529.
5. 961.	6. 1369.	7. 2209.	8. 2809.
9. 5329.	10. 6241.	11. 7921.	12. 9409.

SUPPLEMENTARY EXAMPLES.

Re-work Exercise 99, 1-15, by the General Method of Art. 271.

EXAMPLE 2. *Find the square root of* 56644.

We first mark off the digits of 56644 in periods of *two*, beginning at the right hand.

There are 3 periods, and therefore 3 digits in the square root.

```
      2  3  8
    ------------
    | 5,66,44
    | 4
 43 | 166
    | 129
468 | 3744
    | 3744
    ------------
```

Step i. The root nearest to that of the first period 5 is 2. The square of 2 is 4. Subtracting 4 from 5, the remainder is 1. We bring down the second period, and we get 166 for the next dividend.

Step ii. The *trial* divisor is 2×2 or 4, and the *trial* dividend is 16. The true quotient is 3. We place 3 as the second digit in the root, and we also annex 3 to the trial divisor.

Step iii. In the next step the *trial* divisor is 46, obtained by doubling the 23 in the root; and the *trial* dividend is 374.

EXERCISE 275.

Find the square root of

1. 14161.	2. 22201.	3. 22801.	4. 64009.
5. 146689.	6. 226576.	7. 249001.	8. 253009.

H′ (3). The Square Root of a Decimal.

272. To find the square root of a decimal, we proceed exactly in the same way as in finding the square root of a whole number, but we mark off the periods *from the decimal point* both ways, right and left.

If the given decimal has an *odd* number of decimal places, 0 may be added to make a complete period, and we may then add 00 as many times as we please.

EXAMPLE 1. *Find the square root of* 1127·6164.

Square Root = 3 3·5 8

11,27,·61,64
9
63 | 227
189
665 | 3861
3325
6708 | 53664
53664

NOTE. Before bringing down the period 61, which comes next after the decimal point, put a decimal point in the root.

EXERCISE 276.

Find the square root of

1. 2·89.	2. 5·29.	3. 79·21.
4. 2·2801.	5. 2490·01.	6. ·001369.
7. ·011881.	8. 64·6416.	9. ·023409.
10. 170·3025.	11. 121·2201.	12. ·00000961.

EXAMPLE 2. *Find the square root of* ·144 *correct to* 2 *decimal places.*

Square Root = 0 · 3 7 9

0,·14,40,00
9
67 | 540
469
749 | 7100
6741
359

NOTE. To find the correct second decimal place, we must work to three places.

Reqd. square root = 0·38.

EXERCISE 277.

Find, correct to 2 decimal places, the square root of

1. 57·127. 2. ·016. 3. ·823.

Find, correct to 3 decimal places, the square root of

4. 1·6. 5. ·4. 6. ·002. 7. 64·34.

Find, correct to 3 significant figures, the square root of

8. 57·127. 9. ·016. 10. ·823. 11. ·002.

Find, correct to 1 decimal place, the square root of

12. $\frac{3}{10}$. 13. $\frac{7}{16}$. 14. $3\frac{1}{4}$. 15. $5\frac{1}{8}$.

EXERCISE 278.

1. The area of the top of a square table is 14 sq. ft. 9 sq. in.; find the length of a side.

2. The area of a square field is 10 acres; how many yards of fencing are required to enclose it?

3. How long will it take a man to walk round the field (Question 2) at the rate of a mile in $10\frac{3}{8}$ minutes?

4. The area of a square is equal to the area of a rectangle whose adjacent sides are 36 feet and 7 feet; find the length of a side of the square to the nearest foot.

Draw diagrams in connection with the solution of Ques. 5 *and* 6.

5. The area of a playground, which is twice as long as it is wide, is 338 sq. yards; find the dimensions of the playground.

6. The area of the floor of a room is 56 sq. yds. 8 sq. ft., and the room is twice as long as it is wide; find the dimensions of the floor.

J'. THE PARALLELOGRAM.

273. Any four-sided figure (*quadrilateral*), having its opposite sides equal and parallel, is called a **parallelogram.**

A straight line joining opposite angles of a parallelogram is called a **diagonal.**

A **rectangle** is a parallelogram in which all the angles are right angles (Art. 123).

A **square** is a rectangle with all its sides equal (Art. 121).

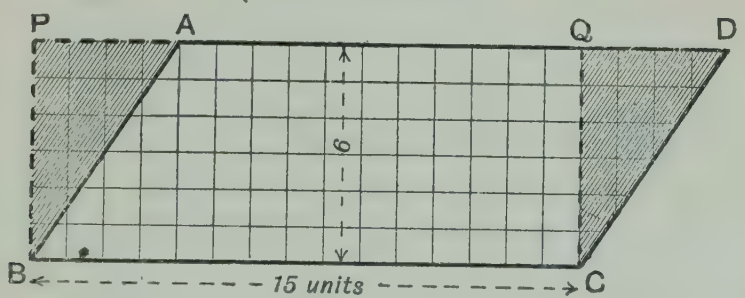
A **rhombus** is a parallelogram with all its sides equal, while its angles are not right angles.

274. We have learnt how to find the area of a rectangle (Art. 124); by the aid of squared paper we may learn how to find the area of a parallelogram.

EXERCISE 279.

SQUARED PAPER EXERCISE.

On squared paper draw a line BC 15 *units long, and on it construct a rectangle* PBCQ 6 *units high.*



1. How many small squares does the figure PBCQ contain?

From P *measure off* 4 *units to the right to* A, *and join* BA; *from* Q *measure off* 4 *units to the right to* D, *and join* CD.

NOTE. In counting the squares, omit parts less than half of a square and count parts greater than half as whole squares.

2. How many small squares does the figure ABCD contain?

3. Count the number of squares in each of the shaded figures APB and DQC.

4. What do you infer as to the area of the parallelogram ABCD as compared with that of the rectangle PBCQ ?

5. Verify this by drawing the parallelogram and the rectangle separately on squared paper and counting the number of squares in each.

6. Which line in the diagram represents the base of the figures ?

7. Which lines in the diagram represent the altitude of the figures ?

8. How can the area of any parallelogram be found ?

9. On squared paper draw a rectangle PBCQ 5 cm. by 4 cm., and on the same base BC, a parallelogram ABCD of the same height. Repeat the processes 1-5 above.

10. On squared paper draw a parallelogram ABCD, having BC = 6 cm., BP = 5 cm. and AP = 3 cm. Find its area.

275. *To calculate the area of a parallelogram : Multiply the number of units in the base by the number of units in the perpendicular height.*

Any side may be taken as **base**. The **height** is the perpendicular distance between the base and the side parallel to it.

EXERCISE 280

Check each result.

Calculate, in sq. ft. and sq. in., the area of a parallelogram whose base and perpendicular height are respectively

1. 5 ft. 9 in. and 1 ft. 9 in.
2. 6 ft. 9 in. and 4 ft. 6 in.
3. 5 ft. 6 in. and 3 ft. 9 in.
4. 7 ft. 6 in. and 5 ft. 9 in.

5. A gardener plants 55 seedlings in a bed which is in the form of a parallelogram, allowing 4 sq. ft. for each. Find the perpendicular distance between two parallel sides, each of which measures 20 ft.

SUPPLEMENTARY EXAMPLES.

Re-work Exercises **158** (1-9) and **197** (7-14).

K′. TRIANGLES.

276. A **triangle** is a plane figure bounded by three straight lines.

Any side of the triangle may be called its **base.**

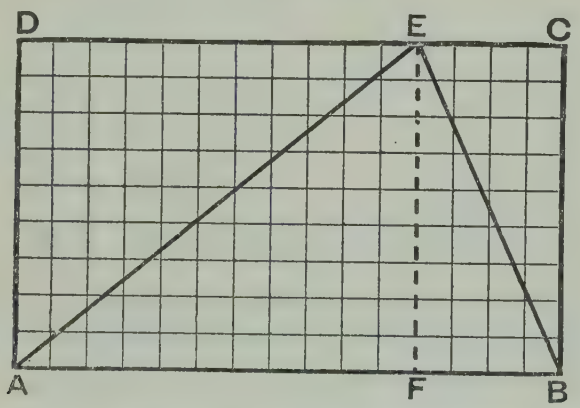
The **altitude** or **height** of a triangle is the perpendicular drawn to the base from the opposite angular point (the *apex* or *vertex*).

K′ (1). The Area of a Triangle, given the Base and Altitude.

EXERCISE 281.

SQUARED PAPER EXERCISE.

On squared paper draw a line AB 15 *units long; on* AB *as base construct a rectangle* ABCD 9 *units high.*



1. How many small squares does the figure ABCD contain?

From B *and* C *measure off* 4 *units to the left to* F *and* E *respectively, and join* EF *by means of a dotted line. Join* EA *and* EB.

2. How many small squares does the triangle ABE contain?

3. What do you infer as to the area of the triangle ABE as compared with that of the rectangle ABCD?

4. Which line in the diagram represents the *altitude* both of the triangle and of the rectangle? Which line represents the base of both figures?

5. By means of tracing paper, or otherwise, show that figures BCE and BEF are equal triangles; similarly, show that figures AEF and ADE are equal triangles.

277. *To calculate the area of a triangle, when the base and height are given : Multiply the number of units in the base by the number of units in the perpendicular height, and divide the product by* 2.

Using the letters A, b and h to denote the *number of units* in the *area, base* and *height* of the triangle respectively,

$$A = \tfrac{1}{2}bh.$$

EXERCISE 282.

Check each result.

NOTE. Give answers **1-2** in sq. cm., **3-6** in sq. ft. and sq. in., **7-10** in sq. yds. and sq. ft.

Calculate the area of a triangle whose base and height are respectively

1. 2·5 cm. and 1·5 cm.
2. 18·4 cm. and 10·5 cm.
3. 2 ft. 8 in. and 1 ft. 6 in.
4. 3 ft. 6 in. and 8 in.
5. 3 yds. and 4 ft. 6 in.
6. 11 ft. and 3 ft. 6 in.
7. 25 yds. and 8 ft.
8. 50 ft. and 3 yds.
9. 4 yds. 1 ft. and 5 yds. 1 ft.
10. 8 yds. 2 ft. and 6 yds. 2 ft.

11. The area of a triangular piece of wood is 5 sq. ft. 39 sq. in., and the altitude is 3 ft. 10 in. ; find the length of the base.

12. A gardener plants seedlings in a bed in the form of a triangle, the base of which is 2 ft. 8 in. and the perpendicular height 1 ft. 6 in ; if each seedling occupies a space 4 in. square, how many are planted ?

K′ (2). Right-angled Triangles.

278. A triangle having one of its angles a right angle is called a **right-angled triangle** ; either of the sides containing the right angle may be considered as the *base* of the triangle and the other as the *height*.

The side opposite the right angle is called the **hypotenuse.** In the figure below ABC is a right-angled triangle having the right angle at C ; the side AB is the hypotenuse.

EXERCISE 283.

NOTE. Give answers 1-2 in sq. cm., 3-6 in sq. ft. and sq. in.

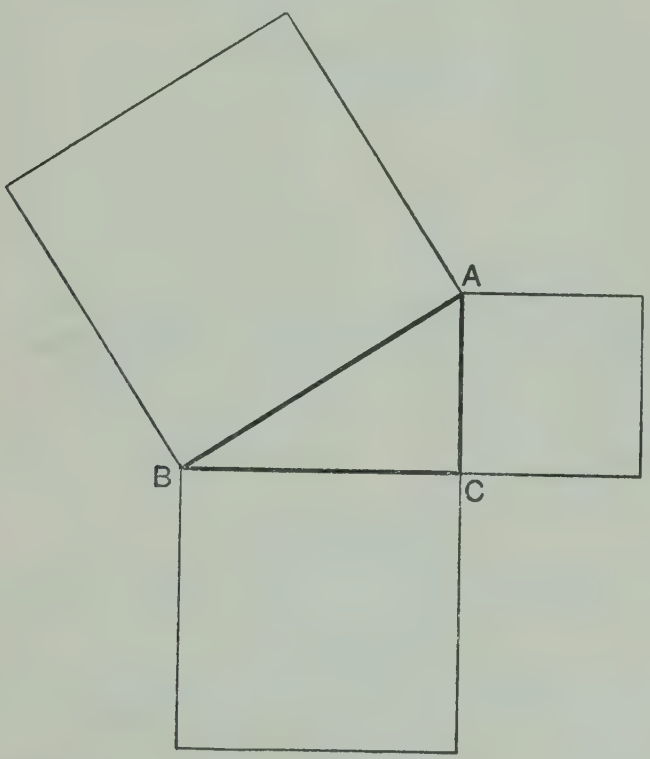
Find the area of a right-angled triangle when the two sides containing the right angle are respectively

1. 7 cm. and 5 cm.
2. 5·4 cm. and 2·5 cm.
3. 2 ft. 8 in. and 1 ft. 6 in.
4. 8 ft. 9 in. and 7 ft. 6 in.
5. 3 ft. 4 in. and 2 ft. 3 in.
6. 18 ft. 4 in. and 10 ft. 3 in.

EXERCISE 284.

SQUARED PAPER EXERCISE.

On squared paper draw a line BC 4 *cm. long, and at point* C *draw a straight line* CA, 3 *cm. long, at right angles to* BC. *Join* AB. ABC *is a right-angled triangle.*



Describe a square upon each of the three sides of the triangle, as shown in the figure above.

Measure the area of each of the squares, in every way you know. Write down your results thus :

1. Square on AC = ... sq. cm.
2. Square on BC = ... sq. cm.

3. Sum of results 1 and 2 = ... sq. cm.
4. Square on AB = ... sq. cm.
5. Difference between 4 and 3 = ... sq. cm.

Repeat the above with several right-angled triangles.

279. The important property of right-angled triangles, which we have just learnt, is known as the **Theorem of Pythagoras** from the name of the Greek mathematician who lived about 525 B.C.:

In any right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other sides.

EXERCISE 285.

Check each result.

Calculate the length of the hypotenuse of a right-angled triangle, if the other sides are respectively

1. 6 cm. and 8 cm.
2. 5 cm. and 12 cm.
3. 8 in. and 15 in.
4. 15 in. and 20 in.
5. 2·1 in. and 2·8 in.
6. 2·7 in. and 3·6 in.

Find the length of one side of a right-angled triangle, of which the hypotenuse and the other side are respectively

7. 13 cm. and 5 cm.
8. 25 in. and 7 in.
9. 4·5 cm. and 2·7 cm.
10. 3·5 in. and 2·1 in.

EXERCISE 286.

Diagrams should be drawn in connection with the solutions of the following questions.

1. Find, to the nearest hundredth of an inch, the length of the diagonal of a square, the sides of which are 5 inches long.

2. The diagonal of a rectangle is 25 inches long, and one of the sides is 16 inches long; find, to the nearest tenth of an inch, the length of the other side.

3. A footpath goes along two adjacent sides of a rectangular playing field 52 yards long and 39 yards wide. Find the distance which would be saved if the path were straight across the ground from corner to corner.

4. The foot of a ladder is 7 ft. away from a vertical wall, and the height of the top of the ladder, which rests against the wall, is 24 ft. Find the length of the ladder.

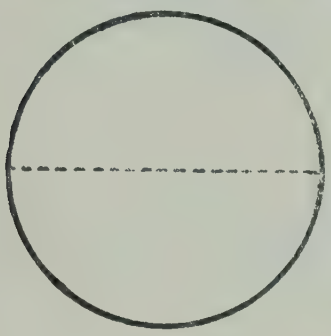
5. A ladder 45 ft. long reaches a window 36 ft. from the ground. Find the distance of the foot of the ladder from the side of the house.

6. Find the cost of painting the gable end of a house at 3*s.* per sq. yard. The width of the house is 25 ft., the ridge of the roof rises 10 ft. above the eaves and the eaves are 32 ft. from the ground.

L′. THE CIRCLE.

Pupils should be led to discover the processes, or rules, by their own measurements based upon a model of a large wooden cylinder, such as is used in the Art Room.

280. A **circle** is a plane figure bounded by one line called the **circumference,** and is such that all straight lines drawn to the circumference from a certain point called the **centre** are equal to one another.



A line drawn through the centre, and terminated at both ends by the circumference, is called a **diameter** of the circle.

A straight line drawn from the centre of a circle to the circumference is half a diameter, and is called a **radius.**

L′ (1). The Length of the Circumference.

EXERCISE 287.

EXPERIMENTAL EXERCISE.

1. Measure the diameter of a cylinder as accurately as you can in centimetres or inches.

2. Fold a piece of paper tightly round the cylinder until the edges overlap. Prick through the overlapping paper with a pin. Unroll the paper, spread it out flat and measure the distance between the pinholes in centimetres or inches.

3. From your measurements find how many times result 2 is contained in result 1. Work out the quotient correct to one place of decimals.

4. Measure the diameter of a halfpenny.

5. Roll the coin along a straight line on which inches and tenths of inches are marked. Find how many times the circumference of a halfpenny contains the diameter.

N.B. Repeat processes **4-5** with a penny and other circular objects.

6. Draw on thin cardboard a circle of a moderately large radius. Measure the length of the circumference in any way you please. (Take at least three measurements and use the average of the three.) Then write down your result thus:

$$\frac{\text{Length of circumference}}{\text{Length of diameter}} = \ldots\ldots\ldots$$

7. Repeat this process with several circles of different radii (inches and centimetres).

281. From these experiments we find that the result is always the same: the length of the circumference of a circle is always equal to the length of the diameter multiplied by some definite number which is a little more than 3, very nearly equal to $3\frac{1}{7}$, more nearly to 3·1416.

This **constant ratio** which the circumference of any circle bears to its diameter is denoted by the symbol π (a Greek letter called *pi*); that is to say

$$\frac{\textit{circumference}}{\textit{diameter}} = \pi,$$

or

$$\textit{Circumference} = \textit{diameter} \times \pi.$$

EXERCISE 288.

(*The value of π is to be taken as* 3·1416.)

NOTE. Give the answers in yards and decimals of a yard.

Find the circumference of a circle whose radius is

1. 20 yds.	2. $1\frac{1}{2}$ yds.	3. 5 yds. 1 ft. 6 in.
4. 5 ft.	5. $10\frac{1}{2}$ ft.	6. 6 ft. 3 in.

Find the diameter of a circle whose circumference is

7. 31·416 yds.	8. 125·664 yds.	9. 43·9824 yds.
10. 10·9956 yds.	11. 35·343 yds.	12. 27·489 yds.

EXERCISE 289.

(*The value of π is to be taken as $\frac{22}{7}$.*)

1. The spoke of a wheel measures 1 ft. 9 in. from the centre to the rim. What is the measurement all round the wheel?

2. How many times will a wheel, 4 ft. 8 in. in diameter, turn round in travelling $1\frac{1}{2}$ miles?

How many revolutions will a wheel half the above diameter make in travelling a distance five times as great as the above?

3. The measurement round a circular table is 18 ft. 4 in. What is the width of the table across its widest part ?

4. The circumference of a wheel is 27 ft. 6 in. What is the distance from the centre of the wheel to the rim ?

5. A locomotive engine is travelling at the rate of 45 miles an hour, and the diameter of the driving wheel is 7 ft. How many complete revolutions will the wheel make in a second ?

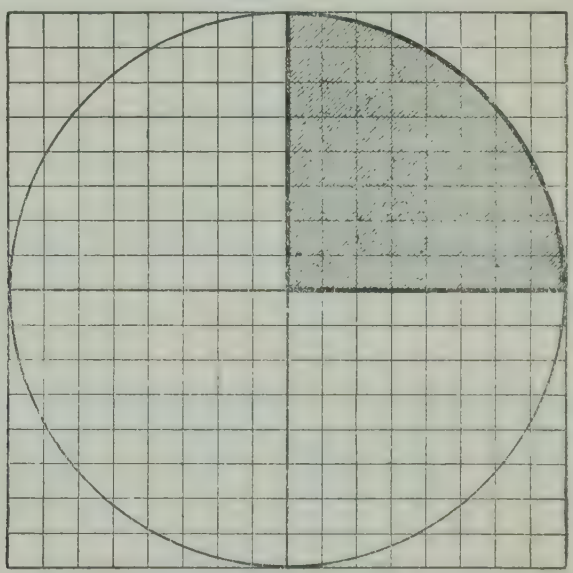
6. The driving wheel of a locomotive engine is 7 ft. in diameter and makes 120 revolutions a minute. What is the speed of the engine expressed in (i) feet per second, (ii) miles per hour ?

L′ (2). The Area of a Circle.

EXERCISE 290.

SQUARED PAPER EXERCISE.

Draw on squared paper a circle having a radius of 1 *inch. By drawing along the lines on the paper, describe a square each side of which touches the circle.* (See figure below.)



NOTE. The shaded portion is one-quarter of the circle, and is called a **quadrant.**

1. Find, by counting the number of small squares, the area of (*a*) the quadrant, and of (*b*) the square described on the radius of the circle. Write down your results, and calculate the value of the quotient :

$$\frac{\text{Area of quadrant}}{\text{Area of square}} = \frac{\qquad}{\qquad} = \ldots\ldots$$

2. From result 1 (*a*) write down the area of the circle.

3. Fill in the calculations :

$$\frac{\text{Area of circle}}{\text{Area of square described on radius}} = \frac{\qquad}{\qquad} = \ldots\ldots\ldots$$

Repeat the exercise with circles of different radii (inches and centimetres : say 3 in., 5 in., 3 cm., 5 cm.).

The ratio which the area of a circle bears to the area of the square on the radius is found in every case to be the same as that which the length of the circumference bears to the length of the diameter. The exact value of this ratio we have denoted by π.

282. To calculate the area of a circle: *Multiply the area of the square on the radius by* π.

Using the letters A and r to denote the *number of units* in the *area* and *radius* respectively,

$$A = \pi r^2.$$

EXERCISE 291.

(*The value of* π *is to be taken as* 3·1416.)

NOTE. Give the answers in sq. yds. and decimals of a sq. yd.

Find the area of a circle whose radius is

1. 20 yds.
2. $1\frac{1}{2}$ yds.
3. 5 yds. 1 ft. 6 in.
4. 5 ft.
5. $10\frac{1}{2}$ ft.
6. 6 ft. 3 in.

EXERCISE 292.

(*The value of* π *is to be taken as* $\frac{22}{7}$ *unless otherwise stated.*)

1. Find the area of a circular disc, the diameter of which is 14 inches.

2. Find the area of a circular table, if the measurement across the widest part is 3 ft. 6 in.

3. Find the area of grass that a horse can graze over, if he is fastened to a stake in a paddock by a rope 28 ft. long.

4. Find, to three significant figures, the radius of a circular disc whose area is 100 sq. centimetres.

5. Find the length of rope needed to tether a goat in order that it may feed on 154 sq. yards of grass.

6. The side of a square is 10 cm. in length, and a circle is inscribed in the square ; find the area of the space between them. ($\pi = 3{\cdot}1416$.)

283. A **sector** of a circle is a figure bounded by two radii and the part of the circumference (called an *arc*) between them.

284. The area of a circle may be looked at from another point of view.

EXERCISE 293.

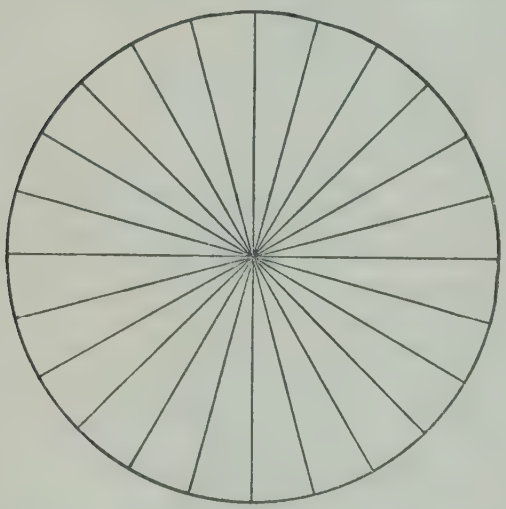
EXPERIMENTAL EXERCISE.

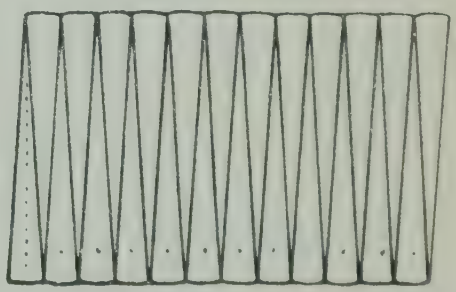
(i) *Draw on thin cardboard a circle of a moderately large radius.*

(ii) *Draw two diameters at right angles.*

(iii) *Divide the half circumference into* 12 *equal parts.*

(iv) *From the different points draw radii, so that the circle will be divided into* 24 *equal parts* (*sectors*).





(v) *Cut out the sectors and arrange them alternately up and down as shown in the figure.*

(vi) *Repeat processes* (i)-(v) *with a circle divided into* 32 *equal sectors.*

285. The above figure is almost exactly a parallelogram, the length of the base being equal to half the length of the circum-

ference of the circle and the height being equal to the radius of the circle; therefore the area of the circle is approximately equal to the area of the parallelogram.

NOTE. The greater the number of sectors, the more nearly will the arcs be straight lines and the figure become more exactly a rectangle.

The area of the circle can also be found by considering the sectors as triangles. Each of the sectors is nearly an isosceles triangle (Ex. 293), having radii for two sides and a part of the circumference for base.

286. To calculate the area of a circle: *Multiply half the circumference by half the diameter.*

Since half the circumference is πr and half the diameter is r,

$A = \pi r \times r = \pi r^2$, as before. [Art. 282.]

EXERCISE 294.

(*The value of π is to be taken as $\frac{22}{7}$.*)

Find the area of a circle whose radius and circumference are respectively

1. $3\frac{1}{2}$ in. and 11 in.
2. 7 in. and 3 ft. 8 in.
3. 1 ft. 9 in. and 3 yds. 2 ft.
4. 2 ft. 4 in. and 14 ft. 8 in.

5. Find the area in square yards of a circular grass plot which is surrounded by a fence 132 ft. long, and which has a path 21 ft. long across the widest part.

6. Find the area of a circular flower-bed which is 4 ft. 1 in. across the widest part, and measures 12 ft. 10 in. round.

L′ (3). The Area of a Circular Ring.

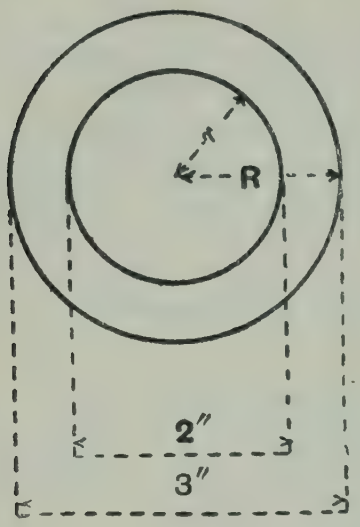
287. When two circles are described having the same centre, but with different radii, they are called **concentric circles**, and the space between their circumferences is called a **circular ring.**

The area of a circular ring is clearly equal to the difference between the area of the outer circle and that of the inner circle.

Using the letters A, R and r to denote the *number of units* in the *area*, *outer radius* and *inner radius* respectively,

$$A = \pi(R^2 - r^2) = \pi(R + r)(R - r).$$

EXAMPLE. *A circular ring has an outer diameter of* 3 *inches and an inner diameter of* 2 *inches. Find the area.* ($\pi = 3{\cdot}142$.)



$$\text{Area} = 3{\cdot}142(1{\cdot}5 + 1)\,(1{\cdot}5 - 1) \text{ sq. in.}$$
$$= 3{\cdot}142 \times 2{\cdot}5 \times {\cdot}5 \text{ sq. in.}$$
$$= \underline{3{\cdot}9275 \text{ sq. in.}}$$

EXERCISE 295.

(*The value of* π *is to be taken as* 3·1416.)

NOTE. Give the answers in square inches and decimals of sq. in.

Find the area of a circular ring whose outer and inner radii are respectively

1. 5 in., 3 in.
2. 4 in., 3 in.
3. 6 in., 5 in.
4. $4\frac{1}{2}$ in., $3\frac{1}{2}$ in.
5. 2 in., $1\frac{1}{2}$ in.
6. $3\frac{1}{4}$ in., $2\frac{1}{2}$ in.

EXERCISE 296.

(*The value of* π *is to be taken as* $\frac{22}{7}$.)

1. Find the area in square yards of a circular running track whose outer radius is 140 yards, and whose width is 7 yards.

2. A circular fish-pond has a radius of 56 ft., and is contained in a circular enclosure whose radius from the centre of the pond to the outer edge of the enclosure is 70 ft. ; find in square yards the area of the path round the pond.

3. A circular flower-bed 21 ft. in diameter is bordered by a gravel path $3\frac{1}{2}$ ft. wide ; find the area of the path in square feet.

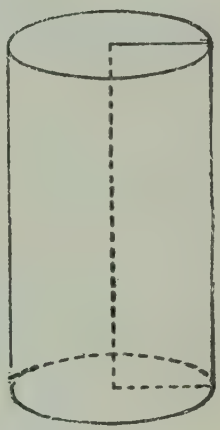
4. A metal ring is $2\frac{1}{2}$ in. wide, and the internal diameter is 8 in. ; find its value at a penny per square inch.

5. A wall encloses a circular area 4 ft. in diameter and is surrounded by a wooden platform 3 ft. wide ; find the cost of the platform at 10*d.* per square foot.

M′. THE CYLINDER.

M′ (1). Surface of a Cylinder.

288. A **right cylinder** is a solid figure, described by the revolution of a rectangle about one side which remains fixed (called the *axis*). The ends (or *bases*) of the cylinder are circles.



The **axis** of a cylinder is therefore a line joining the centres of the two ends and is at right angles to each end.

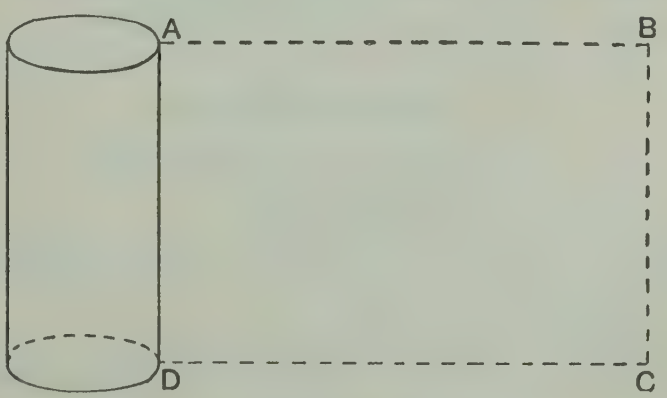
EXERCISE 297.

EXPERIMENTAL EXERCISE.

1. Examine a cylinder carefully and describe its shape.

2. Take a tracing of one end and test the other end with it.

3. Cut a piece of paper so that it will exactly cover the *curved surface* of a cylinder. What is the shape of the paper ?



4. Referring to the above figure, if ABCD represents the surface thus opened out, what are the lengths of AB and of BC ?

5. If the radius of an end is r, what is
   (i) the circumference of a circular end ?
   (ii) the area of a circular end ?

6. What is the least number of measurements which need be taken so that the *whole surface area* of a cylinder may be calculated ?

289. To calculate the curved surface of a cylinder : *Multiply the circumference of the base by the height.*

Using the letters A, r and h to denote the *number of units* in the *area, radius* of base and *height* of the cylinder respectively,

$$A = 2\pi rh.$$

The whole surface of a cylinder = *the curved surface + area of the ends* :

$$A = 2\pi r^2 + 2\pi rh = 2\pi r(r + h).$$

EXERCISE. *Find* (i) *the area of the curved surface, and* (ii) *the area of the whole surface of a cylinder whose height is* 10 *in. and the radius of whose base is* 7 *in.* ($\pi = \frac{22}{7}$.)

(i) Radius = 7 inches ; therefore diameter = 14 inches ;
$\therefore$ circumference of base = $14 \times \frac{22}{7}$ inches = 44 inches ;
$\therefore$ area of curved surface = 44×10 sq. inches = 440 sq. inches
= **3 sq. ft. 8 sq. in.**

(ii) Area of the two ends = $2 \times \frac{22}{7} \times 7 \times 7$ sq. inches = 308 sq. in.
= 2 sq. ft. 20 sq. in. ;
$\therefore$ total surface area = 3 sq. ft. 8 sq. in. + 2 sq. ft. 20 sq. in.
= **5 sq. ft. 28 sq. in.**

EXERCISE 298.

(*The value of π is to be taken as $\frac{22}{7}$.*)

NOTE. Give answers 1-6 in sq. yds., etc.

Find the area of the curved surface of a cylinder of which the following dimensions are given :

1. Circumference of base = 2 ft. 11 in. ; height = 6 ft.
2. Diameter of base = 1 ft. 9 in. ; height = 3 ft.
3. Radius of base = 1 ft. 2 in. ; height = 5 ft.

Find the area of the whole surface of a cylinder of the following dimensions :

4. Circumference of base = 14 ft. 8 in. ; height = 6 ft.
5. Diameter of base = 5 ft. 10 in. ; height = 3 ft.
6. Radius of base = 1 ft. 2 in. ; height = 9 ft.

7. The diameter of the end of a roller is 1 ft. 9 in., and the roller is 3 ft. long ; how many revolutions will it make in going over a grass-plot $\frac{1}{4}$ acre in extent ?

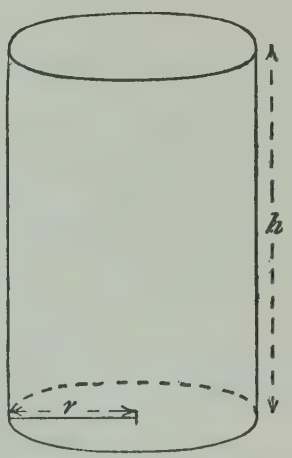
8. The circumference of a roller is 11 ft., and the roller is 6 ft. long. It makes 30 revolutions in passing from one end of a cricket ground to the other ; what is the area of the ground, if it is rolled in 22 journeys ?

M′ (2). Volume of a Cylinder.

290. To calculate the volume of a cylinder : *Multiply the area of the base by the height.*

Using the letters V, r and h to denote the number of units in the *volume, radius* of the circular base and *height* of the cylinder respectively,

$$V = (\pi r^2) \times h$$
$$= \pi r^2 h.$$



EXAMPLE. *Find the volume of a cylinder whose height is* 10 *in. and the radius of whose base is* 7 *in.* ($\pi = \frac{22}{7}$.)

Area of base $= \frac{22}{7} \times 7 \times 7 = 154$ sq. inches ;

$\therefore$ volume $= 154 \times 10$ c. inches

$= \underline{1540 \text{ c. inches.}}$

291. The **cross section** of a cylinder is a section at right angles to the axis, and is equal and parallel to the bases.

EXAMPLE. *The area of the cross-section of a cylinder is* 27·4 *sq. cm., and the length of the cylinder is* 15·6 *cm. ; find the volume.*

Volume $= 27{\cdot}4 \times 15{\cdot}6$ c.cm.

$= \underline{427{\cdot}44 \text{ c.cm.}}$

EXERCISE 299.

(The value of π is to be taken as $\frac{22}{7}$.)

Find the volume of a cylinder of which the following dimensions are given :

1. Area of base	$=58\frac{1}{4}$ sq. in. ;	height $=8$ in.
2. Area of base	$=10{\cdot}25$ sq. cm. ;	height $=5{\cdot}6$ cm.
3. Radius of base	$=3\frac{1}{2}$ in. ;	height $=8$ in.
4. Radius of base	$=7{\cdot}7$ cm. ;	height $=14{\cdot}5$ cm.
5. Diameter of base	$=1$ ft. 2 in. ;	height $=10$ in.
6. Diameter of base	$=8{\cdot}4$ cm. ;	height $=16{\cdot}8$ cm.
7. Circumference of base	$=1$ ft. 10 in. ;	height $=1$ ft. 6 in.
8. Circumference of base	$=17{\cdot}6$ cm. ;	height $=25{\cdot}4$ cm.

EXERCISE 300.

(The value of π is to be taken as $\frac{22}{7}$, unless another value is given.)

1. Find how many cubic feet of earth must be dug out in order to make a well 14 ft. in diameter and 50 ft. deep.

2. In digging a well 14 ft. in diameter, 114 c. yds. 2 c. ft. of earth were excavated ; calculate the depth of the wall.

3. Find how many gallons of water can be contained in a cylindrical boiler 16 ft. long and 7 ft. in diameter, if $6\frac{1}{4}$ gallons of water occupy 1 cubic foot.

4. The interior diameter of a rain gauge is 14·2 cm., and the volume of the rainfall registered is $37\frac{1}{2}$ cubic centimetres ; find the depth of the rainfall in millimetres.

5. Calculate the weight of a metal cylinder whose diameter is 7 cm. and whose length is 10 cm., if the weight of a cubic centimetre of the metal weighs 7·5 grams.

6. A cubic foot of copper is drawn out into a wire $\frac{1}{10}$ inch in diameter ; find to the nearest yard the length of the wire.

7. The area of the cross-section of a bronze cylinder is 27·4 sq. cm., the length of the cylinder is 15·6 cm. ; find the weight of the cylinder, assuming that bronze is 8·96 times as heavy as water, and that 1 c.cm. of water weighs 1 gram. Give the result in grams correct to 3 significant figures.

M′ (3). Volume of a Hollow Cylinder.

292. Gas pipes, chimney-pots and the ordinary garden roller are familiar examples of a hollow cylinder.

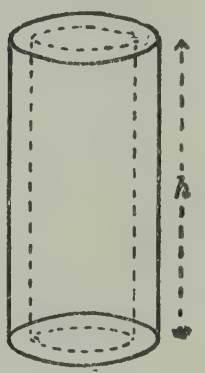
293. To calculate the volume of a hollow cylinder.

Using the letters V, R, r and h to denote the number of units in the *volume*, *outer radius*, *inner radius* and *height* of the cylinder respectively,

$$V = \pi(R^2 - r^2)h = \pi(R + r)(R - r)h.$$

EXAMPLE. *The external diameter of a metal pipe is* 3 *inches and the internal diameter is* 2 *inches. If the pipe is* 12 *inches long, find the volume of metal in it.* ($\pi = 3{\cdot}142$.)

$$\begin{aligned}\text{Volume} &= 3{\cdot}142\,(1{\cdot}5 + 1)\,(1{\cdot}5 - 1) \times 12 \text{ c. in.}\\ &= 3{\cdot}142 \times 2{\cdot}5 \times {\cdot}5 \times 12 \text{ c. in.}\\ &= \underline{47{\cdot}13 \text{ c. in.}}\end{aligned}$$



EXERCISE 301.

(*The value of* π *to be taken as* $\frac{22}{7}$.)

1. The outer diameter of a metal pipe is 7 cm., the inner diameter is 6 cm., and the length of the pipe is 14 cm.; how many cubic centimetres of metal are there in it?

2. A hollow cylindrical tube open at both ends is made of metal 1 in. thick. The exterior diameter is 8 in. and the length of the pipe 1 foot; how many cubic inches of metal are there in it?

3. The external diameter of a hollow cylinder open at both ends being 44 cm., thickness 2 cm. and height 25 cm., what is the volume of the metal?

4. The external diameter of a cylindrical shell is $17\frac{1}{2}$ in., the internal diameter is 14 in., and the height of the shell is 4 ft. What is the volume of the shell?

5. What is the solid content of a hollow iron roller, the external diameter of which is 4 ft. 8 in., the thickness of the iron $3\frac{1}{2}$ in. and the length of the roller 8 ft. ?

6. What is the weight of a copper pipe the exterior diameter of which is 5 cm. and the interior diameter is 3·8 cm. The length of the pipe is 25 cm., and 1 cubic centimetre of copper weighs 8·8 grams. Give the result in grams correct to the nearest gram.

N'. MENSURATION FORMULAE.

NOTE. $\pi = 3{\cdot}1416$ (or $\frac{22}{7}$).

1. To find the Area of a Triangle :

A = area, b = base, h = height ; $A = \frac{1}{2}bh$.

2. To find the Circumference of a Circle :

C = circumference, d = diameter, r = radius.

$$C = \pi d \text{ or } 2\pi r.$$

3. To find the Area of a Circle :

A = area, r = radius ; $A = \pi r^2$.

4. To find the Area of a Plane Circular Ring :

A = area, R = outer radius, r = inner radius.

$$A = \pi(R^2 - r^2) = \pi(R + r)(R - r).$$

5. To find the Area of the Curved Surface of a Cylinder :

A = area, r = radius of circular base, h = height of cylinder.

$$A = 2\pi rh.$$

6. To find the Volume of a Solid Cylinder :

V = volume, r = radius of circular base, h = height of cylinder.

$$V = \pi r^2 h.$$

7. To find the Volume of a Hollow Cylinder :

V = volume, R = external radius, r = internal radius,
h = height of cylinder.

$$V = \pi h(R^2 - r^2) = \pi h(R + r)(R - r).$$

P′. SIMPLE BUSINESS CALCULATIONS.

'P′ (1). Explanation of Business Terms.

294. **Debt** is a term generally applied to an amount due or payable from one person to another in return for goods or other obligation.

A **Debtor** is one who owes value (money or goods) to another.

A **Creditor** is one to whom value (money or goods) is owing.

A **Personal Account** is a record of the business carried on between two persons, *e.g.* buying or selling goods, receiving or paying money.

Balance is the term used to denote the difference (if any) between the two sides of an account.

P′ (2). How to keep a Cash Book.

295. The **Cash Book** is a record of receipts of cash and payments made by cash.

Cash (money) comprises coin, bank-notes, cheques or other documents which can be immediately converted into cash.

The entries are made on two pages facing each other ; receipts are recorded on the left-hand side and payments on the right-hand side.

The entries are made in this order :

1. The date at which cash is received OR paid.
2. The name of the person *to* whom the owner of the Cash Book is indebted for cash received, OR the name of the person *by* whom the owner of the Cash Book is credited for cash paid.
3. The amount of cash received OR paid.

The Cash Book is balanced periodically, and the Balance (if any) is brought down to the opposite side to form the first entry for the next period.

The Balance of a Cash Account always denotes a surplus of receipts over payments, because it is impossible to pay out more Cash than is received.

The Balance can be checked at any time, and ought always to agree with the actual amount of Cash in hand.

EXAMPLE. *On Jan. 1st I start the week with £10 in hand; on Jan. 2nd I receive £15 from H. House for rent due to me; on Jan. 2nd I pay £1. 15s. 4d. to T. Figg, grocer; on Jan. 3rd I pay £2. 8s. 10d. to A. Lamb, butcher; on Jan. 4th I pay £6. 19s. 11d. to D. Sox, outfitter; on Jan. 5th I receive £25 from A. Cashier for interest due to me; on Jan. 6th I pay £9. 8s. 3d. to T. Brass, ironmonger. Show all the entries as they appear in a Cash Book, and state how much I have in hand at the end of the week (Jan. 6th).*

Cash Account.

For the week ending January 6th, 19—

Dr. Cr.

DATE.		RECEIPTS.	£.	s.	d.	DATE.		PAYMENTS.	£.	s.	d.
19—						19—					
Jan.	1	Balance	10	–	–	Jan.	2	T. Figg	1	15	4
,,	2	H. House	15	–	–	,,	3	A. Lamb	2	8	10
,,	5	A. Cashier	25	–	–	,,	4	D. Sox	6	19	11
						,,	6	T. Brass	9	8	3
						,,	6	Balance c/f	29	7	8
		£	50	–	–			£	50	–	–
Jan.	8	Balance b/f	29	7	8						

296. Explanation of Abbreviations:

Cr. = Creditor; refers to the credit or *right-hand* side of an Account.
Dr. = Debtor; refers to the debit or *left-hand* side of an Account.
C/D = Carried down. C/F = Carried forward.
B/D = Brought down. B/F = Brought forward.

HINTS.

1. *Noughts must not be inserted in the money columns ; insert a dot or dash.*

2. *The totals of the two sides of an account must be written level with each other.*

EXERCISE 302.

Rule *six* Cash Book forms on behalf of yourself and enter the transactions specified below. Supply dates at which payment was made and names of persons to whom payment was made. Balance each account.

1-6. Re-work Exercise 117, 1-6 ; and arrange as directed above.

EXERCISE 303.

1. Rule a Cash Book on behalf of yourself, and enter the following transactions, balancing on 6th January :

19—		£.	*s.*	*d.*
Jan. 1.	Cash in hand	10	–	–
„ 2.	Paid Samuel Weller	3	16	0
„ 4.	Paid Thomas Pinch	2	10	9
„ 6.	Paid Martin Chuzzlewit	1	17	14

2. Bring down the Balance on 8th January, and enter the following transactions, balancing on 13th January :

19—		£.	*s.*	*d.*
Jan. 9.	Received from Caleb Plummer	10	10	–
„ 10.	Paid Mark Tapley	2	7	11
„ 11.	Paid Oliver Twist	2	16	8
„ 12.	Paid Tracy Tupman	1	19	7

3. Bring down the Balance on 15th January, and enter the following transactions, balancing on 20th January :

19—		£.	*s.*	*d.*
Jan. 15.	Paid Newman Noggs	3	8	4
„ 16.	Received from Dodson & Fogg	15	–	–
„ 18.	Paid Samuel Pickwick	1	19	5
„ 20.	Paid Nathaniel Winkle	1	17	6

4. Bring down the Balance on 22nd January, and enter the following transactions, balancing on 27th January :

19—		£.	s.	d.
Jan. 22.	Paid David Copperfield - - -	2	17	4
„ 23.	Received from Uriah Heep - - -	15	15	–
„ 26.	Paid Nicholas Nickleby - - -		18	5
„ 27.	Paid Daniel Peggotty - - - -	4	17	3

P′ (3). Invoices ; Receipts for Money.

(*Refer to Art.* 95.)

EXERCISE 304.

Make out the invoices specified below and show how payment is receipted on each invoice. Assume *you* are the purchaser of the goods.

NOTE. Receipts for amounts of £2 and upwards require a 2*d.* stamp affixed.

	Invoice.	Date.	Seller of the Goods.	Date of Payment.
1.	Ex. 123 (2)	27th Dec. 19—	T. Figg	2nd Jan. 19—
2.	„ „ (4)	28th Dec. „	A. Lamb.	3rd Jan. „
3.	„ „ (6)	29th Dec. „	D. Sox	4th Jan. „
4.	„ „ (7)	30th Dec. „	T. Brass	6th Jan. „

NOTE. The totals of these invoices are entered as *Payments* in the Cash Account (p. 251).

P′ (4). Cash Discount.

(*Refer to Arts.* 245 *and* 248.)

297. The term **Gross Amount** means the amount *before* any deduction for discount is made. The term **Net Amount** means the amount *after* deduction for discount has been made.

EXERCISE 305.

Calculations to be made correct to the nearest penny.

Supply the figures required in the last two columns :

Name of Account.	Gross Amount. £. s. d.	Rate of Discount.	Discount. £. s. d.	Net Amount. £. s. d.
1. T. Figg -	1 15 4	$1\frac{1}{4}\%$		
2. A. Lamb -	2 8 10	$2\frac{1}{2}\%$		
3. D. Sox -	6 19 11	5 %		
4. T. Brass -	9 8 3	$7\frac{1}{2}\%$		

P′ (5). Rates and Taxes.

298. **Rates** are payments levied by a local authority on property situated in its district to defray expenses under its control, *e.g.* expenditure on police, roads, etc.

Taxes are payments levied by the Imperial Parliament to defray the general expenses of government, *e.g.* expenditure on the navy, army, etc.

Taxes levied on certain imported goods, *e.g.* tea, tobacco, etc., are called *Customs duties.*

Taxes levied on certain goods produced and consumed at home, *e.g.* the manufacture and sale of spirits and beer, are called *Excise duties.*

299. The value of a person's property, as estimated by the authorities, for the purposes of calculating his share of rates and taxes, is called the **assessment.**

Rates and taxes are calculated on the assessment at so much in the £.

300. No new principles are involved in the examples which follow.

EXAMPLE 1. *A person pays income-tax at* 3*s. in the* £ *on an assessment of* £220. 10*s.*; *find the amount of income-tax payable.*

(Solution by **Ratio Method.**)

$3s. = \frac{3}{20}$ of £1.

Income tax $= \frac{3}{20}$ of the assessment;

∴ Amount of tax $= \frac{3}{20}$ of £220. 10*s.*

$=$ **£33. 1s. 6d.**

EXAMPLE 2. *Find the amount of a rate at* 15*s. in the* £ *when the assessment of a house and premises is* £40. 10*s.*

(Solution by **Practice.**)

£40·5 = assessment;

10*s.* $= \frac{1}{2}$ of £1. | 20·25 = amount at 10*s.* in the £.;

5*s.* $= \frac{1}{2}$ of 10*s.* | 10·125 = ... 5*s.* .....;

∴ Amount of rate = £30·375 = **£30. 7s. 6d.**

Alternative Solution. Apply Ratio Method.

EXERCISE 306.

1. A person's taxable income is £336. He has to pay 3*s.* in the £ on £225 and 6*s.* in the £ on the remainder; find the total amount of income-tax payable.

2. HOUSE DUTY.

	In the £.
Houses of an annual value of £20, and not exceeding £40 -	3*d.*
„ „ „ „ exceeding £40, and not exceeding £60 - - - -	6*d.*
„ „ „ „ exceeding £60 - - - -	9*d.*

Find from the above table the House Duty payable on rents as under:

(*a*) £24. (*b*) £38. (*c*) £46. (*d*) £52. (*e*) £65.

3. The assessment (or the rateable value) of the property in a town is £180000. If the local authority needs £135000 for public purposes, find how much in the £ the rate will be.

4. Find the amount payable for rates on property assessed as under:

	(*a*)	(*b*)	(*c*)	(*d*)
Assessment -	£62. 10*s.*	£56. 15*s.*	£48. 5*s.*	£23. 12*s.*
Rate in the £	15*s.*	16*s.*	13*s.* 4*d.*	18*s.* 4*d.*

5. The following table gives the purposes for which a rate is made and the amount in the £ levied for each purpose. Fill in for each purpose the amounts payable on property assessed at (*a*) £40. 10*s.*; (*b*) £62. 10*s.*

	s.	*d.*	(*a*) £. *s.* *d.*	(*b*) £. *s.* *d.*
Relief of the Poor - -	2	4		
County Council Expenses	4	–		
Education - - -	2	6		
Police - - - -	1	4		
Highways - - -	1	6		
Lighting - - -		8		
Scavenging - - -	1	10		
Miscellaneous - -		10		
Total - - - -	....	....	....	... .

Fill in the last line in the above table.

REVISION PAPERS.

Numbers less than 100: The Four Rules.

PAPER 1.

1. One book has sixteen pages, another twenty-four pages, and another forty-eight pages ; how many pages are there in the three books ?

2. George is eighteen years old ; how old was he five years ago ?

3. Tom made 39 runs in a cricket match, and Harry made 13 less than Tom ; how many runs did they make altogether ?

4. Sixteen ounces make a pound ; how many ounces make 5 pounds ?

5. How many apples will you receive, if I give you the third part of 96 apples ?

PAPER 2.

1. In a certain school there are fifteen boys in the first class, twenty-seven in the second, twenty-nine in the third, and eighteen in the fourth ; how many boys are in the school ?

2. Willie is fourteen years old ; in how many years' time will he be twenty-nine ?

3. Tom has 28 marbles and Harry has 15 marbles. If Tom gives 4 of his marbles to Harry, how many will each then have ?

4. Twenty-four hours make a day ; how many hours are there in 3 days ?

5. If 58 apples are equally divided among 5 boys, how many will each boy get, and how many apples will be left over ?

PAPER 3.

1. In a wood there are 27 elm trees, 28 oak trees, 19 beech trees, and 14 pine trees ; what is the total number of trees ?

2. By how many is 56 greater than 41 ? And 56 is how many less than 69 ?

3. Three feet make a yard ; how many feet long is a piece of string nineteen yards long ?

4. Tom has 18 marbles and George has three times as many ; how many have they between them ?

5. There are 92 pears in 4 bags and an equal number in each ; how many pears are there in each bag ?

PAPER 4.

1. Write in figures in a vertical column, and add together : seven, twenty-three, seventeen, thirty-five.

2. George has nineteen marbles and Tom has thirty-five ; how many more has Tom than George ?

3. A boy has two copy-books ; one has in it 30 pages and the other 24 pages. If he has filled up 18 pages in the first book already, how many more pages has he to write to fill both the books ?

4. Write down the answers to the following :

$$(a)\ 9+3-2+4\ ;\quad (b)\ 9+3-(2+4).$$

5. A dairyman had a quarter-dozen boxes, each containing a score of eggs. He sold a dozen eggs to each of four customers ; how many eggs altogether has he now ?

PAPER 5.

1. Find the sum of the following numbers arranged as they are :

$$35+3+16+8+19.$$

2. Tom has thirty-five marbles ; how many more does he want to make fifty ?

3. Write down the answers to the following :

$$(a)\ 5+3\times 4-2\ ;\quad (b)\ (5+3)\times(4-2).$$

4. Find the sixth part of 3 times 24.

5. In a school there were 85 pupils last term. One-fifth of them left at the end of the term, but 19 new pupils came after the holidays ; how many pupils are now in the school ?

PAPER 6.

1. Write down the sum of the following numbers arranged as they are :

$$47+19+25.$$

2. George has nineteen marbles ; how many more does he want to make ninety-one ?

3. There are 20 boys in a class. Eight of them did 3 sums each, and the rest did 4 sums each ; how many sums were worked by this class ?

4. Find the answers to the following :

$$(a)\ 14\times 3-18\times 2\ ;\quad (b)\ 45\div 3+36\div 2.$$

5. In a bag there are 54 nuts. Of these, one-third are chestnuts, one-half of the remainder are filberts, and the rest are walnuts ; how many nuts of each kind are there in the bag ?

Numbers less than 1000: The Four Rules.

PAPER 7.

1. A school cricket team makes ninety-three runs in the first innings and eighty-five in the second ; what is the total score ?

2. A farmer has one hundred and eighteen sheep and lambs together. He has sixty-nine sheep ; how many lambs has he ?

3. Three boys together have one hundred and twenty marbles. The first has sixty-five, and the second has twenty-eight less than the first; how many marbles has the third boy ?

4. In a school there are 6 classes and 25 boys in each class. Nineteen boys, however, are absent; how many are present ?

5. A bag contains 114 nuts, a second one-third as many, and a third a sixth as many; how many nuts are there in the first bag more than in the second and third together ?

PAPER 8.

1. I have in my garden eighty-five gooseberry bushes, fifty-eight black-currant bushes, and seventy-nine other currant bushes; how many bushes have I altogether ?

2. A train carries 366 passengers. Of these 77 ride first-class and the rest third-class; how many third-class passengers are there ?

3. A case of eggs contains 275. Of these, 29 are bad and 57 are cracked; how many eggs are sound ?

4. A sack of potatoes weighs 168 pounds; how many pounds will 5 sacks weigh ?

5. A bunch of bananas consisted of 280 pods. After cutting off one-eighth of the pods, how many pods should remain on the bunch ?

PAPER 9.

1. A railway train carries 37 first-class passengers, 59 more second-class than first, and 196 more third-class than first-class; what is the total number of passengers ?

2. In a cricket match one team made 250 runs, and won the match by 62; how many runs did the other team make ?

3. On each of 8 shelves there are 49 books, and on each of 7 other shelves there are 68; how many books are there on all the shelves ?

4. How often can I cut off 9 inches from a length of string containing 324 inches ?

5. A dealer collects 12 dozen eggs from one farm, 8 score from another, and 101 from a third. He sells the lot at 9 for a shilling; how many shillings does he receive ?

PAPER 10.

1. One book has twenty-eight pages, another one hundred and ninety-two, a third ninety-six, and a fourth two hundred and eighty-eight; how many pages are there in the four books ?

2. A steamer has eight hundred and fourteen persons on board. The crew number one hundred and twenty-five; how many passengers are there ?

3. On board another steamer the passengers and the crew together number 851, and the crew alone number 98; how many are the passengers more than the crew ?

4. Sixteen ounces make a pound; how many ounces are there in sixty-one pounds of chocolate ? And how many pounds are there in three hundred and eighty-four ounces of chocolate ?

5. There are twenty-eight rows of boys, with fourteen in a row. If we reduce the number of rows by one and add one boy to each row, what is the difference in the total number ?

PAPER 11.

1. Write in figures in a vertical column, and add together: five, twenty-three, one hundred and thirty-one, two hundred and forty-seven, five hundred and forty-six.

2. In a school of five hundred and seventy boys, two hundred and seventy-seven are in the upper school; how many boys are in the lower school?

3. How many pages are there in forty-eight books, if each book contains sixteen pages?

4. If 719 sticks are arranged in groups of 18 sticks, how many groups are there, and how many sticks will be left over?

5. Brown makes the following scores in eight cricket matches: 86, 37, 25, 0, 19, 36, 73, 108; what is his average score per match?

PAPER 12.

1. At the beginning of the year there are 816 boys in a school. During the year 198 leave and 99 new boys are admitted; how many boys are there in the school at the end of the year?

2. In another school there are thirty-five boys in the preparatory class, six times as many boys in the lower forms, and five times as many in the upper forms; how many boys are there in the school?

3. Find the answers to the following:

$$(a)\ 18\times 40;\quad (b)\ 580\div 40.$$

4. Brown saves 68 pence and Smith saves 7 times as much as Brown. Jones saves a quarter as much as Smith. How many pence are saved altogether by the three boys?

5. In eight innings Brown makes the following numbers of runs: 41, 27, 18, 0, 59, 13, 5, 29; what is his average score? How many more runs should he have made to increase his average by 3 runs?

Numbers less than 10000: The Four Rules.

PAPER 13.

1. Find the sum of $319+1968+2394+69+5064$.

2. Find the product of 132 and 72, and check your result by multiplying in a different manner.

3. How often can 28 be subtracted from 8540?

4. How many must be added to 869 to make 8693?

5. A man was born in 1833, and died at the age of 77. When he died his son was 51 years of age; in what year was the son born?

PAPER 14.

1. Find the total of $25+408+1687+395+4610$.

2. Find the difference between nine hundred and eight and nine thousand and eighty.

3. How many times is 36 contained in 7416?

4. Find the product of 125 and 24, and check your result by multiplying in a different manner.

5. A man died in 1912 at the age of 94: how old was he in 1843?

PAPER 15.

1. Find the amount of $4613+37+1091+7+938$.

2. At a school treat 93 boys and 75 girls are present; if twenty-one cakes are provided, into how many equal pieces should each cake be cut so as to give each child a piece?

3. Divide nine thousand nine hundred and eighteen into fifty-seven equal parts.

4. Find the excess of nine thousand two hundred over three thousand five hundred and twelve.

5. A man was 29 years old in 1912; in what year will he be 65 years old?

PAPER 16.

1. Add together 8, 1809, 46, 4613 and 867.

2. How many must be taken from 6387 to leave 718?

3. What is the nineteenth part of eight thousand six hundred and sixty-four?

4. Divide the difference between 365 dozen and 135 score by 42.

5. A man died when 70 years old; 31 years after, his son died, aged 60; how old was the father when his son was born?

Numbers less than 100000: The Four Rules.

PAPER 17.

1. Find the total of $693+7894+6473+1069$.

2. What is the remainder when the difference between 7908 and 8619 is subtracted from the sum of 986 and 4723?

3. A carriage wheel turns round 264 times in going a mile; how many times will it turn in going and coming between X and Y, two towns which are 37 miles apart?

4. Walnuts are sold at 14 for a penny and chestnuts at 20 a penny; what is the total number of pence which must be given for 1940 chestnuts and 1372 walnuts?

5. How often is the 26th part of 2704 contained in 26936?

PAPER 18.

1. Find the sum of $5216+3947+865+8906$.

2. A has collected 3912 stamps, B has collected 986 less than A, and C has collected 379 more than A; how many has B less than C?

3. How many passengers are carried during a day by a train which makes 18 trips, carrying on an average 317 on the up journey and 296 on the return journey?

4. A number increased by sixty-three times itself amounts to 31248; what is the number?

5. A newspaper boy sells six dozen papers each day on the four days of the week commencing Monday, and twice as many each day on Friday and Saturday; how many papers does he sell in a year (52 weeks)?

PAPER 19.

1. Find the sum of $121+32005+4867+50201+208$.

2. What number must be added to the difference between nine hundred and seventy-nine and four thousand six hundred and eighteen to make nine thousand ?

3. What is the product of 43 and 1047 ? Prove the result by long division

4. How many times must 181 be added to 21607 to make 25770 ?

5. Find the average of the following :

(a) 19, 11, 12, 13, 30 ; (b) 14, 24, 18, 27, 12.

PAPER 20.

1. Add together 4762, 35670, 117, 20134 and 23016.

2. What number must be taken from the sum of three thousand six hundred and twenty-one and nine hundred and eighteen to make a number equal to their difference ?

3. Add the product of 197 and 506 to their difference.

4. How many times 75 can be deducted from 6075 to leave a remainder equal to itself ?

5. Find the difference between $9\times7+4\times6$ and $(9\times7+4)\times6$.

Numbers less than 1000000: The Four Rules.

PAPER 21.

1. Find the sum of $37420+56+89201+7502+342+729865+6345$; and write the answer in words.

2. What number must be added to the difference between 4832 and 691 to make a number equal to their sum ?

3. Divide 867760 by 57, and subtract the remainder from the quotient.

4. How often is 25 contained in 45 times 2345 ?

5. Write down the answers to the following :

(a) $92-29-37+43$; (b) $22-3\times6+7$.

PAPER 22.

1. Add together 8476, 29, 134209, 37561, 421, 587642 and 709 ; and write the answer in words.

2. What number subtracted from one hundred thousand leaves seventy thousand and fifty-eight ?

3. Divide 695457 by 83, and the quotient by 57 ; what is the second quotient ?

4. How many times 249 must be deducted from 461008 to reduce it to 399007 ?

5. In an election in a town of 3628 voters there were two candidates ; 1915 electors did not vote, and the " Blue " candidate has a majority of 115 over the " Red " ; find the number of votes given to each.

PAPER 23.

1. Find the total of $56201 + 323 + 9992 + 476500 + 1291 + 189765 + 18652$; and write the answer in words.

2. Find the difference between the product of 4083 and 97, and the product of 180 and 569.

3. How many times 587 must be added to 5309 to make 372184 ?

4. Show that the sum of 54306 and 81459 is equal to five times their difference.

5. A has 103 post cards ; B has twice as many as C, who has 19 less than A ; how many more than A has B ?

PAPER 24.

1. Add together 62018, 3789, 808, 99076 and 50891 ; and write the answer in words.

2. By how many is the difference between fifty thousand and 12209 less than the difference between 23658 and one hundred thousand ?

3. What number multiplied by 97 will give the same product as 582 multiplied by 393 ?

4. How many letters are there in a book of 365 pages, of which 87 pages are pictures, and each of the others contains 36 lines, and each line, on an average, 47 letters ?

5. Find the value of the following :

(*a*) $712 - 308 - 267 + 368 + 405$;

(*b*) $(12 \times 6 - 3) \times 8 - 12 \times 6 - 3 \times 8$.

Factors and Prime Numbers.

PAPER 25.

1. Separate 3675 into prime factors, using the index-form.

2. Find the H.C.F. of 45 and 60 ; and the L.C.M. of 24 and 30.

3. Multiply 8172 by 25 and 987 by 125, using the shortest method in each case.

4. If 731 is multiplied by a certain number, it is increased by 17544 ; what is the number ? Prove your result.

5. At the pit's mouth 368 tons of coal are loaded into railway trucks holding 9, 8 and 6 tons respectively, and the trucks of each size are equal in number ; how many of each kind of truck are there ?

PAPER 26.

1. Separate 6480 into prime factors, using the index-form.

2. Find the L.C.M. of 14, 20, 28, 30 ; and the H.C.F. of 24 and 27.

3. Multiply 832 (i) by 25, and (ii) by 125, using the shortest method in each case. Prove your results.

4. A is 75 miles distant from D ; C is 40 miles distant from A, and B is 50 miles distant from D. If A, B, C, D are situated in a straight line in this order, find their distances apart.

5. The sum of the ages of a father and a son is 75 years. The father was 25 years old when his son was born ; what is the age of each ?

PAPER 27.

1. Find by factorisation the square root of 1764.

2. Find the L.C.M. of 11, 22, 45 and 55 ; and the H.C.F. of 169 and 325.

3. Write down the largest number that can be formed with the digits 3, 8, 9, 5, 2, and the smallest number ; find the difference between them.

4. Divide 368975 by 25, and prove your result, also by division.

5. Twelve years ago a father was five times as old as his son, and his son is now 19 years old ; what is the father's present age ?

PAPER 28.

1. Find by factorisation the square root of 5625.

2. Find the H.C.F. of 324 and 720 ; and the L.C.M. of 8, 15, 45 and 30.

3. Write down all the numbers of four digits that can be formed with the digits 3, 6, 0, 4, and add them together.

4. Divide 794375 by 125, and prove your result, also by division.

5. In a cricket match A, B and C scored between them 65 runs. A and B together made 40, and B and C together made 46 ; how many runs did each boy make ?

PAPER 29.

1. Find by factorisation the cube root of 2744.

2. Find the H.C.F. of 450 and 196 ; and the L.C.M. of 14, 35, 36 and 45.

3. Write down all the numbers of four digits that can be formed with the digits 7, 0, 8, 5, and add them together.

4. The divisor is 48, the quotient 596, and the remainder 10 ; what is the dividend ? Prove your result.

5. In a cricket match A, B and C together scored 280 runs. If A and B together made 192, and C made 11 more than B, how many runs did each make ?

PAPER 30.

1. Find by factorisation (i) the square root, and (ii) the cube root of 729.

2. Find by the shortest method the difference between 36 times 4725 and 47 times the same number.

3. Find the least number which will leave a remainder 3 when divided by either 5, 7, 9 or 11.

4. What is the smallest number which must be taken from 40584 so that the remainder may be exactly divisible by 48 ? Prove your result.

5. A man, being asked how old his son is, replies, "Twenty-three years hence he will be as old as I was when he was born, and I shall be fifty-eight." What is the present age of the son ?

Numbers less than 100. Sums of Money less than £1.

PAPER 31.

1. Find the sum of the following :

   (*a*) 16, 37, 9 and 32 ; (*b*) 1*s.* 4*d.*, 7*s.* 9*d.*, 3*s.* 6*d.* and 2*s.* 11*d.*

2. Which is the greater, and by how many, 40 – 25 or 61 – 36 ?

3. Find the value of x when $63 - 37 - (53 - 36) = x$.

4. What will be the cost of 3 pairs of gloves at 1*s.* 7*d.* per pair ? And how much change shall I get out of half-a-sovereign ?

5. A pair of boots and a pair of gloves together cost 19*s.* 2*d.* The gloves cost 3*s.* 11*d.* ; how much did the boots cost more than the gloves ?

PAPER 32.

1. Add together the following :

(*a*) 4, 12, 19, 36 and 18 ; (*b*) 3*s.* 6*d.*, 8*s.* 7*d.*, 1*s.* 9*d.* and 3*s.* 4*d.*

2. Which is the greater, and by how much, 18*s.* 6*d.* – 7*s.* 11*d.* or 15*s.* 2*d.* – 3*s.* 5*d.*?

3. Find the value of x when $96 \div 6 + 70 \div 5 + 84 \div 4 = x$.

4. What will be the total cost of 4 toys at 11*d.* each and 2 books at 1*s.* 11*d.* each ?

5. A boy has 7*s.* 9*d.*, and his uncle gives him 10*s.* 6*d.* He buys a paint-box for 11*s.* 9*d.* and a book for 4*s.* 11*d.* ; how much money remains ?

PAPER 33.

1. Find the total of the following :

(*a*) $9 + 36 + 23 + 0 + 17$; (*b*) 6*s.* 3*d.* + 2*s.* 6*d.* + 3*s.* 6*d.* + 6*s.* 8*d.*

2. From a bag containing 95 pennies, a boy took 19, then 27, then 36 ; how many were left ?

From a purse containing 16*s.* 8*d.* he took 14*s.* 11*d.* : how much money was left ?

3. Find the value of x when $13 \times 3 + 14 \times 4 = x$.

4. How much more shall I pay for 6 books at 1*s.* 8*d.* each than for 5 toys at 1*s.* 3*d.* each ?

5. If 2 pots of jam cost 1*s.* 10*d.* and 3 jars of marmalade cost 4*s.*, what is the difference in price between a pot of jam and a jar of marmalade ?

PAPER 34.

1. Find the sum of the following :

(*a*) 51, 19, 16 and 8 ; (*b*) 2*s.* 6*d.*, 3*s.* 9*d.*, 8*s.* 9*d.*, 1*s.* 6*d.* and 9*d.*

2. How many pennies must be added to 49 to make 81 ? And how much money must be taken from 8*s.* 1*d.* to leave 4*s.* 9*d.*?

3. Find the value of x when $14 \times 6 - 13 \times 6 + 78 \div 6 = x$.

4. What is the total cost of 5 cakes at 10*d.* each, 4 boxes of chocolate at 11*d.* each and 6 pots of jam at 9*d.* each ?

5. I have a bag containing 91 nuts, of which I keep 16 for myself. I share the remainder equally among 3 boys and 2 girls ; how many nuts does each child receive ?

Numbers less than 1000. Sums of Money not greater than £10.

PAPER 35.

1. Divide 857 by 7×8, and give the complete remainder.

2. Find the value of x when $x + £1.\ 12s.\ 4\frac{1}{2}d. = £10.$

3. Divide £8 by 56, and give the value of the remainder in farthings. Verify your result by multiplication.

4. Find the value of $15\times 7\times 8 - 18\times 9\times 4.$

5. Make out in proper form a bill for 12 lb. of tea at 1*s.* 4*d.* per lb., 10 lb. of coffee at 1*s.* 6*d.* per lb., 3 lb. of raisins at $7\frac{1}{2}d.$ per lb., and 18 lb. of sugar at $3\frac{1}{2}d.$ per lb.

How much change will there be out of a £5 note ?

PAPER 36.

1. Divide 912 by 8×9, and give the complete remainder.

2. Find the value of x when £3. 19*s.* $4\frac{1}{2}d. + x = £8.\ 14s.\ 7\frac{3}{4}d.$

3. Divide £6. 14*s.* $11\frac{1}{2}d.$ by 24, and give the value of the remainder. Verify your result by using different factors.

4. Find the value of $17\times 6\times 8 - 18\times 5\times 7.$

5. What should be the cost of 7 bags of potatoes, if 8 bags can be bought for £3. 3*s.* 4*d.*?

PAPER 37.

1. Divide 912 by 7×9, and give the complete remainder.

2. Find the remainder when 8 times 15*s.* $9\frac{3}{4}d.$ is subtracted from £8. 13*s.* 6*d.*

3. Divide £9. 12*s.* $11\frac{3}{4}d.$ by 88, and give the value of the remainder. Verify your result by multiplication.

4. Find the value of $19\times 2\times 7 + 29\times 3\times 5.$

5. Make out in proper form a bill for 5 fenders at 11*s.* 3*d.* each, 4 tea-trays at 12*s.* 6*d.* each, 7 hoes at 1*s.* $11\frac{1}{2}d.$ each, and 9 spades at 2*s.* $4\frac{1}{2}d.$ each.

How much change will there be out of eight sovereigns ?

PAPER 38.

1. Divide 807 by 25, using factors ; and give the complete remainder.

2. I have £7. 19*s.* 6*d.* ; how much more do I need to pay tradesmen's bills for £2. 19*s.* 1*d.*, £1. 14*s.* $7\frac{1}{2}d.$ and £4. 10*s.* 11*d.*?

3. Divide £7. 0*s.* 6*d.* by 36, and give the value of the remainder in pence. Verify your result by using different factors.

4. Find the value of $28\times 3\times 4 + 8\times 7\times 6.$

5. What is the cost of 8 chairs, if 5 of them cost £4. 8*s.* 4*d.*?

Numbers greater than 1000. Sums of Money greater than £10.

PAPER 39.

1. A has £12. 19*s.* 9*d.* and B has £12. 18*s.* 7*d.* They divide the total amount equally between them ; how much must A give B ?

2. If 9 copies of a picture cost £10. 18*s.* 3*d.*, what will 10 copies of the picture cost ?

3. A tram-car carries 1052 people on Monday, 2106 on Tuesday, 1001 on Wednesday, 2093 on Thursday, 1064 on Friday, and 2560 on Saturday; how many more passengers are there during the last three days than during the first three days ?

4. Multiply £3. 12*s.* 4½*d.* by 21; and divide 3917 by 48. Prove both results.

5. A man earns in a week as much as two women. If 3 men and 3 women together earn £8. 16*s.* 3*d.* in a week, what are the weekly earnings of a woman ?

PAPER 40.

1. A man bought a horse and two carts for thirty-three guineas. One cart cost £7. 15*s.*, the other cost £1. 17*s.* less than this; what did the horse cost ?

2. Multiply £1. 16*s.* 10½*d.* by 39; and divide 2461 by 23. Prove both results.

3. A and B have £1. 3*s.* 11*d.* between them, but B has 4*s.* 6½*d.* more than A; how much has each ?

4. If 17 boxes of chocolate cost £1. 2*s.* 8*d.*, how much must be paid for 21 boxes ?

5. Bob has as many florins as Harry has half-crowns. Harry has £13. 2*s.* 6*d.*; how much money has Bob ?

PAPER 41.

1. Find the sum of nineteen pounds nineteen shillings and ninepence, twelve pounds fourteen shillings and tenpence farthing, seventeen pounds seven shillings and sixpence, and nineteen shillings and elevenpence three farthings.

2. From 8 times a certain number I subtract 614. The result is 5714; find the first number.

3. A has 3*s.* 11*d.* less than B, and C has half as much as A. If B has 18*s.* 10½*d.*, how much has C ?

4. If 11 cricket bats cost £9. 12*s.* 6*d.*, how much will 19 bats cost ?

5. Fred saves sixpence for every shilling George saves. Fred has two guineas; how much has George saved ?

PAPER 42.

1. What is the least number which can be subtracted from 9136 to make it divisible, without remainder, by 75 ?

2. Divide £26. 11*s.* between A and B so that B may have £5. 17*s.* 8*d.* more than A.

3. If 18 birthday cakes cost £4. 1*s.*, what will 13 of the cakes cost ?

4. Find the total of eight pounds thirteen shillings and fourpence, twelve pounds sixteen shillings and elevenpence halfpenny, nineteen pounds seventeen shillings and fourpence farthing, and ten pounds four shillings and tenpence farthing.

5. A horse and two cows cost £94. 13*s.* 9*d.*, and the horse costs three times as much as each cow; what does each cow cost ?

Measures of Time.

PAPER 43.

1. The time by a clock is a quarter-past 10; what time will the clock indicate 35 hrs. 24 min. later ?

2. Find the eighth part of 2 days 10 hrs. 41 min. 20 sec.; and verify the result.

3. Add together seventeen pounds six shillings and threepence halfpenny, ten pounds fourteen shillings and fivepence, eighteen pounds five shillings and sixpence farthing, nine pounds twelve shillings and sevenpence three farthings.

4. How many boxes of matches can be bought for £1. 1*s.* 3*d.*, at the rate of $2\frac{1}{2}$*d.* for a dozen boxes ?

5. A has £4. 3*s.* 4*d.* less than B, and C has half as much as A; if B has 14 guineas, how much money has C ?

PAPER 44.

1. Subtract 74 days 11 hrs. 7 min. 20 sec. from 11 weeks 3 days 15 hrs. 17 sec.; and verify the result.

2. If a cricket ball costs 6*s.* 11*d.*, how many can be purchased for £2. 8*s.* 5*d.*?

3. I have £6. 10*s.* in my purse. If I pay one-eighth of it to A and then one-third of the remainder to B, how much money shall I have left ?

4. Find the value, by the shortest method known to you, of 865 times $11\frac{3}{4}$*d.*

5. A man earns 9*d.* per hour; how much will he earn in 50 weeks, if he works $5\frac{1}{2}$ days per week and, on an average, 8 hours per day ?

Measures of Weight.

PAPER 45.

1. A van and its load weighs 15 cwt. 2 qrs. 18 lb.; what would 72 such vanloads weigh ?

2. If 3 pennies weigh an ounce, how many pennies will weigh $18\frac{1}{2}$ cwt.?

3. The dividend of a sum is £279. 18*s.* The quotient is £3. 17*s.* 9*d.*, and there is no remainder. What is the divisor ? Verify the result, using factors of the divisor.

4. The contents of a bag containing 53 crowns, 53 half-crowns, 50 shillings and 15 threepenny pieces are equally divided among 19 persons; how much money will each receive ?

5. How many copies of a book can be bought for £4. 14*s.* 8*d.*, if 17*s.* 9*d.* will buy three copies ?

PAPER 46.

1. Find the sixth part of 7 tons 7 cwt. 2 qrs. 16 lb. Verify the result.

2. Find the sum of twelve pounds five shillings and threepence, eighteen shillings and sevenpence, eight pounds nineteen shillings and elevenpence halfpenny, seventeen pounds and fourpence, thirteen pounds sixteen shillings and ninepence farthing, and sixteen shillings and tenpence.

3. A boy multiplied £19. 13*s.* $3\frac{1}{4}$*d.* by 13 instead of by 31; by how much was the product wrong ?

4. I pay an account for £2. 8*s.* with equal numbers of sixpences, shillings, florins and half-crowns ; how many of each coin do I use ?

5. I bought some coal at 1*s.* 2*d.* per cwt. and had 19*s.* 2*d.* left out of 3 sovereigns ; what weight of coal did I buy ?

Measures of Capacity.

PAPER 47.

1. From a cask containing 40 gallons of wine, 5 gall. 3 qts. 1 pt. are drawn off ; how much wine remains in the cask ?

2. I change £10 for equal numbers of shillings, threepenny pieces and pennies ; how many have I of each coin ?

3. If £7749 has to be divided fairly between 77 persons, find as nearly as possible what each should receive, and how much money is left undivided.

4. Find, by the shortest method known to you, the value of 119 times £4. 7*s.* 6*d.*

5. What is the cost of 46 gallons of oil at three halfpence per pint ?

PAPER 48.

1. If 16 jars of equal capacity are filled out of a cask containing 46 gallons of wine, how much will each jar contain ?

2. How many gallons of water are there in 56 jugs, if each jug holds 2 qts. 1½ pt. ?

3. How many times can you take 1*s.* 9½*d.* out of £1 ? What sum of money will remain ?

4. Find the cost of 796 pints of spirit at 12*s.* 7*d.* per gallon.

5. A horse and a cart together cost £74. 16*s.* 8*d.* The horse costs three times as much as the cart ; what does the horse cost ?

Measures of Length.

PAPER 49.

1. From a coil of 20 yards of wire, 7 yds. 1 ft. 11 in. is cut off ; what length now remains ?

2. What is the top line of this sum ?

yds.	ft.	in.
*	*	*
3		
71		
12		
861		

3. Find the sum of eight pounds and eightpence, fourteen pounds and sevenpence halfpenny, ten pounds nineteen shillings and elevenpence three farthings, nineteen pounds ten shillings and tenpence halfpenny, and seventeen pounds seventeen shillings and sevenpence three farthings.

4. A man earns 8½*d.* per hour ; how much will he earn in 9 weeks, if he works 6 days per week and 9 hours per day ?

5. If 4 cwt. 1 qr. 11 lb. of coffee is mixed with 1 qr. 18 lb. of chicory, how many ½-lb. packets of the mixture can be made ?

PAPER 50.

1. Five desks of equal length placed end to end extend 13 yds. 2 ft. 3 in.; how long is each desk ?

2. Fill in the missing line :

yds.	ft.	in.
13	1	9
**	*	*
11	2	5
7	1	11
2	0	6
46	0	6

3. I have 4 sovereigns, 5 half-sovereigns, 2 florins, 3 sixpences and a number of half-crowns in my purse. Altogether the contents of the purse amount to £9. 3*s.* ; how many half-crowns are there ?

4. A return railway ticket from X to Y costs 1*s.* 7*d.* Supposing a man has to do the journey six times each week, how much will he save in a year by taking a season ticket for 19 guineas ?

5. If milk costs 2½*d.* per pint, how many gallons can be purchased for £1. 11*s.* 8*d.* ?

PAPER 51.

1. The Severn tunnel is 4 miles 2 furlongs 184 yds. long and the Mersey tunnel is 1 mile 940 yds. long ; how much longer is the former than the latter ?

2. Add together thirteen pounds thirteen shillings and threepence, fourteen pounds fourteen shillings and fourpence, fifteen pounds fifteen shillings and elevenpence farthing, eighteen pounds eight shillings and tenpence three farthings, and nineteen shillings and twopence three farthings.

3. How many times does a wheel, whose circumference is 3 yds. 2 ft. 7 in., turn in rolling 1946 yards ?

4. Divide £157. 10*s.* between A, B and C so that A shall have £3. 10*s.* more than B, and B £1. 10*s.* more than C. Verify the result.

5. A piece of wire 3 miles long costs £121 ; what is the price per yard ?

PAPER 52.

1. The total length of three roads is 1 mile 60 chains. One road is 55 chains 14 yds. long and a second is 43 chains 16 yds. long ; what is the length of the third ?

2. Using the shortest method, find the value of

(*a*) 326×997 ; (*b*) $69750 \div 250$.

3. Find the cost of 7½ gross of eggs at 16 for a shilling.

4. A railway train travels at the rate of 45 miles per hour : how many yards does it travel in 10 minutes ?

5. If 6 men earn £2. 12*s.* 6*d.* in 10 hours, find the average wages of a man per hour, and thence find the wages of 8 men for 12 hours.

Measures of Area.

PAPER 53.

1. Multiply 1 sq. ft. 116 sq. in. by 7, and give the answer in sq. yds., sq. ft. and sq. in.

2. If a rectangular hall covers 80 sq. yds. 5 sq. ft., and is 25 ft. wide, how many feet is it in length ?

3. How much tobacco will be needed to make 39 quarter-pound tins, 36 two-ounce packets, 191 ounce packets and 381 half-ounce packets ?

4. A and B had equal amounts of money. A gave B 11*s.* 5½*d.*, and then B had £3. 5s. ; how much has A got now ?

5. If 3 yds. 1 ft. 6 in. of canvas cost 15*s.* 9*d.*, what is the cost per yard ?

PAPER 54.

1. Find, in square feet, the areas of the floor and the walls of a room 20 ft. long, 18 ft. wide and 16 ft. high.

2. How many square inches are there in 17 sq. yds. 8 sq. ft. 71 sq. in. ?

3. Multiply 804 by itself and add 8040 to the product. Show that the result is the same as 804×814.

4. If milk is sold at 2½*d.* per pint, how many gallons can be purchased for £1. 11*s.* 8*d.* ?

5. How many pounds of butter at 1*s.* 4½*d.* per lb. will cost the same amount of money as 3 cwt. of sugar at 2¾*d.* per lb. ?

Measures of Larger Area (Land).

PAPER 55.

1. Find the acreage of a garden occupying 43560 sq. feet.

2. Each side of a square field is 10 chains long ; into how many allotments, each containing one-quarter of an acre, can the field be divided ?

3. Find, in feet, the length of a rectangular garden, 25 ft. wide, whose area is 80 sq. yds. 5 sq. ft.

4. If 19 yds. 2 ft. 8 in. of copper wire cost £5. 4*s.* 5*d.*, what is the price per foot ?

5. Divide 1381 marbles among four boys so that three of them shall have equal shares and the fourth 143 less than each of the others.

PAPER 56.

1. Find, in square miles, the size of a parish 108374 acres in extent.

2. The area of a rectangular piece of ground is 12 acres, and the length of one side is 176 yards ; how many yards long is the adjacent side ?

3. Two straight rods together measure a yard, and one is 8 inches longer than the other ; what is the length of each rod ?

4. If land is sold at the rate of £1331 per acre, how much per square yard is this ?

5. A room 30 ft. long and 24 ft. wide is covered with carpet at 5*s.* per sq. yard ; what is the cost of the carpet ?

Measures of Volume and Space.

PAPER 57.

1. A block of marble contains 2 c. yards 17 c. ft. ; what is the content of another block which is six times as large ?

2. What is the volume, in cubic feet, of a rectangular block 1 yd. 1 ft. long, 1 ft. 6 in. broad and 3 ft. high ?

3. What is the greatest number less than 20000 that contains 391 an integral number of times ?

4. If 12 gall. 2 qts. of milk cost 16*s.* 8*d.*, what will 19 gall. 3 qts. cost ?

5. My gas bill for one quarter amounted to £1. 15*s.* and the gas cost 2*s.* 11*d.* per thousand cubic feet ; how many cubic feet of gas did I pay for ?

PAPER 58.

1. In digging out the ground for the cellars and foundations of a house, 686 c. yds. 26 c. ft. 1026 c. in. of earth was excavated by 9 men ; how much was dug out by each, on an average ?

2. If a cubic foot of water contains $6\frac{1}{4}$ gallons, how many gallons of water are there in a cistern 2 yds. 2 ft. long, 2 yds. wide and 4 ft. deep ?

3. A hall can accommodate 120 persons, allowing 70 cubic feet of space for each. The room is 35 ft. long and 16 ft. wide ; what is its height ?

4. My gas bill for one quarter amounted to £1. 19*s.* 7*d.* and the gas cost 4*s.* 2*d.* per thousand cubic feet ; how many cubic feet of gas did I pay for ?

5. How many boys can be accommodated in a class-room 21 ft. 4 in. long, 12 ft. wide and 10 ft. high, allowing 80 cubic feet of space for each boy ?

Decimal Measures ; Decimals : The Four Rules.

PAPER 59.

1. (*a*) How many times does 1 metre contain 5 centimetres ?

(*b*) How many times is 5 centimetres contained in 2 decimetres ?

(*c*) Express 6 dm. 0 cm. 5 mm. in cm. and decimals of a cm., and 850 mm. in dm. and cm.

2. (*a*) Multiply by 10 : (i) 6·5 cm. ; (ii) ·045 cm.

(*b*) Divide by 10 : (i) 9·4 dm. ; (ii) ·65 dm.

3. Find the sum of the following lengths :

5·8 cm., 8·5 cm., ·8 cm., 6·5 cm.

4. Perform the following operations :

(*a*) 700771 + 4044 + 50805 ; (*b*) £504. 17*s.* 6*d.* ÷ 42.

5. A crop of damsons, comprising 124 bushels, is owned equally by two men. One takes 74 bushels, paying the other 18*s.* ; what is the value of the damsons per bushel ?

PAPER 60.

1. (*a*) Multiply by 100 : (i) 1·25 dm. ; (ii) ·2 dm. ; (iii) ·05 dm

(*b*) Divide by 100 : (i) 15·2 dm. ; (ii) 3·8 dm. ; (iii) ·5 dm.

2. (*a*) Find the sum of the following lengths : ·3 dm., ·26 dm., ·08 dm., ·72 dm., ·65 dm.

(*b*) What length must be added to 1·05 dm. to make 5·5 dm. ?

3. What decimal of a metre is each of the following ?

(*a*) 15 cm. ; (*b*) 5 cm. ; (*c*) 15 mm. ; (*d*) 5 mm.

4. Perform the following operations :

(*a*) 17 cwt. 3 qrs. 26 lb. ÷ 19 ; (*b*) £175. 15*s*. $3\frac{3}{4}$*d*. ÷ 21.

5. A stack of straw is sold for £2. 15*s*. per wagon-load. The purchaser finds a wagon holds 1 ton 2 cwt. and the stack weighs 14 tons 6 cwt. ; how much does he give for the straw ?

PAPER 61.

1. Add together 3·4 Kg., 4·09 Kg., 0·885 Kg. and 0·063 Kg.
What weight must be added to 3·375 Kg. to make 10·2 Kg. ?

2. Express as francs and add 12 f. 76 c., 21 f. 80 c., 19 f. 5 c., 1 f. 4 c.
Subtract 5 centimes from 5 francs. Write both answers in francs and centimes.

3. Three strips of wood placed end to end in a straight line extend 10·5 inches. The first is 4·75 inches long and the second 2·8 inches ; how long is the third strip ?

4. Perform the following operations :

(*a*) £98. 18*s*. $4\frac{1}{2}$*d*. × 13 ; (*b*) £7. 15*s*. $7\frac{1}{2}$*d*. × 129.

5. In a cricket match, in the first innings A team made 314 runs and B team made 237. In the second innings A team made 246 runs and won by 43 ; how many runs did B team make in the second innings ?

PAPER 62.

1. Express as kilometres and add 2 Km. 985 m., 3 Km. 29 m., 5 Km. 6 m.
Express as kilometres and subtract 900 m. from 5 Km. 10 m.
Write both answers in kilometres and metres.

2. Express as dollars and add $39. 16 c., $4. 5 c., $173. 90 c.
Express as dollars and subtract 5 c. from $5.
Write both answers in dollars and cents.

3. A merchant has 12 hectolitres of wine. If he sells 1·5 Hl. to one customer, 2·75 Hl. to a second customer and 4·8 Hl. to a third, how many hectolitres of wine remain unsold ?

4. If out of £100, thirty-one persons receive £3. 3*s*. 6*d*. each, how much is left ?

5. A pint of pure water weighs a pound and a quarter. Find, in pounds, the weight of 85 gallons 3 quarts of distilled water.

PAPER 63.

1. Multiply by 5 : (*a*) 302 m. ; (*b*) 30·2 m. ; (*c*) 3·02 m. ; (*d*) ·302 m.

2. Find the value of (*a*) ·054 × 70 ; (*b*) ·008 × 72 : (*c*) ·605 × 12 × 8.

3. Simplify the expressions :

(*a*) 70·54 + 1·23 + ·075 + 18·21 ; (*b*) 4·7062 − ·94836.

4. How many hat-bands, each requiring 23 inches of ribbon, can be cut from a roll of ribbon 30 yards long ? How much ribbon will be left over ?

5. Last year I bought 35 tons of coal for £42 ; this year my coal account was £54. Assuming I always pay the same price for coal, how many tons did I buy this year ?

PAPER 64.

1. Multiply (*a*) 6·3 by 4·9 ; (*b*) 5·5 by 3·6 ; (*c*) 8·4 by 2·5.

2. Find the following products :

(*a*) 7·6 × ·27 ; (*b*) 13·2 × ·014 ; (*c*) 13 × ·013.

3. Find the sum of 77·77, 9·393 and ·234, and subtract from it the sum of 15·15 and 3·131.

4. Perform the following operations :

(*a*) £7106. 19*s*. 3*d*. ÷ 396 ; (*b*) £307. 6*s*. 4½*d*. × 196.

5. The product of three numbers is 1672. The product of the first two is 209 and the product of the last two is 88 ; what are the three numbers ?

PAPER 65.

1. Divide (*a*) 12·935 by 5 ; (*b*) 5·736 by 12 ; (*c*) ·531 by 9 ; (*d*) ·76 by 8.

2. Find the following quotients :

(*a*) 7·56 ÷ 20 ; (*b*) 293·4 ÷ 90 ; (*c*) 4·48 ÷ 80.

3. Find the cost of 315 articles at £1. 4*s*. 8*d*. each, and the cost of one article when 365 cost £453. 11*s*. 9¼*d*.

4. Express 23 hrs. 27 min. 11 sec. in seconds ; and 1851 pints in gallons, etc.

5. Find, in francs and centimes, the value of 6 dollars 36 cents, if 1 dollar can be exchanged for 5 francs 25 centimes.

PAPER 66.

1. Divide (*a*) 4·15 by ·5 ; (*b*) ·25 by ·05 ; (*c*) ·32 by ·5 ; (*d*) ·003 by ·05.

2. Find the following quotients :

(*a*) 1·71 ÷ 19 ; (*b*) 1·71 ÷ ·19 ; (*c*) 1·71 ÷ ·019 ; (*d*) 7·619 ÷ ·0019.

3. Perform the following operations :

(*a*) £894. 16*s*. 4¼*d*. + £305. 5*s*. 11½*d*. + 18*s*. 5¾*d*. + £4. 5*s*. 8½*d*. :
(*b*) £31075. 11*s*. 11½*d*. − £2183. 0*s*. 7¾*d*.

4. If 47 articles cost 145 francs 70 centimes, what is the price of the articles per dozen ?

5. A fruiterer sold oranges at 9*d*. per dozen. He had bought them at 4*s*. 2*d*. per hundred ; how much did he gain on each dozen sold ?

Approximation.

PAPER 67.

1. Divide £87251. 19*s.* 2*d.* into 53 equal parts, to the nearest penny.

2. Divide ·566 by ·0807 and 7·04 by ·0847, correct to four places of decimals in each case.

3. Add together 76 kilograms, 486 hectograms, 39857 decagrams, 4729600 grams, and give the answer, correct to the nearest kilogram.

4. Find, correct to the nearest centime, the cost of 287 m. 7 cm. of cloth at 5 f. 70 c. per metre.

5. If 235 m. 50 cm. of silk cost 659 f. 25 c., find, correct to the nearest centime, the price per metre.

PAPER 68.

1. The sum of 60 guineas is divided, to the nearest penny, among 100 persons ; how much does each receive ?

2. If £1 is worth 4 dollars 85 cents, express 100 dollars in English money, to the nearest penny.

3. If 77 metres of wire can be bought for 10 centimes, what will be the cost, to the nearest centime, of 1 kilometre of wire ?

4. Divide 56·372 by 19·3 and ·07354 by ·826, to 2 significant figures in each case.

5. Assuming 1 metre = 1·09364 yards, express 2500 metres in yards, to the nearest yard ; and given that a yard = ·914376 of a metre, express 4400 yards in kilometres and metres, to the nearest metre.

Decimals of Concrete Quantities.

PAPER 69.

1. Express (*a*) $7\frac{1}{2}d.$ as the decimal of 1*s.* ; (*b*) 7*s.* 6*d.* as the decimal of £1 ; and (*c*) £1. 17*s.* as the decimal of £1.

2. Express 11*s.* $4\frac{1}{2}d.$ as the decimal of £1, and £1. 17*s.* 6*d.* as the decimal of £3.

3. Multiply 5·49 by ·0137 ; and divide 1·53 by ·068.

4. Convert $\frac{3}{5}+\frac{7}{250}+\frac{7}{5000}$ to a decimal, using the shortest method known to you.

5. If I walk 5·58 kilometres in an hour, how many centimetres do I walk in 4 seconds ?

PAPER 70.

1. Express 1 qr. 14 lb. as the decimal of 1 cwt., and 7 cwt. 3 qrs. as the decimal of 1 ton.

2. What decimal of 1 ton is 1 cwt. 2 qrs. 14 lb. ?

3. Divide ·005 by 6·25, and 798·1 by ·23.

4. If 450 lb. of coffee are bought for $163. 80 c. and are sold at 43 cents per lb., what is the profit on the outlay ?

5. If 3 Hl. 6 l. of wine cost 2376 francs, what should be the cost of 13 Hl. 9 l. ?

PAPER 71.

1. Find the value of (*a*) ·875 of 1*s.*; (*b*) ·2875 of £1; and (*c*) ·54375 of £1.

2. Express £2·96875 in pounds, shillings and pence; and find the value of ·053125 of £40.

3. Divide 2789·44 by ·0368, and 2·78944 by 368.

4. When eggs cost 30 centimes each, how many eggs can be bought for 8 francs 40 centimes?

5. Given 1 metre = 39·37 inches, express 5 miles in kilometres and metres, correct to the nearest metre.

PAPER 72.

1. Find the value of (*a*) ·375 of 1 cwt., and (*b*) ·3875 of 1 ton.

2. Express 8·325 of 5 cwt. in tons, etc., and 7 lb. as the decimal of 1 ton.

3. Divide 105·6 by ·0275, and ·001056 by 27·5.

4. Express (*a*) 12·47 shillings as the decimal of £1, and (*b*) 5*s.* 6*d.* as the decimal of £2.

5. If 3·5 metres of silk cost 4 francs 55 centimes, what will be the cost of 5·5 metres of the same silk?

PAPER 73.

1. Express as decimals of £1, correct to 3 places: (*a*) 3*s.* 1½*d.*, and (*b*) 15*s.* 4½*d.*

2. Express in £ *s.* *d.*, correct to the nearest farthing: (*a*) £·2813, and (*b*) £2·9719.

3. Express ·891 of £1 in shillings and pence, correct to the nearest penny, and ·0729 of 1 ton in pounds, correct to the nearest pound.

4. Divide 19·924 by 3400, and ·19924 by ·034.

5. Find the cost of 97 m. 4 cm. of cloth at 3 f. 75 c. per metre; and the value of 7 dozen books at $9. 37 c. each.

Decimal Measures of Area and Volume.

PAPER 74.

1. (*a*) A rectangle is 27·3 cm. long and 20·3 cm. wide. Find its area (i) in square centimetres, (ii) in square metres.

(*b*) The area of a rectangle is 5·35 sq. metres, and its length is 11·2 metres; find its width, to the nearest centimetre.

2. Express ·5347 of an acre in square yards, correct to the nearest square yard. Find it also correct to the nearest square foot.

3. Express $\frac{1}{80}$ as a decimal, and ·025 as a vulgar fraction in its lowest terms.

4. Express 7·8125 of 8 cwt. in pounds; and find the value of ·78125 of 1 cwt. in qrs. lb. and oz.

5. A pleasure ground is 200 ft. long and 150 ft. wide, and a carriage drive 10 ft. wide runs round it inside the fence; what is the area of the drive?

PAPER 75.

1. The floor of a room is 6·5 metres long and 4·4 metres wide; how many square metres does it contain? How many francs would it cost to cover the floor with a carpet at 5 centimes the square decimetre?

2. Express ·91875 of an acre in roods and perches; and 3·45 times 3 roods 23 perches, correct to the nearest perch.

3. Divide 991·488 by ·0256, and 9·91488 by 2560.

4. Express 25000 feet in miles, furlongs, yards and feet; and ·7187 of a mile in yards, correct to the nearest yard.

5. If a piece of land can be divided up into 75 allotments each of 300 sq. yards, into how many allotments of 250 sq. yards could it be divided?

PAPER 76.

1. (*a*) Find, in cubic metres, the volume of a rectangular block 42 m. 25 cm. long, 4 m. 60 cm. wide and 12 mm. deep.

(*b*) Find, in metres, the length of a rectangular block containing 13 cubic metres, if the width is 5 m. and the height 80 cm.

2. Given that the litre is the volume of 1 cubic decimetre, how many litres will be contained in a rectangular tank 3 m. 50 cm. long, 1 m. 80 cm. wide and 42 cm. deep?

3. If a litre of water weighs 1 kilogram, what is the weight of 26 hectolitres 46 litres of water?

4. If a gallon of water weighs 10 lb. and a cubic foot of water weighs 1000 ounces, how many quarts of water are there in a cubic foot?

5. Find the area of the material required to make a box (without a lid) 9·2 inches long, 4·2 inches wide and 4·2 inches deep.

Vulgar Fractions: First Notions; The Four Rules.

PAPER 77.

1. (*a*) What fraction of a foot is—

(i) 5 inches; (ii) 6 inches; (iii) 9 inches?

(*b*) How many quarters are there in—

(i) 5 inches; (ii) $6\frac{3}{4}$ inches; (iii) $9\frac{1}{2}$ inches?

(*c*) Express as mixed numbers: (i) $\frac{9}{2}$; (ii) $\frac{11}{4}$; (iii) $\frac{13}{8}$.

(*d*) Express as improper fractions: (i) $3\frac{1}{2}$; (ii) $2\frac{3}{4}$; (iii) $4\frac{5}{8}$.

(*e*) Express in their lowest terms: (i) $\frac{5}{10}$; (ii) $\frac{6}{8}$; (iii) $\frac{6}{4}$.

2. (*a*) Which is the longer—(i) $\frac{3}{4}$ in. or $\frac{7}{8}$ in.? (ii) $\frac{7}{10}$ in. or $\frac{1}{2}$ in.?

(*b*) What length do you get by adding—

(i) $\frac{1}{2}$ in. and $\frac{3}{4}$ in.? (ii) $\frac{3}{8}$ in. and $\frac{1}{2}$ in.?

(*c*) What length do you get by adding—

(i) $\frac{9}{10}$ in. and $\frac{7}{10}$ in.? (ii) $\frac{5}{8}$ in. and $\frac{3}{4}$ in.?

3. Arrange in order of magnitude, beginning with the smallest:

(*a*) $\frac{2}{3}$ shilling, $\frac{3}{4}$ shilling, $\frac{5}{12}$ shilling; (*b*) $\frac{3}{4}$ inch, $\frac{7}{8}$ inch, $\frac{3}{10}$ inch.

4. Find the least number divisible by all the numbers 1 to 8 inclusive.

5. Find the prime factors of 105, 168 and 147; hence find the highest common factor of the three numbers.

PAPER 78.

1. Write $5\frac{5}{12}$ as an improper fraction, $\frac{91}{12}$ as a mixed number and $\frac{40}{48}$ in its lowest terms.

2. What fractions with denominator 24 are equal to $\frac{7}{8}$ and $\frac{5}{6}$? Which is the greater, $\frac{5}{6}$ or $\frac{7}{8}$?

Express as a fraction of a shilling five-sixths of a shilling added to seven-eighths of a shilling.

3. Find the L.C.M. of 4, 10 and 12.

Reduce $\frac{3}{4}$, $\frac{7}{12}$, $\frac{9}{10}$ to their lowest common denominator, and arrange them in ascending order of magnitude.

4. Resolve 324 and 720 into their prime factors; hence find the H.C.F. of 324 and 720. Reduce $\frac{324}{720}$ to its lowest terms.

5. Find the smallest sum of money which is an exact multiple of each of the following: 1*s.* 4*d.*, 2*s.*, 1*s.* 8*d.* and 1*s.* 3*d.*

PAPER 79.

1. Add together (*a*) $\frac{7}{10}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{1}{2}$; (*b*) $1\frac{7}{12}$, $\frac{2}{3}$, $4\frac{7}{8}$, $\frac{1}{2}$.

2. Find the value of (*a*) $\frac{1}{10}-\frac{1}{12}$; (*b*) $\frac{3}{4}-\frac{1}{8}$; (*c*) $\frac{4}{5}-\frac{1}{2}$.

3. What length must be added to (i) $\frac{5}{8}$ inch to make 1 inch; (ii) $\frac{1}{2}$ inch to make $3\frac{1}{4}$ inches; (iii) $2\frac{7}{8}$ inches to make $4\frac{1}{2}$ inches?

4. Find the L.C.M. of 10, 12, 15, 30, 42 and 105.

5. A certain number of pennies is more than £5 in value, and less than £5. 12*s.*, and can be divided equally among 153 boys; what is the amount each receives?

PAPER 80.

1. Find the value of

(*a*) $3\frac{1}{2}+2\frac{1}{3}+2\frac{1}{4}+3\frac{1}{6}$; (*b*) $5\frac{2}{3}-\frac{3}{4}$; (*c*) $5\frac{3}{8}-2\frac{7}{10}$.

2. Simplify the following expressions:

(*a*) $5-\frac{7}{8}-\frac{3}{4}$; (*b*) $7\frac{5}{12}-6\frac{14}{15}+1\frac{7}{20}$.

3. Find the prime factors of 196, 350 and 924; hence find the greatest common measure of the three numbers.

4. Reduce $\frac{1960}{3500}$ and $\frac{3500}{9240}$ to their lowest terms.

5. A flock of sheep can be counted by fours, fives and sixes. It contains less than 250 sheep and more than 200; how many are there in the flock?

PAPER 81.

1. (*a*) Multiply (i) $\frac{1}{10}$ inch by 7; (ii) $\frac{5}{12}$ inch by 3; (iii) $2\frac{3}{8}$ inches by 4.

(*b*) Divide (i) $\frac{3}{4}$ inch by 3; (ii) $\frac{1}{2}$ inch by 4; (iii) $1\frac{1}{4}$ inches by 5.

2. Add $3\frac{7}{22}$ to $5\frac{31}{33}$; and subtract $11\frac{5}{12}$ from $15\frac{2}{9}$.

3. Simplify the following expressions:

(*a*) $3\frac{1}{4}+2\frac{3}{8}-5\frac{1}{6}-\frac{1}{3}$; (*b*) $8\frac{5}{8}+\frac{5}{96}-5\frac{7}{16}+\frac{3}{32}$.

4. Find the greatest number which will divide 36, 84, 120 and 216, and the least number which they will divide, in each case without remainder.

5. If a man spends £413. 13*s.* 4*d.* in a year of 365 days, what is his average daily expenditure?

PAPER 82.

1. (*a*) What length is (i) $\frac{1}{5}$ of $\frac{1}{2}$ in.; (ii) $\frac{3}{4}$ of $\frac{1}{2}$ in.; (iii) $\frac{2}{5}$ of $\frac{3}{4}$ in.?

(*b*) How often is $\frac{3}{8}$ in. contained in 6 in.?

(*c*) How many pieces of string $1\frac{3}{4}$ in. long can be cut from 14 in.?

(*d*) How many times is $1\frac{3}{8}$ in. contained in $2\frac{3}{4}$ in.?

2. How many inches are there in $3\frac{3}{5}$ lengths of string each $4\frac{7}{12}$ inches long?

How many pieces of string, each $4\frac{5}{8}$ in. long, can be cut from $18\frac{1}{2}$ inches?

3. Simplify the following expressions:

$$(a)\ 2\tfrac{7}{8}-3\tfrac{5}{6}+8\tfrac{5}{24}-5\tfrac{11}{12}; \quad (b)\ 4\tfrac{1}{4}-5\tfrac{7}{9}+8\tfrac{1}{3}-2\tfrac{5}{6}.$$

4. Find the value of (*a*) $\frac{4}{5}\times\frac{5}{6}\times\frac{2}{3}$; (*b*) $4\frac{7}{12}\times 3\frac{3}{5}$.

5. Find the average of 36, 15, 22, 0, 38, 6, 11, 51, 42, 3, 19 and 57.

PAPER 83.

1. Multiply $2\frac{3}{35}$ by $1\frac{18}{73}$; and divide $3\frac{1}{5}$ by $4\frac{4}{7}$.

2. Divide the sum of $\frac{2}{3}$ and $3\frac{1}{8}$ by $2\frac{7}{16}$; and divide the product of $4\frac{2}{3}$ and $5\frac{4}{7}$ by $3\frac{1}{4}$.

3. Simplify the following expressions:

$$(a)\ 10\tfrac{1}{2}\times 6\tfrac{3}{8}\times 1\tfrac{1}{7}\times\tfrac{1}{17}\times 3\tfrac{1}{5}; \quad (b)\ 1\tfrac{2}{3}\times 3\tfrac{3}{8}\div 1\tfrac{5}{16}.$$

4. Find the highest common factor of 444 and 703; and reduce $\frac{444}{703}$ to its lowest terms.

5. The weights of eight men are respectively 9 st. 5 lb., 12 st., 11 st. 5 lb., 12 st. 8 lb., 13 st. 3 lb., 11 st. 10 lb., 12 st. 2 lb. and 11 st. 5 lb.; find the average weight per man.

Brackets and Complex Fractions.

PAPER 84.

1. Simplify the following expressions:

$$(a)\ (35-21+14)\div 7; \quad (b)\ \left(\tfrac{1}{3}-\tfrac{1}{5}+\tfrac{1}{7}\right)\div 1\tfrac{8}{21}.$$

2. Compare the sum and the difference of $\frac{42}{75}$ and $\frac{5}{8}$.

3. Multiply $1\frac{5}{21}$ by $\frac{63}{65}$; and divide $\frac{17}{96}$ by $1\frac{5}{12}$.

4. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{9}$ of $\frac{27}{28}$ to a simple fraction.

Subtract $5\frac{3}{8}$ of $1\frac{3}{5}$ from $2\frac{2}{3}$ of $5\frac{1}{4}$.

5. Express ·375 as a vulgar fraction in its lowest terms; and express $\frac{15}{203}$ as a decimal, correct to 3 places.

PAPER 85.

1. Add $\frac{1}{1}\frac{3}{8}$ and $\frac{2}{3}$ together; subtract $1\frac{1}{5}$ from the sum; multiply the result by $2\frac{1}{1}\frac{1}{7}$.

2. What number must be added to $\frac{5}{8}$ of $\frac{3}{7}$ to make $\frac{9}{1\,0}$ of $3\frac{3}{4}$?

3. Simplify the following expressions:

$$(a)\ (3\tfrac{1}{4}-2\tfrac{1}{3})\div(4\tfrac{1}{4}+2\tfrac{1}{6});\quad (b)\ (\tfrac{3}{4}+\tfrac{4}{5}+\tfrac{5}{6})\times(\tfrac{1}{1}\tfrac{1}{3}-\tfrac{7}{1\,1}).$$

4. Multiply and divide ·006175 by 32·5.

5. What is the cost, at 2*d.* per 1000 gallons, of filling a swimming bath 80 ft. long and 30 ft. wide with water to the depth of 5 ft., assuming that $6\frac{1}{4}$ gallons of water occupy 1 cubic foot?

PAPER 86.

1. Express in their lowest terms:

$$(a)\ \frac{\frac{7}{1\,0}}{3\frac{1}{2}};\qquad (b)\ \frac{10}{2\frac{1}{2}};\qquad (c)\ \frac{\frac{2}{5}\text{ of }5}{\frac{3}{5}\text{ of }1\frac{4}{5}}.$$

2. Reduce to a simple fraction $2\frac{1}{2}+5\frac{1}{9}$ of $\frac{4}{2\,3}-\frac{4}{1\,5}$ of $3\frac{1}{8}$.

3. Divide $\frac{4}{7}$ of $5\frac{3}{5}$ by $\frac{1}{2}$ of $\frac{1}{3}$ of $1\frac{4}{5}$.

4. Simplify the following expressions:

$$(a)\ \tfrac{8}{4\,5}\times\frac{5\frac{1}{8}}{8\frac{1}{5}}\times5\tfrac{1}{7}\times1\tfrac{1}{6};\quad (b)\ (1\tfrac{1}{3\,6}\times6\tfrac{1}{4})\div(11\tfrac{9}{1\,6}\times\tfrac{2\,0}{2\,1}).$$

5. How often does the sum of 2064 and 1892 contain their difference? How many times does the sum of 58·48 and 65·79 contain their difference?

PAPER 87.

1. Reduce to its lowest terms $\dfrac{\frac{3}{4}\text{ of }\frac{1}{8}\text{ of }5\frac{1}{2}}{4\frac{1}{2}\text{ of }8\frac{1}{4}}$.

2. Find the quotient of $\dfrac{20\frac{3}{4}}{3}$ by $\dfrac{41\frac{1}{2}}{4}$.

3. Simplify the following expressions:

$$(a)\ \frac{5\frac{2}{5}\times7\frac{2}{9}}{8\frac{7}{2\,4}-3\frac{5}{1\,2}};\quad (b)\ (3\tfrac{1}{4}+1\tfrac{1}{3})\times(\tfrac{2}{5}-\tfrac{1}{1\,1}).$$

4. Divide the sum of $6\frac{5}{7}$ and $4\frac{3}{5}$ by their difference; and divide the difference between $10\frac{6}{7}$ and $13\frac{4}{7}$ by their sum.

5. If 1 kilogram = 2·2 lb., how many kilograms and grams are there in $35\frac{3}{4}$ lb.?

Fractions of Concrete Quantities.

PAPER 88.

1. How much money is (*a*) $\frac{7}{8}$ of 19*s.* 8*d.* and (*b*) $\frac{5}{1\,6}$ of 19*s.* 4*d.*?

2. Multiply £47. 17*s.* 8*d.* by $5\frac{3}{4}$; and divide 17*s.* 11*d.* by $3\frac{1}{3}$.

3. What fraction of 1*s.* is $9\frac{3}{4}$*d.*, and what fraction of £1 is 16*s.* 3*d.*?

4. Simplify the following expressions :

(*a*) $2(7\times315-200)+106\times5$; (*b*) $(\cdot437\times4\cdot8)+(\cdot056\times\cdot4)$.

5. If 1 lb. = 453·6 grams, how many pounds are equivalent to 56·7 kilograms ?

PAPER 89.

1. Express 3*s.* 9*d.* as the fraction of £2 in its lowest terms ; and convert this fraction to a decimal.

2. What fraction of 17 tons 15 cwt. is 5 tons 6 cwt. 2 qrs. ?

3. Add together $\frac{2}{7}$ of ·0987 and ·045 of $1\frac{2}{9}$, expressing the result as a decimal.

4. Simplify the following expressions :

$$(a)\ \frac{3\frac{1}{4}-1\frac{1}{2}}{3\frac{1}{3}-1\frac{1}{6}}\times1\tfrac{1}{4}\ ; \qquad (b)\ \frac{4\frac{2}{7}+1\frac{1}{2\,1}}{9\frac{5}{1\,8}-2\frac{1}{6}}.$$

5. If a swimmer takes 22 strokes a minute and each stroke takes him $2\frac{1}{2}$ yards, how many minutes will it take him to swim half a mile ?

Practice.

PAPER 90.

1. Find, by Practice, the cost of 765 tons of cheese at £2. 17*s.* 8*d.* per ton.

2. Express 162, 588 and 1008 in prime factors, and write in prime factors their highest common factor and lowest common multiple.

3. Express £3. 4*s.* $10\frac{1}{2}d.$ as the decimal of £4 ; and £4. 7*s.* 2*d.* as the fraction of £9. 16*s.* $1\frac{1}{2}d.$

4. Simplify the following expressions :

$$(a)\ \frac{2-1\frac{1}{1\,6}}{2\frac{7}{2\,4}+\frac{5}{6}}\ ; \qquad (b)\ \frac{250}{0\cdot8}-\frac{22\cdot4}{0\cdot25}.$$

5. If a motor-car travels 660 yards in 1 min. 15 sec., what is its speed in miles per hour ?

PAPER 91.

1. Find, by Practice, the cost of 170 miles of telegraph wire at £14. 16*s.* 8*d.* per mile.

2. Divide 56 by 2187·5 ; and multiply 163840 by ·00625.

3. Find the sum of ·8125 of £14. 15*s.* and $\frac{7}{1\,5}$ of £3. 2*s.* $9\frac{3}{4}d.$

4. Simplify the following expressions :

$$(a)\ \frac{4\frac{2}{3}\times8\times5\frac{1}{7}}{28\frac{1}{2}\times\frac{8}{5\,7}}\ ; \qquad (b)\ \frac{\cdot034\times\cdot081}{1\cdot53}.$$

5. If a train travels 1408 yards in 48 seconds, how many minutes will it take to travel a mile and a half ?

PAPER 92.

1. Find, by Practice, the cost of repairing 27 yds. 2 ft. 11 in. of road at £2. 4*s.* 3*d.* per yard.

2. Divide the product of 61·93 and 0·117 by 1·43.

3. Add together $\frac{7}{12}$, $2\frac{1}{6}$ and $\frac{5\frac{1}{2}}{4\frac{2}{5}}$.

Express £1. 2*s.* $4\frac{1}{2}$*d.* as the fraction of £5. 4*s.* 5*d.*

4. Express (*a*) ·000007 kilogram in milligrams; and (*b*) $\frac{1}{4}$ pint as the decimal of 1 gallon.

5. The area of a rectangular room is 287·5 sq. metres; if one side measures 125 decimetres, what is the length of the other in metres?

PAPER 93.

1. Find, by Practice, the value of 16 bags of coal, each containing 1 cwt. 3 qrs. 7 lb., at £1. 7*s.* 6*d.* per ton.

2. Simplify $\frac{5}{21}$ of $3\frac{1}{2} - 2\frac{1}{6} \div 8\frac{2}{3} + \frac{5}{8}$ of $3\frac{2}{3} \times \frac{9}{10}$.

3. Express ·00752 of 1 ton in ounces, correct to the nearest ounce; and reduce 3 cwt. 2 qrs. 10 lb. 8 oz. to the decimal of 5 cwt.

4. Find the value, in francs and centimes, of 213750 grams of sugar at 96 centimes per kilogram.

5. The circumference of a wheel is $3\frac{1}{7}$ times the diameter. How many revolutions per minute will the driving wheel of a locomotive, 5 ft. 3 in. in diameter, make when the engine is travelling at the rate of 45 miles per hour?

Fractions: Problems.

PAPER 94.

1. I spend $\frac{7}{80}$ of the money in my purse on a theatre ticket, $\frac{2}{5}$ of it on shopping and lose 6*d.* If I have then one-half of the original sum left, how much did the purse contain at first?

2. Express the product of 20, 63 and 108 in prime factors. What additional factors are required to make the product a square?

3. Multiply ·0719 by ·8203; and divide ·8521 by 92·7 to 3 significant figures.

4. Find the value of 1·08 of 5*s.* + ·03 of 7*s.* 6*d.* + ·0875 of £1. 10*s.*

5. The rainfall on a certain day was 2 inches; find the number of gallons of water which fell on an acre, assuming a cubic foot of water occupies $6\frac{1}{4}$ gallons.

PAPER 95.

1. A and B are engaged to do a piece of work for £1. 8*s.* A could do it alone in 6 hours, B in 8 hours. How long will they take working together, and what money should each receive?

2. A, B and C pay for the cost of making a road. A pays one-half, B pays one-third; what fraction of the cost does C pay? If C's share is £43. 6*s.* 8*d.*, what is the total cost of the road?

3. If 125 men can do a piece of work in 120 days, in how many days should 150 men do it ?

4. A can run a mile in 5 minutes, and B can run a mile in 5 min. 20 sec. ; how many yards has B run when A has run a mile, if both start together ?

5. If tea costs 8 f. 25 c. per kilogram, what is the price per lb. in English money ? (25 francs = £1 ; $2\frac{1}{2}$ lb. = 1 Kg.)

PAPER 96.

1. If $\frac{7}{11}$ of an estate is worth £4655, what is the value of the remainder ?

2. If $5\frac{5}{8}$ lb. of tea cost 11*s.* 3*d.*, what is the cost of 1 lb. ? And what will $4\frac{9}{16}$ lb. cost ?

3. A woman, after spending $\frac{1}{5}$ of her money at one shop and $\frac{1}{4}$ of the remainder at another, has £2. 8*s.* left ; how much had she at first ?

4. Find the value of 15 tons 3 cwt. 2 qrs. 16 lb. of coffee at £1. 1*s* per cwt.

5. If 3 cwt. 2 qrs. of butter cost £18. 15*s.* 8*d.*, what is the cost per cwt. ?

Fractions : Areas and Volumes.

PAPER 97.

1. Find the area of a rectangle 3 ft. 6 in. long and 2 ft. 4 in. wide. Express the area (i) in sq. feet and the fraction of a sq. foot ; (ii) in sq. feet and sq. inches.

2. If $\frac{8}{15}$ of an estate contains 23984 acres, what is the extent of the whole estate ?

3. Express 2 acres 3 roods 15 perches as the decimal of an acre.

4. Find the prime factors of 7056 ; hence find the square root of 7056. What will it cost to make a fence round a square plot of land containing 7056 square yards at 3*s.* 4*d.* per yard ?

5. A garden, 60 yards long and 24 yards wide, has a path round it, within the fence, $4\frac{1}{2}$ feet wide ; what is the area of the path ?

PAPER 98.

1. Find, in sq. ft. and sq. in., the area of a rectangle 5 ft. 4 in. long and 3 ft. 3 in. wide ; and find, in sq. yards, etc., the area of a rectangle 4 yds. 2 ft. 9 in. long and 3 yds. 3 in. wide.

2. What length of board 4 inches wide will be required to panel a room 23 ft. long and 15 ft. 8 in. wide to a height of 5 ft. 6 in.?

3. Find the rent of 12 acres 2 roods 17 perches of land at £14 per acre ; and the rent of 30 hectares 75 ares of land at 3 francs 5 centimes per are.

4. If the rent of a rectangular piece of ground 35 yards wide and 36 yards long is 5 guineas, what is the rent per acre ?

5. If a field of 15 acres 3 roods 10 perches provides pasture for 46 sheep, for how many sheep will a similar field of 33 acres suffice ?

PAPER 99.

1. Find the cubic content of a box 3 ft. 4 in. long, 2 ft. 5 in. wide and 1 ft. 7 in. deep. Express the volume (i) in cubic feet and the fraction of a cubic foot; (ii) in cubic feet and cubic inches.

2. How many cubic inches are there in $\frac{17}{243}$ of a cubic yard ?

3. Simplify (*a*) $\dfrac{4\frac{2}{3}-1\frac{5}{6}\times\frac{3}{2}}{3\frac{3}{4}+2\frac{1}{2}\times1\frac{1}{2}}$; (*b*) $\dfrac{12{\cdot}5+9{\cdot}7}{{\cdot}05-{\cdot}013}$.

4. A pond of 4 acres is frozen over with ice whose total weight is 1089 tons; if a cubic foot of ice weighs 896 ounces, find the thickness of the ice in inches.

5. The circumference of a wheel is $3\frac{1}{7}$ times the diameter. How many revolutions will a motor-car wheel, 750 millimetres in diameter, make in travelling 33 kilometres ?

PAPER 100.

1. Given that the litre is the volume of 1 cubic decimetre, find how many litres will be contained in a rectangular tank 1 metre long, 2 decimetres wide and 13 centimetres deep.

2. A class-room has 4 windows, and each window has 12 panes of glass measuring 1 ft. 7 in. by 11 in.; what is the total area of glass, in sq. feet and sq. inches ?

3. The area of a surface is 0·293 square metres, and its length is 0·62 metre; find its width to 2 significant figures.

4. A coal cellar is 3 ft. wide and 3 ft. 9 in. long. To what depth, to the nearest inch, would it be filled by a ton of coal spread uniformly, if a cubic foot of coal weighs 80 lb. ?

5. The circumference of a wheel is $3\frac{1}{7}$ times the diameter. What is the diameter of a wheel which turns 288 times in travelling half a mile ?

PAPER 101.

1. If a cubic foot of water weighs 1000 ounces, what is the weight of a cubic yard of water, expressed in hundredweights and pounds ?

2. A picture frame, 1 m. 5 cm. long and 74 cm. wide, is made of wood 6 cm. wide; how many square centimetres of wood are there in the frame ?

3. If 6 men can build a wall in 10 days, (*a*) how many men would be needed to build the wall in 12 days ? (*b*) how many days would 8 men take to build the wall ?

4. Multiply 0·763 by 0·0703; and divide 27·376 by 54·3, correct to three places of decimals.

5. In a rectangular pleasure ground, 96 ft. long and 84 ft. wide, there are four oblong grass-plots, each 22 ft. 6 in. long and 18 ft. wide: what will be the cost of covering the remaining portion with gravel to the depth of 2 in. at 6*s.* per cubic yard ?

EXAMINATION PAPERS.

HINTS TO STUDENTS.

(1) *Number each answer; note carefully questions which have sub-sections, and therefore require more than one answer.*

(2) *Show all the working of the answers.*

(3) *In answering questions such as* **4-8** *show clearly the different steps of your work.*

TEST 1.

Time allowed, 40 minutes.

1. Write down the answers to these addition sums :

(i)

2931
4168
3798
2342
5119

(ii)

£.	*s.*	*d.*
1868	7	9
257	13	10
2387	14	7
34	5	3

2. (i) Multiply 27·03 by 3·5.

(ii) Divide 0·284 by 0·92, correct to 3 decimal places.

(iii) Simplify (*a*) $\frac{3}{8}+\frac{5}{12}-\frac{3+5}{8+12}$; (*b*) $2\frac{3}{4}-\frac{2}{3}-\frac{1\frac{2}{3}}{2\frac{1}{2}}$.

3. Copy the following, filling up the blank spaces :

1 linear yard =	ft.	1 linear foot =	inches.
1 square yard =	sq. ft.	1 square foot =	sq. in.
1 cubic yard =	c. ft.	1 cubic foot =	c. in.

4. If 10 copies of a book cost £3. 15*s.*, (i) what will 4 copies cost, (ii) how many copies can be bought for £2. 5*s.*?

5. The cost of making up a road is to be paid by three people, A, B and C. A pays twice as much as C, and B pays three times as much as C. If the whole cost is £180, what does each pay?

6. Find the area (i) of the top, (ii) of the front, of a box which is 2 ft. 6 in. long, 1 ft. 6 in. wide and 1 ft. 4 in. high.

Give the area in each case in sq. ft. and sq. in.

7. Out of a total of 560 marks given at an examination a boy obtains 402; what percentage of the total does he obtain? Give the result correct to 3 significant figures.

8. A ladder 25 ft. long stands upright against a wall. Find how far the bottom of the ladder must be pulled out from the wall to lower the top 1 ft.

TEST 2.

Time allowed, 40 minutes.

1. Write down the answers to these subtraction sums:

(i)

From	863247
Take	91878

(ii)

£.	*s.*	*d.*
56271	15	7
2463	18	10

2. (i) Multiply 27·63 by 5·07.
(ii) Divide 0·003572 by 0·0037, correct to 2 decimal places.
(iii) Simplify $7\frac{5}{6} - 4\frac{1}{2} + 2\frac{1}{4} - 3\frac{5}{8}$.

3. Write down the answers to the following:

(i) $0{\cdot}012 \times 1000$.
(ii) $172 \div 1000$.
(iii) $0{\cdot}01 \times 0{\cdot}001$.
(iv) $0{\cdot}02 \div 0{\cdot}001$.

4. If 5 men can complete a piece of work in 12 days, (i) how many men would be required to complete it in 10 days, (ii) how many days would 8 men take to complete it?

5. The cost of making up a road is paid by three people, X, Y and Z. X pays one-half, Y pays one-third; what fraction does Z pay? If Z's share is £43. 6*s.* 8*d.*, what is the total cost?

6. The area of a certain rectangle is 5·35 sq. metres, and one side measures 11·2 metres. Find the length of the other side to the nearest centimetre.

7. In a certain town the number of deaths last year was 10·9 per thousand inhabitants. If the population of the town was 32350, find the number of deaths. Express the death-rate as a percentage of the population.

8. (i) Write down the formula for the area of a circle, r being the number of units in the radius.
(ii) Find, to 3 significant figures, the radius of a circle whose area is 100 sq. inches. ($\pi = \frac{22}{7}$.)

TEST 3.

Time allowed, 40 minutes.

1. Write down the answers to these two sums:

Multiply (i) 870563 by 7; (ii) £6403. 17*s.* 4*d.* by 7.

2. (i) Multiply 0·865 by 25·4.
(ii) Divide 0·8521 by 92·7, correct to 3 significant figures.
(iii) Simplify (*a*) $\dfrac{2 \times 3 \times 3 \times 7}{3 \times 7 \times 7}$; (*b*) $4\frac{1}{12} \times 4\frac{4}{7} \div 2\frac{1}{10}$.

3. Copy the following, filling up the blank spaces :

1 yard =	inches.	1 metre =	centimetres.
1 ton =	pounds.	1 kilogram =	grams.
1 gallon =	pints.	1 hectolitre =	litres.
1 mile =	yards.	1 kilometre =	metres.

4. A brass tube 4 ft. in length weighs 5 lb. ; what length of the tube will weigh $7\frac{1}{2}$ oz. ?

5. Two men P and Q rent a grazing field for £6. 10*s*. P puts in 14 sheep and Q 6 sheep ; how much rent ought each to pay ?

6. Find the volume of a wall 100 yards long, 8 feet high and 9 inches thick. Give the result (i) in cubic feet, (ii) as a compound quantity in cubic yards and cubic feet.

7. (i) The number of pupils attending a certain school A is $12\frac{1}{2}$ per cent. more than last term and is now 243 ; what was the number last term ?

(ii) The number of pupils attending another school B is $12\frac{1}{2}$ per cent. less than last term and is now 210 ; what was the number last term ?

8. In an experiment to find the value of π, a garden roller was marked with some whitewash and then rolled straight across a lawn ; the roller left eleven white marks on the ground. How many times did it turn round ? If the distance between the end marks was 46 ft. 10 in. and the diameter of the roller was 18 inches, find the value of π, correct to 1 decimal place.

TEST 4.

Time allowed, 40 minutes.

1. Write down the answers to these two sums :

Divide (i) 325116 by 12 ; (ii) £27653. 17*s*. 6*d*. by 8.

2. (i) Multiply 17·36 by 0·248.

(ii) Divide 82·78 by 11·6, correct to 2 decimal places.

(iii) Simplify $4\frac{1}{4} - \frac{1}{2}(3\frac{3}{8} - 2\frac{5}{12})$.

3. Write down the cost of

(i) 3 dozen eggs at $5\frac{1}{2}d$. each.

(ii) 5 lb. of tea at 2*s*. 10*d*. per lb.

(iii) $2\frac{1}{2}$ yds. of ribbon at 1*s*. 3*d*. per yd.

4. In an experiment on a spiral spring it is found that a pull of 10 lb. stretches the spring 1·5 inches. Find the stretch produced by a pull of 21 lb.

5. (i) Reduce 16*s*. $4\frac{1}{4}d$. to the decimal of £1, correct to 4 decimal places.

(ii) Express £10·358 in £. *s*. *d*., correct to the nearest penny.

6. A room 20 ft. by 15 ft. has a carpet in the middle and a stained border 2 ft. wide all round the carpet. Find the area of the carpet and the area of the border.

7. (i) A shopkeeper sells for £3. 15*s.* an article which cost him £3 ; what percentage profit does he make on his outlay ?

(ii) The price of an article has just been put up 25 per cent. Formerly the price was £2. 12*s.* What will be the new price ?

8. Find the number of revolutions made by a wheel 56 inches in diameter in travelling $1\frac{1}{2}$ miles ? ($\pi = 3\frac{1}{7}$.)

What will be the number of revolutions of a wheel of half the above diameter if the distance to be travelled is 5 times as great ?

TEST 5.

Time allowed, 40 minutes.

1. (i) Multiply 3275 by 509.

(ii) Divide £173. 18*s.* 9*d.* by 23.

2. (i) Multiply 0·0719 by 0·8203.

(ii) Divide 0·07354 by 0·826, correct to 2 significant figures.

(iii) Simplify $\frac{5}{21}$ of $3\frac{1}{2} - 2\frac{1}{6} \div 8\frac{2}{3} + \frac{5}{8}$ of $3\frac{2}{3} \times \frac{9}{10}$.

3. Write down the cost of

(i) 4 tons of coal at 3*s.* 3*d.* per cwt.

(ii) 3 cwt. of potatoes at $1\frac{1}{2}d.$ per lb.

(iii) 1 gross boxes of matches at $1\frac{1}{4}d.$ per box.

4. The scale of a certain map is 6 inches to the mile. What length on the map will represent a road 620 yards long ? Give the answer in inches, correct to 3 significant figures.

5. (i) Express 13*s.* $4\frac{1}{2}d.$ as a decimal of £1, correct to 3 decimal places.

(ii) Reduce £5·695 to £. *s.* *d.*, correct to the nearest penny.

6. Find the area of the metal required to make a biscuit box (without a lid) measuring 9·2 inches long, 4·2 inches wide and 4·2 inches deep.

7. A shopkeeper marks goods for sale to realize a profit of $33\frac{1}{3}\%$ on his outlay.

(i) The cost price of a certain article was 7*s.* 6*d.* ; what is the selling price ?

(ii) The selling price for another article is 7*s.* 6*d.* ; what was the cost price of this article ?

8. A carriage wheel of diameter 7 feet makes 60 revolutions per minute. Find the speed of the carriage (i) in feet per second, (ii) in miles per hour. ($\pi = 3\frac{1}{7}$.)

TEST 6.

Time allowed, 40 minutes.

1. (i) Multiply £1. 7*s.* 2*d.* by 16.

(ii) Divide 102718 by 29.

2. (i) Multiply 23·07 by 30·51.

(ii) Divide 0·03752 by 0·813, correct to 4 decimal places.

(iii) Simplify $(3\frac{5}{6} + 2\frac{2}{3}) \div (3\frac{5}{6} - 2\frac{3}{4})$.

3. (i) Express 1386 and 1764 each as the product of prime factors.
(ii) What is the largest number by which both 1386 and 1764 are exactly divisible ?
(iii) What is the square root of 1764 ?

4. If 4·3 metres of wire weigh 28·9 grams, find the weight of 3·5 metres of the same wire. Give the answer correct to 3 significant figures.

5. (i) Which is the greater, $8\frac{1}{2}$ inches or 0·7 feet ?
(ii) Express the difference as the decimal of a foot correct to 4 decimal places.

6. (*a*) A rectangular field measures 550 yds. by 330 yds. ; express its area in acres. (*b*) The ground is to be divided into allotments of 10 sq. poles each ; (i) how many allotments will there be to the acre ; (ii) how many allotments will there be in the above field ? [4840 sq. yds. = 1 acre, 40 sq. poles = 1 rood, 4 roods = 1 acre.]

7. If a Parisian shopkeeper buys 36 metres of silk for 403 f. 20 c. and sells it at the rate of 12 f. 32 c. per metre, what percentage profit does he make on his outlay ?

8. (i) Write down the formula for the volume of a cylinder, h being the number of units in the height, d being the units in the diameter of the circular base.
(ii) Find the volume of a cylinder if $d = 7$ cm. and $h = 2{\cdot}5$ cm. ($\pi = 3{\cdot}14$.)

TEST 7.

Time allowed, 40 minutes.

1. (i) Multiply 5683 by 307.
(ii) Divide £564. 2*s*. 2*d*. by 139.

2. (i) Multiply 82·1 by 0·8031.
(ii) Divide 0·06251 by 73·2, correct to 3 significant figures.
(iii) Simplify $3\frac{1}{7} \times (3\frac{1}{4} + 1\frac{1}{2})(3\frac{1}{4} - 1\frac{1}{2})$.

3. (i) Express $\frac{13}{17}$ and $\frac{11}{19}$ as decimals, correct to 2 significant figures.
(ii) Express 0·85 and 0·0625 as the simplest possible vulgar fractions.

4. A piece of ground can be divided up into 75 allotments each of 300 sq. yards ; into how many allotments of 250 sq. yards each could it be divided ?

5. Find the length of each side of a square field whose area is 7 acres. Give the result in yards, correct to the nearest yard.

6. Assuming that a cubic foot of water weighs 1000 ounces, find what weight of water falls on a rectangular playground measuring 70 yards by 40 yards if there is a fall of 1 inch of rain. Give the result in tons, cwt. and lb.

7. Find, to the nearest penny, the simple interest on £135. 10*s*. for 2 years at $5\frac{1}{2}\%$ per annum.

8. Find the weight of a solid metal cylinder of diameter 7 cm., length 10 cm., weight per cubic centimetre 7·5 grams. ($\pi = 3\frac{1}{7}$.)

TEST 8.

Time allowed, 40 minutes.

1. (i) Divide 223858 by 43.
(ii) Multiply £29. 15*s*. 6*d*. by 29.

2. (i) Multiply 35·27 by 0·0358.
(ii) Divide 56·372 by 19·3, correct to 3 significant figures.
(iii) Simplify $\dfrac{4\frac{2}{3}-1\frac{5}{6}\div\frac{2}{3}}{3\frac{3}{4}+2\frac{1}{2}\times1\frac{1}{2}}$.

3. Complete the following table, using the index notation :

(i) 1 square metre =	sq. cm. =	sq. mm.,
	1 sq. cm. =	sq. mm.
(ii) 1 cubic metre =	c. dm. =	c. cm.,
	1 c. dm. =	c. cm.

4. If a rectangular sheet of metal measuring 50 cm. by 40 cm. weighs 2000 grams, what will be the weight of a sheet measuring 30 cm. by 24 cm. ?

5. Reduce 13*s*. $10\frac{1}{2}$*d*. to the decimal of £1.
Hence write down the cost of 1000 articles at 13*s*. $10\frac{1}{2}$*d*. each.

6. A rectangular cistern is 2 m. 50 cm. long, 1 m. 40 cm. wide and 1 m. 72 cm. deep ; how many buckets of water will it contain, if a bucket holds 10 litres ? [1 litre = the volume of 1 cubic decimetre.]

7. Find, to the nearest centime, the simple interest on 4868 francs 75 centimes for 3 years at $5\frac{1}{4}\%$ per annum.

8. A metal pipe 50 cm. long has an outside diameter 4 cm. and an inside diameter 2·4 cm. Find (i) what volume of water the pipe would hold, (ii) what would be the weight of the empty pipe if each cubic centimetre of the metal weighed 10 grams ? ($\pi = 3\frac{1}{7}$.)

ANSWERS.

1.

1. 10.	2. 13.	3. 31.	4. 20.	5. 24.	6. 42.
7. 50.	8. 56.	9. 65.	10. 80.	11. 87.	12. 78.

2.

1. Fourteen.	2. Forty-one.	3. Twenty-five.	4. Fifty-two.
5. Sixty-eight.	6. Eighty-six.	7. Thirty-seven.	8. Seventy-three.
9. Ninety-four.	10. Forty-nine.		

3.

1. 33.	2. 36.	3. 36.	4. 39.	5. 38.
6. 35.	7. 61.	8. 60.	9. 58.	10. 60.

4.

1. 58.	2. 88.	3. 57.	4. 31.	5. 96.
6. 94.	7. 93.	8. 91.	9. 79.	10. 91.

5.

1. 97.	2. 93.	3. 40.	4. 62.	5. 84.	6. 57.
7. 65.	8. 75.	9. 88.	10. 71.	11. 59.	12. 81.
13. 95.	14. 83.	15. 98.	16. 74.	17. 99.	18. 85.
19. 74.	20. 90.	21. 84.	22. 75.	23. 98.	24. 99.

6.

1. 93.	2. 81.	3. 70.	4. 95.	5. 81.
6. 83.	7. 80.	8. 74.	9. 81.	10. 80.

7.

1. 1.	2. 1.	3. 1.	4. 2.	5. 3.
6. 5.	7. 11.	8. 12.	9. 12.	10. 13.
11. 12.	12. 12.	13. 13.	14. 15.	15. 14.
16. 14.	17. 12.	18. 12.	19. 13.	20. 12.

8.

1. 6.	2. 15.	3. 5.	4. 4.	5. 21.
6. 12.	7. 12.	8. 12.	9. 12.	10. 34.

9.

1. 16.	2. 61.	3. 22.	4. 45.	5. 43.
6. 43.	7. 53.	8. 12.	9. 57.	10. 22.

10.

1. 39.	2. 19.	3. 6.	4. 7.	5. 62.
6. 45.	7. 18.	8. 13.	9. 15.	10. 54.

11.

1. 69.	**2.** 54.	**3.** 49.	**4.** 26.	**5.** 39.	**6.** 34.
7. 35.	**8.** 19.	**9.** 38.	**10.** 69.	**11.** 57.	**12.** 12.
13. 72.	**14.** 18.	**15.** 59.	**16.** 38.	**17.** 90.	**18.** 82.

12.

1. 7.	**2.** 54.	**3.** 39.	**4.** 88.	**5.** 24.
6. 58.	**7.** 77.	**8.** 7.	**9.** 25.	**10.** 29.

13.

1. 11.	**2.** 12.	**3.** 8.	**4.** 17.	**5.** 5.	**6.** 27.
7. 28.	**8.** 7.	**9.** 25.	**10.** 15.	**11.** 18.	**12.** 17.

14.

1. 43.	**2.** 26.	**3.** 29.	**4.** 32.	**5.** 27.
6. 13.	**7.** 8.	**8.** 6.	**9.** 6.	**10.** 18.

15.

1. 18.	**2.** 18.	**3.** 14.	**4.** 4.	**5.** 14.
6. 4.	**7.** 29.	**8.** 37.	**9.** 19.	**10.** 37.
11. 3.	**12.** 37.	**13.** 13.	**14.** 21.	**15.** 19.
16. 31.	**17.** 29.	**18.** 37.	**19.** 19.	**20.** 7.

16.

1. 6*d.*	**2.** 9*s.*	**3.** 12.	**4.** 8.	**5.** Tom, 8; Bob, 17.
6. George, 14; Harry, 17.	**7.** 53.	**8.** 19.	**9.** 38.	**10.** 7.

17.

1. 3.	**2.** 16.	**3.** 15.	**4.** 15.	**5.** 24.	**6.** 10.	**7.** 20.
8. 20.	**9.** 18.	**10.** 72.	**11.** 24.	**12.** 24.	**13.** 24.	**14.** 24.
15. 24.	**16.** 40.	**17.** 60.	**18.** 36.	**19.** 27.	**20.** 64.	**21.** 16.

18.

1. 36.	**2.** 68.	**3.** 96.	**4.** 24.	**5.** 54.
6. 36.	**7.** 72.	**8.** 96.	**9.** 96.	**10.** 48.

19.

1. (*a*) 30; (*b*) 34; (*c*) 52; (*d*) 70; (*e*) 74; (*f*) 92.
2. (*a*) 42; (*b*) 48; (*c*) 45; (*d*) 72; (*e*) 78; (*f*) 75.
3. (*a*) 52; (*b*) 60; (*c*) 56; (*d*) 68; (*e*) 64; (*f*) 72.
4. (*a*) 60; (*b*) 70; (*c*) 65; (*d*) 80; (*e*) 75; (*f*) 85.
5. (*a*) 60; (*b*) 72; (*c*) 84; (*d*) 78; (*e*) 96; (*f*) 90.

20.

1. 26.	2. 36.	3. 38.	4. 24.	5. 27.	6. 75.
7. 57.	8. 45.	9. 19.	10. 75.	11. 30.	12. 84.
13. 40.	14. 4.	15. 21.	16. 45.	17. 37.	18. 31.
19. 8.	20. 10.	21. 8.	22. 24.	23. 75.	24. 39.

21.

1. 85.	2. 56.	3. 49.	4. 94.	5. 15.
6. 15.	7. 53.	8. 1.	9. 72.	10. 58.

22.

1. 64.	2. 96.	3. 54.	4. 49.	5. 51.
6. 99.	7. 48.	8. 16.	9. 72.	10. 71.

23.

1. 6.	2. 4.	3. 3.	4. 2.	5. 5.	6. 3.	7. 12.
8. 8.	9. 6.	10. 4.	11. 24.	12. 16.	13. 12.	14. 8.
15. 9.	16. 6.	17. 3.	18. 6.	19. 8.	20. 9.	21. 9.
22. 5.	23. 5.	24. 8.	25. 8.	26. 12.		

24.

1. 34.	2. 21.	3. 32.	4. 11 ; 4.
5. 16.	6. 16 ; 4.	7. 17.	8. 17.
9 15.	10. 25.	11. 14.	12. 13.
13. 13.	14. 14.	15. 17.	16. 15 ; 4.

25.

1. (*a*) 23 ; (*b*) 24 ; (*c*) 42 ; (*d*) 32 ; (*e*) 43.
2. (*a*) 13 ; (*b*) 21 ; (*c*) 32 ; (*d*) 23 ; (*e*) 31.
3. (*a*) 6, rem. 1 ; (*b*) 14, rem. 1 ; (*c*) 39, rem. 1 ; (*d*) 47, rem. 1 ; (*e*) 23, rem. 1.
4. (*a*) 6, rem. 2 ; (*b*) 11, rem. 2 ; (*c*) 19, rem. 2 ; (*d*) 29, rem. 1 ; (*e*) 31, rem. 1.
5. (*a*) 19, rem. 1 ; (*b*) 4, rem. 3 ; (*c*) 6, rem. 3 ; (*d*) 12, rem. 3 ; (*e*) 20, rem. 3.
6. (*a*) 23, rem. 1 ; (*b*) 22, rem. 2 ; (*c*) 16, rem. 2 ; (*d*) 24, rem. 3 ; (*e*) 12, rem. 1.
7. (*a*) 17, rem. 2 ; (*b*) 8, rem. 3 ; (*c*) 12, rem. 1 ; (*d*) 14, rem. 3 ; (*e*) 17, rem. 4.
8. (*a*) 19, rem. 1 ; (*b*) 11, rem. 4 ; (*c*) 14, rem. 2 ; (*d*) 19, rem. 3 ; (*e*) 18, rem. 1.
9. (*a*) 11, rem. 3 ; (*b*) 13, rem. 1 ; (*c*) 11, rem. 5 ; (*d*) 14, rem. 5 ; (*e*) 2, rem. 5.
10. (*a*) 4, rem. 5 ; (*b*) 16, rem. 2 ; (*c*) 7, rem. 5 ; (*d*) 12, rem. 5 ; (*e*) 15, rem. 5.

Supplementary.

3. 2.	4. 3.	5. 4.	6. 4.	7. 5.	8. 5.	9. 6.	10. 6.

26.

1. (*a*) 6; (*b*) 3; (*c*) 9; (*d*) 4; (*e*) 8; (*f*) 2; (*g*) 10.
2. (*a*) 10; (*b*) 5; (*c*) 15; (*d*) 4; (*e*) 8; (*f*) 12; (*g*) 16.
3. (*a*) 9; (*b*) 27. 4. (*a*) 4; (*b*) 12. 5. (*a*) 30; (*b*) 45.
6. (*a*) 18; (*b*) 20. 7. (*a*) 13; (*b*) 39.

27.

1. 4.	2. 6.	3. 5.	4. 15.	5. 13.	6. 13.
7. 12.	8. 8.	9. 9.	10. 12.	11. 9.	12. 11.

28.

1. 51.	2. 59.	3. 4.	4. 22.	5. 4.
6. 21.	7. 65.	8. 21.	9. 3.	10. 19.
11. 52.	12. 61.	13. 5.	14. 18.	15. 10.
16. 24.	17. 22.	18. 25.	19. 68.	20. 9.

29.

1. 23.	2. 15.	3. 15.	4. 39.	5. 16.
6. 8.	7. 12.	8. 18.	9. 50.	10. 57.

30.

1. 12.	2. 19.	3. 18.	4. 22.	5. 29. 6. 84.
7. Tom, 48; Fred, 51.	8. 9.	9. 63.	10. 60.	

31.

1. One hundred.	2. One hundred and forty.
3. Two hundred.	4. Two hundred and thirty.
5. Three hundred and twenty.	6. Four hundred and fifty.
7. Five hundred and sixty.	8. Six hundred and seventy.
9. Seven hundred and eighty.	10. Eight hundred and ninety.
11. Nine hundred.	12. Nine hundred and ten.
13. Two hundred and sixty-nine.	14. Three hundred and fifty-four.
15. Four hundred and ninety-six.	16. Five hundred and forty-three.
17. Five hundred and six.	18. Six hundred and seven.
19. Seven hundred and eight.	20. Eight hundred and nine.

32.

1. 101.	2. 110.	3. 108.	4. 811.	5. 214.	6. 717.
7. 770.	8. 111.	9. 120.	10. 801.	11. 204.	12. 241.
13. 707.	14. 510.	15. 303.	16. 313.	17. 330.	18. 333.

33.

1. 138.	2. 353.	3. 666.	4. 572.	5. 937.
6. 972.	7. 959.	8. 792.	9. 781.	10. 583.

34.

1. 134.	2. 121.	3. 154.	4. 171.	5. 134.	6. 160.
7. 163.	8. 205.	9. 255.	10. 305.	11. 324.	12. 285.
13. 679.	14. 659.	15. 781.	16. 922.	17. 776.	18. 884.
19. 937.	20. 769.	21. 598.	22. 816.	23. 760.	24. 965.

35.

1. 319.	2. 824.	3. 770.	4. 985.	5. 771.
6. 666.	7. 368.	8. 565.	9. 890.	10. 793.

36.

1. 62.	2. 79.	3. 168.	4. 185.	5. 272.
6. 173.	7. 166.	8. 91.	9. 47.	10. 89.

37.

1. 353.	2. 553.	3. 208.	4. 318.	5. 191.	6. 147.	7. 118.
8. 479.	9. 296.	10. 397.	11. 288.	12. 178.	13. 693.	14. 389.
15. 605.	16. 198.	17. 396.	18. 308.	19. 364.	20. 600.	21. 800.

38.

1. 58.	2. 197.	3. 102.	4. 225.	5. 104.	6. 186.
7. 123.	8. 269.	9. 288.	10. 191.	11. 364.	12. 318.
13. 538.	14. 33.	15. 403.	16. 299.		

39.

1. 738.	2. 42.	3. 77.	4. 27.	5. 199.
6. 465.	7. 461.	8. 799.	9. 483.	10. 311.

40.

1. 192.	2. 207.	3. 144.	4. 195.	5. 144.
6. 364.	7. 432.	8. 864.	9. 176.	10. 540.
11. 730.	12. 840.	13. 672.	14. 840.	15. 984.

41.

1. (*a*) 138; (*b*) 230; (*c*) 322.	2. (*a*) 340; (*b*) 408; (*c*) 476.
3. (*a*) 145; (*b*) 203; (*c*) 116.	4. (*a*) 609; (*b*) 696; (*c*) 783.
5. (*a*) 665; (*b*) 760; (*c*) 855.	6. (*a*) 672; (*b*) 756; (*c*) 924.

42.

1. 650.	2. 990.	3. 960.	4. 520.	5. 800.	6. 910.
7. 960.	8. 900.	9. 640.	10. 840.	11. 900.	12. 960.

43.

1. 105.	2. 110.	3. 154.	4. 165.	5. 125.	6. 343.
7. 176.	8. 108.	9. 176.	10. 392.	11. 180.	12. 512.

44.

1. 2×7. **2.** 3×5. **3.** 3×7. **4.** 2×11. **5.** 5×5. **6.** 3×11.
7. 5×7. **8.** 7×7. **9.** 5×11. **10.** 7×11. **11.** 6×7. **12.** 6×11.
13. 3×9. **14.** 9×9. **15.** 4×8. **16.** 8×8. **17.** 4×7. **18.** 5×9.
19. 7×9. **20.** 2×12, or 3×8, or 4×6. **21.** 7×8. **22.** 8×11.
23. 6×9. **24.** 4×12, or 6×8. **25.** 3×12, or 4×9, or 6×6.
26. 8×9, or 6×12. **27.** 2×9, or 3×6. **28.** 4×11.
29. 7×12. **30.** 9×11.

45.

1. 912. **2.** 609. **3.** 612. **4.** 910. **5.** 304.
6. 936. **7.** 812. **8.** 814. **9.** 645. **10.** 846.

46.

1. 288. **2.** 400. **3.** 945. **4.** 972. **5.** 945. **6.** 770.
7. 768. **8.** 945. **9.** 600. **10.** 896. **11.** 980. **12.** 540.
13. 640. **14.** 840. **15.** 900. **16.** 960.

47.

1. 4. **2.** 16. **3.** 36. **4.** 64. **5.** 100. **6.** 144. **7.** 225.
8. 324. **9.** 441. **10.** 484. **11.** 625. **12.** 9. **13.** 81. **14.** 25.
15. 49. **16.** 121. **17.** 196. **18.** 576. **19.** 729. **20.** 784.
21. 256. **22.** 216. **23.** 125. **24.** 343. **25.** 729. **26.** 64.

48.

1. 547. **2.** 785. **3.** 225. **4.** 26. **5.** 430. **6.** 529.
7. 158. **8.** 208. **9.** 273. **10.** 781. **11.** 963. **12.** 264.
13. 756. **14.** 176. **15.** 891. **16.** 900.

49.

1. 920. **2.** 503. **3.** 540. **4.** 212. **5.** 293.
6. 699. **7.** 941. **8.** Increase of 4. **9.** 56. **10.** 28.

50.

1. 493. **2.** 93. **3.** 87. **4.** 87; rem. 2. **5.** 89. **6.** 63; rem. 3.
7. 76. **8.** 39. **9.** 32. **10.** 134. **11.** 67. **12.** 69.
13. 67. **14.** 275. **15.** 149. **16.** 47. **17.** 65. **18.** 92.

51.

1. (*a*) 144; (*b*) 108; (*c*) 96. **2.** (*a*) 72; (*b*) 63; (*c*) 56.
3. (*a*) 88; (*b*) 72; (*c*) 66. **4.** (*a*) 198; (*b*) 110; (*c*) 90.
5. (*a*) 165 rem. 2; (*b*) 99 rem. 2; (*c*) 55 rem. 2.
6. (*a*) 246 rem. 3; (*b*) 197 rem. 2; (*c*) 164 rem. 3.
7. (*a*) 112 rem. 5; (*b*) 98 rem. 5; (*c*) 87 rem. 6.
8. (*a*) 112 rem. 3; (*b*) 96 rem. 3; (*c*) 61 rem. 4.

52.

1. 91.	**2.** 82; rem. 1.	**3.** 73; rem. 2.	**4.** 64; rem. 3.
5. 56; rem. 7.	**6.** 45; rem. 6.	**7.** 75; rem. 4.	**8.** 32; rem. 7.
9. 43; rem. 6.	**10.** 81; rem. 2.		

53.

1. 45; rem. 17.	**2.** 28; rem. 11.	**3.** 19; rem. 35.	**4.** 12; rem. 39.
5. 13; rem. 48.	**6.** 10; rem. 62.	**7.** 7; rem. 46.	**8.** 5; rem. 12.
9. 9; rem. 11.	**10.** 12; rem. 20.	**11.** 9; rem. 54.	**12.** 6; rem. 48.

54.

1. 2; rem. 12.	**2.** 2; rem. 14.	**3.** 2; rem. 8.	**4.** 2; rem. 12.
5. 2; rem. 10.	**6.** 2; rem. 16.	**7.** 2; rem. 16.	**8.** 2; rem. 10.
9. 2; rem. 16.	**10.** 2; rem. 10.	**11.** 28.	**12.** 22.
13. 35.	**14.** 15.	**15.** 19; rem. 16.	**16.** 18; rem. 35.
17. 34; rem. 3.	**18.** 6; rem. 42.	**19.** 23; rem. 16.	**20.** 15; rem. 9.
21. 26; rem. 21.	**22.** 18; rem. 7.	**23.** 16; rem. 14.	**24.** 11; rem. 44.
25. 13; rem. 7.	**26.** 6. rem. 24.		

55.

1. 57.	**2.** 279.	**3.** 56.	**4.** 195.	**5.** 390.
6. 686.	**7.** 78.	**8.** 558.	**9.** 87.	**10.** 615.
11. 82.	**12.** 410.	**13.** 574.	**14.** 902.	**15.** 23.
16. 381.	**17.** 276.	**18.** 45.	**19.** 162.	**20.** 469.

56.

1. 105.	**2.** 183.	**3.** 25.	**4.** 162.	**5.** 174.	**6.** 26.
7. 12.	**8.** 66.	**9.** 90.	**10.** 440.	**11.** 56.	**12.** 46.

57.

1. 67.	**2.** 114.	**3.** 42 himself; 42 each boy.	**4.** 649.	**5.** 199.	
6. 10.	**7.** 588.	**8.** 105.	**9.** 48.	**10.** 22; 16.	

58.

1. One thousand and one.
2. One thousand one hundred.
3. One thousand and ten.
4. One thousand eight hundred and five.
5. Two thousand and forty.
6. Three thousand four hundred and fourteen.
7. Four thousand five hundred and six.
8. Five thousand six hundred and fifty-one.
9. Six thousand one hundred and five.
10. Seven thousand and seventy.
11. Eight thousand seven hundred and eighty.
12. Nine thousand one hundred and ninety.

59.

1. 1011.	**2.** 2001.	**3.** 3054.	**4.** 4806.	**5.** 9005.
6. 8217.	**7.** 1912.	**8.** 7007.	**9.** 7017.	**10.** 1700.

60.

1. 8569.	2. 2901.	3. 6598.	4. 7391.	5. 4851.
6. 3633.	7. 7398.	8. 9036.	9. 4738.	10. 6894.

61.

1. 3963.	2. 3875.	3. 7726.	4. 7496.	5. 6138.	6. 6185.
7. 1845.	8. 2889.	9. 3056.	10. 2036.	11. 657.	12. 4374.

62.

1. 1384.	2. 2187.	3. 3448.	4. 3115.	5. 4476.
6. 6279.	7. 7400.	8. 4347.	9. 8756.	10. 7068.
11. 4830.	12. 9480.	13. 8064.	14. 8040.	15. 9576.
16. 9477.	17. 9198.	18. 9744.	19. 9324.	20. 9779.

63.

1. 7657.	2. 9792.	3. 9253.	4. 9384.	5. 8584.	6. 5673.
7. 9361.	8. 9417.	9. 9853.	10. 9782.	11. 9372.	12. 9047.

64.

1. 1369.	2. 1681.	3. 2809.	4. 4489.
5. 6241.	6. 2704.	7. 4624.	8. 5476.
9. 7225.	10. 2116.	11. 384.	12. 445.
13. 1060.	14. 1485.	15. 980.	16. 2016.

65.

1. 4671.	2. 3035.	3. 1409.	4. 1742.	5. 1372 ; rem. 5.
6. 589 ; rem. 6.		7. 498 ; rem. 7.		8. 805 ; rem. 1.
9. 737 ; rem. 2.		10. 511 ; rem. 6.		11. 631 ; rem. 9.
12. 186.		13. 81 ; rem. 32.		14. 42 ; rem. 68.
15. 408.		16. 252.		17. 487 ; rem. 17.
18. 81 ; rem. 29.		19. 56 ; rem. 6.		20. 45 ; rem. 77.

66.

1. 46.	2. 53.	3. 62.	4. 35.	5. 26.	6. 146.
7. 186.	8. 252.	9. 109.	10. 408.	11. 508.	12. 702.

67.

1. 103.	2. 421.	3. Smith by £273.	4. 1612.	5. 4524.
6. 124.	7. 300.	8. 432.	9. 364.	10. 24.

68.

1. 36.	2. 40.	3. 224.	4. 270.	5. 45.
6. 792.	7. 156.	8. 960.	9. 475.	10. 7.
11. 13.	12. 25.	13. 40.	14. 4.	15. 15.
16. 6.	17. 12.	18. 12.	19. 7.	20. 24.

69.

1. Ten thousand and twenty.
2. Ten thousand two hundred and two.
3. Eleven thousand and eleven.
4. Fifteen thousand and sixty.
5. Fifty thousand and sixteen.
6. Eighteen thousand and eight.
7. Sixty thousand six hundred and six.
8. Ninety thousand and ninety.
9. Seventy thousand nine hundred and seventeen.
10. Forty-one thousand and forty.
11. Eighty-one thousand and eighty-one.
12. Thirty-eight thousand one hundred.

70.

1. 10010.	2. 10101.	3. 11001.	4. 14016.	5. 40060.
6. 17007.	7. 50505.	8. 70070.	9. 60606.	10. 88808.

71.

1. 16464.	2. 24598.	3. 88173.	4. 74454.	5. 49889.
6. 45685.	7. 20904.	8. 78809.	9. 10338.	10. 4688.

72.

1. 46766 ; 57236.	2. 27540 ; 72675.	3. 71904 ; 61632.
4. 46315 ; 28260.	5. 23112 ; 59064.	6. 71100 ; 36972.
7. 25143 ; 72828.	8. 74632 ; 36334.	9. 55566 ; 14175.
10. 44321 ; 52808.	11. 52026 ; 32422.	12. 27370 ; 64124.

73.

1. 34824.	2. 2268.	3. 51480.	4. 37632.	5. 30304.
6. 11451.	7. 16312.	8. 15552.	9. 35532.	10. 92928.

74.

1. 2195 ; rem. 10.	2. 1257 ; rem. 3.	3. 991 ; rem. 4.
4. 4405 ; rem. 7.	5. 1613 ; rem. 21.	6. 1645 ; rem. 18.
7. 682 ; rem. 7.	8. 599 ; rem. 16.	9. 347 ; rem. 7.
10. 1027 ; rem. 26.	11. 526 ; rem. 26.	12. 516 ; rem. 9.

75.

1. 156.	2. 185.	3. 157.	4. 147.	5. 247.
6. 265.	7. 234.	8. 189.	9. 1039.	10. 1047.
11. 2489 ; rem. 3.	12. 3331 ; rem. 11.		13. 2473 ; rem. 20.	
14. 2162 ; rem. 6.	15. 259 ; rem. 32.		16. 246 ; rem. 23.	
17. 1189 ; rem. 22.	18. 1279 ; rem. 18.		19. 1333 ; rem. 3.	
20. 1191 ; rem. 32.	21. 1204 ; rem. 36.		22. 1017 ; rem. 29.	
23. 1128 ; rem. 75.	24. 1006 ; rem. 38.			

76.

1. 6974.	2. 3296.	3. 94809.	4. 517.	5. 515.
6. 502.	7. £15274.	8. 567 ; 9.	9. 429 ; 36.	10. 3468

77.

1. Smith, 162; Jones, 189.
2. A, 9; B, 15; C, 21.
3. A, 156; B, 234; C, 312.
4. A, B and C, 108 each; D, 36.
5. A, 120; B, 40; C, 20.
6. Man, 12*s.*; woman, 6*s.*; child, 3*s.*
7. Boy, 8; girl, 4.
8. A, 50; B, 35.
9. 500; 250; 250.
10. A and B, 40 each; the others, 25 each.
11. A, 36; B, 18; C, 9.
12. A, 6; B, 12; C, 18.

78.

1. One hundred thousand one hundred.
2. One hundred and ten thousand and ten.
3. One hundred and one thousand and one.
4. Two hundred and ten thousand and twelve.
5. Three hundred and one thousand and twenty-one.
6. Four hundred and twenty-one thousand and thirty.
7. Five hundred thousand and fifty.
8. Five hundred and five thousand and fifteen.
9. Six hundred and sixty thousand six hundred and six.
10. Seven hundred and seven thousand seven hundred and seven.
11. Eight hundred and eight thousand and eighty.
12. Nine hundred and nineteen thousand nine hundred and nine.

79.

1. 300030.	2. 470070.	3. 909009.	4. 880080.	5. 517017.
6. 690900.	7. 718078.	8. 200292.	9. 192019.	10. 919009.

80.

1. 234521.	2. 180696.	3. 544307.	4. 674885.	5. 419107.
6. 900467.	7. 594307.	8. 882779.	9. 714199.	10. 341430.

81.

1. 496938.	2. 59949.	3. 72684.	4. 508267.	5. 46275.
6. 355956.	7. 562757.	8. 301257.	9. 215364.	10. 652728.

82.

1. 54780; 109560.	2. 112950; 150600.	3. 299220; 349090.
4. 295840; 332820.	5. 245300; 490600.	6. 473700; 631600.
7. 789000; 946800.	8. 51920; 389400.	
9. 69800; 837600.	10. 107600; 941500.	

83.

1. 406463; 342525.	2. 381416; 295792.	3. 348170; 263480.
4. 428967; 320023.	5. 269404; 372580.	6. 155415; 191280.
7. 174972; 581157.	8. 154713; 333228.	
9. 225170; 190905.	10. 146964; 230376.	

84.

1. 294522.	2. 396460.	3. 187325.	4. 664632.
5. 402325.	6. 195564.	7. 198230.	8. 211640.
9. 458942.	10. 722680.	11. 523827.	12. 954810.

85.

1. 115668.	2. 101088.	3. 151944.	4. 681156.
5. 258112.	6. 320572.	7. 459672.	8. 545058.
9. 369792.	10. 342364.	11. 857696.	12. 557256.

86.

1. 347256.	2. 156400.	3. 613586.	4. 565504.	
5. 133225.	6. 334084.	7. 481636.	8. 755161.	
9. 636056.	10. 103823.	11. 205379.	12. 314432.	13. 405224.

87.

1. 228.	2. 653.	3. 364.	4. 553.	5. 588.	6. 284.
7. 395.	8. 214.	9. 97.	10. 131.	11. 213.	12. 153.

88.

1. 45867 ; rem. 5.	2. 28390 ; rem. 9.	3. 12790 ; rem. 28.
4. 13807 ; rem. 16.	5. 7578 ; rem. 15.	6. 6006 ; rem. 79.
7. 1913 ; rem. 82.	8. 1043 ; rem. 146.	9. 1234 ; rem. 218.
10. 977 ; rem. 173.	11. 514 ; rem. 263.	12. 572 ; rem. 192.

89.

1. 635.	2. 789.	3. 592.	4. 327.	5. 284.	6. 873.
7. 684 ; rem. 287.		8. 794 ; rem. 235.		9. 583 ; rem. 179.	
10. 694 ; rem. 123.		11. 734 ; rem. 97.		12. 756 ; rem. 94.	

90.

1. 358 ; rem. 39.	2. 375 ; rem. 66.	3. 735 ; rem. 101.	4. 168 ; rem. 47.
5. 249 ; rem. 180.	6. 298 ; rem. 63.	7. 152 ; rem. 85.	
8. 741 ; rem. 150.	9. 864 ; rem. 72.	10. 437 ; rem. 432.	

91.

1. 528 ; rem. 3.	2. 758 ; rem. 588.	3. 458 ; rem. 47.
4. 562 ; rem. 290.	5. 561 ; rem. 163.	6. 391 ; rem. 452.
7. 317 ; rem. 447.	8. 1032 ; rem. 6.	9. 2412 ; rem. 171.
10. 1144 ; rem. 513.	11. 2392 ; rem. 134.	12. 1119 ; rem. 38.

92.

1. 86427.	2. 71050.	3. 65930.	4. 79083.	5. 58344.
6. 99330.	7. 986013.	8. 325022.	9. 650076.	10. 133927.
11. 217672.	12. 180348.	13. 71198.	14. 258112.	15. 260952.

93.

1. 14525.	2. 46725.	3. 118075.	4. 13.
5. 343.	6. 3875.	7. 120375.	8. 484250.
9. 532125.	10. 87.	11. 369.	12. 735.

94.

1. 89427.	2. 18094.	3. 403298.	4. 612069.
5. 56025.	6. 23016.	7. 832302.	8. 7.
9. $278489 \div 11$.	10. $606255 \div 80$.		11. $417346 \div 400$.
12. $983495 \div 1200$.	13. $42813 \div 18$.		14. $419421 \div 99$.
15. $423672 \div 21$.	16. $587595 \div 16$.		17. 205×43.
18. 307×98.	19. 917×406.		20. 704×176.
21. $1238 - 456$.	22. $8047 - 7428$.		23. $84569 - 8947$.
24. $12921 - 5144$.	25. 2849×3.		26. 3678×7.
27. 2986×9.	28. 3785×11.		29. $7842 \div 8$.
30. $4956 \div 9$.	31. $67896 \div 5$.		32. $51789 \div 7$.

95.

1. $30 = 2.3.5$.	2. $42 = 2.3.7$.	3. $66 = 2.3.11$.	4. $70 = 2.5.7$.
5. $27 = 3.3.3$.	6. $28 = 2.2.7$.		7. $45 = 3.3.5$.
8. $63 = 3.3.7$.	9. $50 = 2.5.5$.		10. $110 = 2.5.11$.

96.

1. 54 contains 2, 3, 6, 9.		2. 84 contains 2, 3, 4, 6, 12.	
3. 60 ,,	2, 3, 4, 5, 6, 10, 12.	4. 72 ,,	2, 3, 4, 6, 8, 9, 12.
5. 156 ,,	2, 3, 4, 6, 12.	6. 195 ,,	3, 5.
7. 520 ,,	2, 4, 5, 8, 10.	8. 480 ,,	2, 3, 4, 5, 6, 8, 10, 12.
9. 696 ,,	2, 3, 4, 6, 8, 12.	10. 948 ,,	2, 3, 4, 6, 12.

97.

1. $2^2.3.5$.	2. $2.3.13$.	3. $2.3.17$.	4. $2.3.5.7$.
5. $2^2.3^2.7$.	6. $3^2.5.7$.	7. $3.5^2.7$.	8. $2^2.3^2.17$.
9. $3.11.19$.	10. $5.11.13$.	11. 11^3.	12. $2.5.11.13$.
13. $2^4.7.13$.	14. $3^2.5.7.11$.	15. $3.5^2.7^2$.	16. $2^3.3^4.7$.
17. $2^2.3.5.7.11$.	18. $3^2.7^2.11$.	19. $2.3.5^3.7$.	20. $3^2.7.11^2$.

98.

1. 2.	2. 3.	3. 4.	4. 5.	5. 6.	6. 7.
7. 8.	8. 9.	9. 10.	10. 11.	11. 12.	12. 13.

99.

1. 14.	2. 16.	3. 18.	4. 15.	5. 25.
6. 24.	7. 28.	8. 21.	9. 27.	10. 30.
11. 32.	12. 35.	13. 44.	14. 75.	15. 84.
16. 105.	17. 132.	18. 231.	19. 252.	20. 315.
21. 7.	22. 21.	23. 6.	24. 33.	25. 30.
26. 7.	27. 13.	28. 3.	29. 21.	30. 21.

100.

1. 5.	**2.** 6.	**3.** 8.	**4.** 7.	**5.** 9.
6. 14.	**7.** 16.	**8.** 15.	**9.** 21.	**10.** 22.
11. 24.	**12.** 33.	**13.** 45.	**14.** 55.	**15.** 60.
16. 11.	**17.** 3.	**18.** 12.	**19.** 7.	**20.** 11.

101.

1. 2.	**2.** 2.	**3.** 3.	**4.** 4.	**5.** 3.	**6.** 2.	**7.** 2.	**8.** 5.
9. 2.	**10.** 6.	**11.** 2.	**12.** 5.	**13.** 5.	**14.** 9.	**15.** 7.	**16.** 2.
17. 7.	**18.** 9.	**19.** 3.	**20.** 12.	**21.** 4.	**22.** 5.	**23.** 18.	**24.** 14.

102.

1. 4.	**2.** 16.	**3.** 36.	**4.** 15.	**5.** 27.
6. 28.	**7.** 16.	**8.** 63.	**9.** 132.	**10.** 14.
11. 36.	**12.** 11.	**13.** 18.	**14.** 24.	**15.** 34.

103.

1. 4.	**2.** 8.	**3.** 6.	**4.** 6.	**5.** 12.	**6.** 30.
7. 60.	**8.** 105.	**9.** 30.	**10.** 70.	**11.** 30.	**12.** 24.
13. 60.	**14.** 40.	**15.** 60.	**16.** 24.	**17.** 70.	**18.** 90.

104.

1. 120.	**2.** 180.	**3.** 240.	**4.** 360.	**5.** 420.	**6.** 180.
7. 840.	**8.** 480.	**9.** 540.	**10.** 540.	**11.** 288.	**12.** 420.
13. 360.	**14.** 288.	**15.** 240.	**16.** 240.	**17.** 360.	**18.** 432.

105.

1. 2 in.	**2.** 7 in.	**3.** 8.	**4.** 21.	**5.** 6.
6. 24.	**7.** 120.	**8.** 60 in.	**9.** 840.	**10.** 720.

106.

1. 2*s.* 3*d.*	**2.** 2*s.* 6*d.*	**3.** 2*s.* 6*d.*	**4.** 2*s.* 10*d.*	**5.** 3*s.* 2*d.*
6. 7*s.* 10*d.*	**7.** 8*s.* 5*d.*	**8.** 11*s.* 7*d.*	**9.** 14*s.* 9*d.*	**10.** 13*s.* 3*d.*

107.

1. 1*s.* 5*d.*	**2.** 3*s.* 6*d.*	**3.** 3*s.* 3*d.*	**4.** 6*s.* 4*d.*	**5.** 1*s.* 8*d.*
6. 7*s.* 4*d.*	**7.** 2*s.* 9*d.*	**8.** 7*s.* 4*d.*	**9.** 10*s.* 6*d.*	**10.** 3*s.* 8*d.*

108.

1. 10*d.*	**2.** 8*d.*	**3.** 2*s.* 10*d.*	**4.** 4*s.* 4*d.*	**5.** 1*s.* 5*d.*
6. 3*s.* 5*d.*	**7.** 1*s.* 7*d.*	**8.** 8*s.* 9*d.*	**9.** 6*s.* 7*d.*	**10.** 14*s.* 2*d.*

109.

1. 9*s.* 2*d.* **2.** 15*s.* 6*d.* **3.** 14*s.* 8*d.* **4.** 14*s.* 3*d.* **5.** 15*s.* 10*d.* **6.** 1*s.* 6*d.*
7. 19*s.* 1*d.* **8.** 10*d.* **9.** 11*d.* **10.** 1*s.* 3*d.* **11.** 11*s.* 9*d.* **12.** 17*s.* 6*d.*

110.

1. 1*s.* 10*d.*	**2.** 2*s.* 7*d.*	**3.** 3*s.* 11*d.*	**4.** 3*s.* 9*d.*	**5.** 1*s.* 10*d.*
6. 2*d.*	**7.** 6*s.* 5*d.*	**8.** 1*s.* 8*d.*	**9.** 4*s.* 5*d.*	**10.** 3*s.* 10*d.*
11. 2*s.* 11*d.*	**12.** 4*s.* 2*d.*	**13.** 2*s.* 6*d.*	**14.** 7*s.* 11*d.*	**15.** 14*s.*

111.

1. 32.	**2.** 36.	**3.** 42.	**4.** 65.	**5.** 57.	**6.** 153.
7. 190.	**8.** 118.	**9.** 94.	**10.** 175.	**11.** 139.	**12.** 199.
13. 254.	**14.** 358.	**15.** 449.	**16.** 583.	**17.** 784.	**18.** 879.
19. 851.	**20.** 949.	**21.** 522.	**22.** 596.	**23.** 713.	**24.** 999.
25. 384.	**26.** 432.	**27.** 288.	**28.** 504.	**29.** 768.	**30.** 864.
31. 708.	**32.** 990.	**33.** 552.	**34.** 641.	**35.** 751.	**36.** 858.

112.

1. 10*s.* 6*d.*	**2.** 12*s.* 4*d.*	**3.** 16*s.* 2*d.*	**4.** 19*s.* 8*d.*	**5.** £6. 6*s.*
6. £8. 13*s.*	**7.** £7. 8*s.*	**8.** £4. 3*s.* 3*d.*	**9.** £1. 11*s.* 7*d.*	
10. £3. 6*s.* 6*d.*	**11.** £2. 16*s.* 1*d.*	**12.** 17*s.* 1¾*d.*	**13.** 16*s.* 6*d.*	
14. 9*s.* 9¼*d.*	**15.** 16*s.* 6½*d.*	**16.** 11*s.* 2½*d.*	**17.** 18*s.* 11½*d.*	

113.

1. 230.	**2.** 692.	**3.** 40.	**4.** 55.	**5.** 69.
6. 143.	**7.** £7. 16*s.* 6*d.*	**8.** £1. 7*s.* 3*d.*	**9.** £9. 17*s.* 6*d.*	**10.** £9. 12*s.*

114.

1. £3. 10*s.* 1¾*d.*	**2.** £4. 3*s.* 6*d.*	**3.** £3. 8*s.* 2¾*d.*	**4.** £3. 11*s.* 4¼*d.*
5. £2. 18*s.* 6*d.*	**6.** £9. 3*s.* 7¾*d.*		**7.** £9. 10*s.* 2½*d.*
8. £9. 10*s.* 8¼*d.*	**9.** £5. 19*s.* 1½*d.*		**10.** £8. 9*s.* 1¼*d.*
11. £9. 9*s.* 5½*d.*	**12.** £9.		**13.** £7. 19*s.* 8¼*d.*

115.

1. £1. 4*s.* 11*d.*	**2.** £1. 6*s.* 11*d.*	**3.** £1. 3*s.* 1*d.*	**4.** £1. 5*s.* 11*d.*
5. £1. 5*s.* 7*d.*	**6.** £1. 4*s.* 10*d.*	**7.** £1. 10*s.*	**8.** £1. 15*s.* 1*d.*
9. 8*s.* 11*d.*	**10.** 8*s.* 5*d.*	**11.** 9*s.* 11*d.*	**12.** 9*s.* 7*d.*
13. 14*s.* 8*d.*	**14.** 14*s.* 10*d.*	**15.** 16*s.* 11*d.*	**16.** 17*s.* 7*d.*
17. £5. 0*s.* 10*d.*	**18.** 14*s.* 8*d.*	**19.** 12*s.* 9*d.*	**20.** 16*s.* 3*d.*
21. 13*s.* 7*d.*	**22.** 15*s.* 1*d.*	**23.** 14*s.* 4*d.*	**24.** 14*s.* 9*d.*
25. 14*s.* 1*d.*	**26.** £5. 15*s.* 6*d.*		

116.

1. 18*s.* 11*d.*	**2.** £1. 0*s.* 8*d.*	**3.** £1. 10*s.* 11*d.*	**4.** 19*s.* 5½*d.*
5. 15*s.* 11¾*d.*	**6.** £1. 14*s.* 9¼*d.*	**7.** £1. 19*s.* 9½*d.*	**8.** £4. 14*s.* 11½*d.*
9. £6. 19*s.* 11¾*d.*	**10.** 17*s.* 7¼*d.*	**11.** £8. 9*s.* 11½*d.*	**12.** £3. 8*s.* 10½*d.*

117.

1. 17*s.* 4*d.*	**2.** £1. 14*s.* 7*d.*	**3.** £1. 14*s.* 6*d.*	**4.** £1. 11*s.* 1*d.*	**5.** 1*s.* 1*d.*
6. 14*s.* 7*d.*	**7.** 15*s.* 11*d.*	**8.** £1. 9*s.* 9*d.*	**9.** 18*s.* 7*d.*	**10.** 4*s.* 1*d.*

118.

1. £5. 5*s*. 2. £1. 0*s*. 9*d*. 3. £5. 2*s*. 9*d*. 4. £3. 17*s*. 11*d*.
5. £2. 5*s*. 5*d*. 6. £7. 5*s*. 6½*d*. 7. 10*s*. too much. 8. £5.
9. £6. 9*s*. 8*d*. 10. £4. 6*s*. 3*d*. 11. £4. 14*s*. 2*d*. 12. £2. 7*s*. 9*d*.

119.

1. £1. 1*s*. 6*d*.; £1. 12*s*. 3*d*.
2. £2. 3*s*. 6¾*d*.; £2. 18*s*. 1*d*.
3. £2. 13*s*. 7*d*.; £3. 6*s*. 11¾*d*.
4. £3. 4*s*. 4½*d*.; £3. 17*s*. 3*d*.
5. £5. 10*s*. 10½*d*.; £6. 9*s*. 4¼*d*.
6. £5. 7*s*. 7½*d*.; £6. 3*s*. 0*d*.
7. £6. 14*s*. 2*d*.; £7. 10*s*. 11¼*d*.
8. £6. 11*s*. 0*d*.; £9. 16*s*. 6*d*.
9. £6. 14*s*. 3¾*d*.; £8. 19*s*. 1*d*.
10. £7. 14*s*. 2*d*.; £9. 12*s*. 8½*d*.
11. £7. 8*s*. 7¾*d*.; £8. 18*s*. 4½*d*.
12. £7. 15*s*. 3*d*.; £9. 1*s*. 1½*d*.
13. £7. 6*s*. 10¼*d*.; £8. 7*s*. 10*d*.
14. £8. 14*s*. 10*d*.; £9. 16*s*. 8¼*d*.

120.

1. £2. 2*s*. 10½*d*. 2. £4. 19*s*. 3. £7. 2*s*. 6*d*. 4. £3. 18*s*. 4*d*.
5. £5. 15*s*. 6*d*. 6. £8. 12*s*. 7. £7. 12*s*. 8. £6. 6*s*.
9. £5. 4*s*. 10. £5. 10*s*. 3*d*.

121.

1. £1. 6*s*. 10*d*. 2. £5. 18*s*. 9*d*. 3. £5. 7*s*. 7½*d*. 4. £3. 19*s*. 4*d*.
5. £7. 9*s*. 7½*d*. 6. £7. 3*s*. 6*d*. 7. £7. 17*s*. 6*d*. 8. £9. 13*s*. 11½*d*.
9. £9. 2*s*. 3½*d*. 10. £5. 18*s*. 9*d*. 11. £9. 18*s*. 7½*d*. 12. £8. 18*s*. 4*d*.

122.

1. 4*s*. 2. 5*s*. 3*d*. 3. 6*s*. 6*d*. 4. 8*s*. 9*d*. 5. 10*s*. 6*d*.
6. 14*s*. 7. £1. 10*s*. 8. £2. 5*s*. 9. £2. 18*s*. 10. £3. 3*s*.
11. £1. 16*s*. 12. £4. 4*s*. 13. £6. 14. £1. 10*s*. 15. £2. 11*s*.
16. £3. 9*s*. 17. £5. 10*s*. 18. £4. 5*s*. 19. £7. 15*s*. 20. £3. 2*s*. 6*d*.
21. £8. 3*s*. 4*d*. 22. 3*s*. 23. 6*s*. 9*d*. 24. 8*s*. 9*d*. 25. 15*s*.
26. £1. 17*s*. 6*d*. 27. £1. 11*s*. 6*d*. 28. £7. 4*s*. 29. £5. 5*s*. 30. £9. 18*s*.
31. £7. 10*s*. 32. £6. 15*s*. 33. £6.

123.

1. 17*s*. 5½*d*.; £4. 2*s*. 6½*d*.
2. £1. 15*s*. 4*d*.; £3. 4*s*. 8*d*.
3. £2. 0*s*. 1*d*.; £2. 19*s*. 11*d*.
4. £2. 8*s*. 10*d*.; £2. 11*s*. 2*d*.
5. £3. 6*s*. 5½*d*.; £1. 13*s*. 6½*d*.
6. £6. 19*s*. 11*d*.; £3. 0*s*. 1*d*.
7. £9. 8*s*. 3*d*.; 11*s*. 9*d*.
8. £9. 15*s*. 6*d*.; 4*s*. 6*d*.
9. £9. 12*s*. 9½*d*.; 7*s*. 2½*d*.
10. £9. 11*s*. 1¾*d*.; 8*s*. 10¼*d*.

124.

1. 1*s*. 6¾*d*.; 2*s*. 10¼*d*.; 1*s*. 9¾*d*.
2. 1*s*. 6¾*d*.; 2*s*. 7¼*d*.; 1*s*. 9¾*d*
3. 1*s*. 11½*d*.; 2*s*. 8¾*d*.; 3*s*. 7¼*d*.
4. 11¾*d*.; 1*s*. 7½*d*.; 3*s*. 1¼*d*.
5. 10¼*d*.; 2*s*. 0¾*d*.; 1*s*. 9½*d*.
6. 1*s*. 8¾*d*.; 1*s*. 10½*d*.; 1*s*. 4¼*d*.
7. 11¾*d*.; 1*s*. 5¼*d*.; 1*s*. 10½*d*.
8. 1*s*. 8½*d*.; 1*s*. 4¼*d*.; 1*s*. 9¾*d*.
9. 1*s*. 6½*d*.; 1*s*. 8¼*d*.; 1*s*. 3¾*d*.
10. 1*s*. 7½*d*.; 1*s*. 6¾*d*.; 11¾*d*

125.

1. £1. 9*s.* 3¼*d.*; £1. 5*s.* 6¼*d.*; £2. 15*s.* 4¼*d.*
2. £1. 6*s.* 2¼*d.*; £2. 3*s.* 4¼*d.*; £2. 10*s.* 2¼*d.*
3. £1. 4*s.* 2*d.*; 8*s.* 1½*d.*; 15*s.* 2¼*d.*
4. £1. 3*s.* 0½*d.*; 9*s.* 0¼*d.*; £1. 12*s.* 0¾*d.*
5. 6*s.* 1¼*d.*; £1. 5*s.* 1½*d.*; 14*s.* 2¼*d.*
6. 3*s.* 0¼*d.*; £1. 1*s.* 7½*d.*; £1. 6*s.* 0½*d.*
7. £1. 4*s.* 2*d.*; 8*s.* 1½*d.*; 15*s.* 2¼*d.*
8. £1. 2*s.* 2*d.*; £1. 1*s.* 10½*d.*; 3*s.* 7¼*d.*
9. 3*s.* 6½*d.*; 19*s.* 3¼*d.*; 17*s.* 8½*d.*
10. 13*s.* 10¼*d.*; 14*s.* 4¾*d.*; 16*s.* 6½*d.*

126.

1. 1*s.* 10*d.* 2. 8½*d.* 3. 6*s.* 10½*d.* 4. 7¾*d.*
5. 1*s.* 3*d.* 6. 1*s.* 8*d.* 7. 17*s.* 2½*d.* 8. 14*s.* 6*d.*
9. £2. 1*s.* 0½*d.* 10. £2. 19*s.* 6¾*d.* 11. £1. 17*s.* 3½*d.* 12. £5. 0*s.* 6*d.*

127.

1. 1*s.* 11*d.* 2. 5*s.* 7*d.* 3. 9*s.* 4*d.* 4. 3*s.* 9*d.* 5. 5*s.* 5*d.*
6. 2*s.* 10*d.* 7. 1*s.* 11¾*d.* 8. 6*s.* 5¼*d.* 9. 8*s.* 10¼*d.* 10. 3*s.* 6¾*d.*
11. 1*s.* 0¼*d.* 12. 8¾*d.* 13. 1*s.* 5*d.*; rem. 48*d.* 14. 2*s.* 6*d.*; rem. 42*d.*
15. 1*s.* 4*d.*; rem. 5*d.* 16. 1*s.* 4*d.*; rem. 34*d.*
17. 1*s.* 1¾*d.*; rem. 20*f.* 18. 2*s.* 2½*d.*; rem. 12*f.*
19. 1*s.* 11¾*d.*; rem. 35*f.* 20. 2*s.* 6¼*d.*; rem. 39*f.*

128.

1. 3*s.* 11*d.* 2. 4*s.* 3*d.* 3. 5*s.* 8*d.* 4. 2*s.* 11*d.*
5. 10*d.* 6. 7*s.* 5*d.* 7. 7*s.* 6*d.* 8. 1*s.* 4*d.*

129.

1. 14*s.* 4*d.* 2. 4*s.* 10*d.* 3. 2*s.* 6*d.* 4. £1. 7*s.*
5. 1*s.* 6*d.* 6. 6*s.* 3*d.* 7. £1. 13*s.* 8. £2. 1*s.* 8*d.*
9. £9. 5*s.* 10. £5. 4*s.* 5*d.* 11. £6. 6*s.* 12. 6*s.* 4*d.*

130.

1. £13. 15*s.* 1*d.* 2. £23. 6*s.* 3*d.* 3. £61. 16*s.* 5*d.* 4. £65. 13*s.* 1*d.*
5. £76. 6*s.* 1*d.* 6. £88. 5*s.* 11*d.* 7. £187. 15*s.* 1*d.*
8. £216. 19*s.* 11*d.* 9. £130. 15*s.* 4*d.* 10. £219. 4*s.*

131.

1. £12. 12*s.* 6*d.* 2. £16. 19*s.* 8*d.* 3. £2. 9*s.* 10*d.*
4. £6. 16*s.* 10*d.* 5. £16. 17*s.* 6*d.* 6. £10. 16*s.* 8*d.*
7. £21. 10*s.* 7*d.* 8. £49. 13*s.* 6*d.* 9. 18*s.* 11*d.*
10. £14. 11. £14. 1*s.* 3*d.* 12. £11. 3*s.* 11*d.*
13. £2. 19*s.* 9*d.* too much. 14. 10*s.* too much. 15. £47. 14*s.*

132.

1. £21. 9*s.*; £96. 10*s.* 6*d.*
2. £38. 1*s.*; £101. 9*s.* 4*d.*
3. £63. 1*s.* 4*d.*; £110. 7*s.* 4*d.*
4. £83. 3*s.* 9*d.*; £183. 0*s.* 3*d.*
5. £83. 3*s.* 6*d.*; £138. 12*s.* 6*d.*
6. £129. 13*s.* 4¼*d.*; £222. 5*s.* 9*d.*
7. £57. 19*s.* 8¼*d.*; £104. 7*s.* 5¼*d.*
8. £59. 13*s.* 5*d.*; £164. 1*s.* 10¾*d.*
9. £53. 9*s.* 9¾*d.*; £178. 6*s.* 0½*d.*
10. £119. 3*s.* 3*d.*; £178. 14*s.* 10½*d.*

133.

1. £80. 18*s.* 9*d.*; £233. 2*s.*
2. £66. 7*s.* 6*d.*; £123. 18*s.*
3. £233; £356. 15*s.* 7½*d.*
4. £153. 17*s.* 9¾*d.*; £156. 6*s.* 8*d.*
5. £206. 7*s.* 8*d.*; £221. 2*s.* 6*d.*
6. £453. 11*s.* 6¾*d.*; £471. 5*s.*
7. £123. 8*s.* 1*d.*; £713. 19*s.* 7½*d.*
8. £167. 3*s.* 4½*d.*; £179. 2*s.* 2¼*d.*
9. £105. 19*s.*; £231. 15*s.* 3¾*d.*
10. £176. 7*s.* 7½*d.*; £754. 10*s.* 4¾*d.*

134.

1. £87. 5*s.* 3*d.*; £154. 7*s.* 9*d.*
2. £132. 0*s.* 8*d.*; £225. 4*s.* 8*d.*
3. £168. 16*s.* 0¾*d.*; £257. 12*s.* 11¼*d.*
4. £358. 13*s.* 4½*d.*; £630. 1*s.* 10½*d.*
5. £435. 14*s.* 2½*d.*; £924. 11*s.* 1½*d.*
6. £540. 2*s.* 10¼*d.*; £1193. 6*s.* 9¼*d.*
7. £742. 5*s.* 1¾*d.*; £1468. 14*s.* 5¼*d.*
8. £932. 8*s.* 6*d.*; £1595. 17*s.* 7½*d.*
9. £1270. 19*s.* 6¼*d.*; £1422. 14*s.* 8¼*d.*
10. £1424. 12*s.* 8¼*d.*; £1507. 4*s.* 5¼*d.*

135.

1. 6*s.*
2. £29. 7*s.* 2*d.*
3. £2. 6*s.*
4. £14. 2*s.*
5. £8. 16*s.*
6. £39. 8*s.* 6*d.*
7. £67. 11*s.* 8*d.*
8. £152. 7*s.* 6*d.*

136.

1. £159. 5*s.*; £248. 13*s.* 6*d.*
2. £276. 16*s.* 3*d.*; £886. 17*s.* 6*d.*
3. £489. 9*s.* 9*d.*; £1034. 15*s.* 10½*d.*
4. £921. 6*s.* 11¼*d.*; £2051. 15*s.* 10½*d.*
5. £502. 2*s.* 4½*d.*; £1000. 11*s.* 11¾*d.*
6. £1414. 16*s.* 4½*d.*; £2760. 1*s.* 1½*d.*
7. £733. 11*s.* 0¾*d.*; £2456. 11*s.*
8. £1685; £4765. 7*s.* 9¾*d.*
9. £1125. 1*s.* 10½*d.*; £4138. 19*s.* 7½*d.*
10. £612. 2*s.* 9*d.*; £2236. 13*s.* 1½*d.*
11. £6. 5*s.*
12. £13. 10*s.*
13. £23. 15*s.*
14. £27.
15. £15. 4*s.* 2*d.*
16. £24. 6*s.* 8*d.*
17. £45. 12*s.* 6*d.*
18. £109. 10*s.*
19. £141. 8*s.* 9*d.*

137.

1. £36. 3*s.* 3*d.*
2. £31. 8*s.* 4*d.*
3. £189. 11*s.* 8*d.*
4. £185. 5*s.*
5. £46. 5*s.*
6. £261. 12*s.* 5*d.*
7. £174. 15*s.* 2*d.*
8. £2. 18*s.* 6*d.*

138.

1. £13. 17*s.* 4½*d.*
2. £14. 10*s.* 5¼*d.*
3. £24. 12*s.* 6¾*d.*
4. £15. 12*s.* 7¼*d.*
5. £2. 14*s.* 9*d.*
6. £3. 15*s.* 10*d.*
7. £3. 12*s.* 6½*d.*
8. £4. 11*s.* 7¼*d.*
9. £4. 13*s.* 8¾*d.*
10. £5. 10*s.* 4½*d.*
11. £5. 14*s.* 5¾*d.*
12. £6. 17*s.* 9½*d.*

139.

1. £21. 0*s.* 5¼*d.* **2.** £32. 6*s.* 0¾*d.* **3.** £5. 10*s.* 9*d.*
4. £6. 13*s.* 11¼*d.* **5.** £7. 14*s.* 6½*d.* **6.** £8. 15*s.* 10¾*d.*
7. £9. 10*s.* 6½*d.* **8.** £10. 12*s.* 5¾*d.* **9.** £11. 11*s.* 7¼*d.*
10. £7. 19*s.* 4¼*d.* **11.** £4. 16*s.* 5¼*d.* **12.** £5. 17*s.* 10¾*d.*

140.

1. £15. 12*s.* 11½*d.* **2.** £7. 8*s.* 9½*d.* **3.** £5. 11*s.* 5¾*d.* **4.** £8. 11*s.* 9½*d.*
5. £2. 15*s.* 4½*d.* **6.** £5. 5*s.* 6¾*d.* **7.** £17. 12*s.* 10½*d.* **8.** £18. 14*s.* 5¼*d.*
9. £89. 16*s.* 6*d.* **10.** £41. 5*s.* 7¼*d.* **11.** £29. 2*s.* 2¼*d.*
12. £64. 19*s.* 11½*d.* **13.** £39. 6*s.* 7½*d.* **14.** £34. 17*s.* 5¼*d.*
15. £17. 11*s.* 10¾*d.* **16.** £3. 2*s.* 9¾*d.* **17.** £83. 15*s.* 3¼*d.*
18. £85. 16*s.* 4½*d.* **19.** £49. 16*s.* 8½*d.* **20.** £59. 5*s.* 7¼*d.*

141.

1. 17*s.* 6*d.* **2.** £1. 3*s.* 9*d.* **3.** 18*s.* 11*d.* **4.** 2*s.* 6*d.*
5. 4*s.* 11*d.* **6.** 670. **7.** 398. **8.** £27. 10*s.* 8*d.*
9. £337. 10*s.* **10.** £27. 1*s.* **11.** £4. 8*s.* 10½*d.* **12.** On 1*s.* day; £25.
13. 10*d.* **14.** £11. 6*s.* **15.** 5*s.* 3*d.*

142.

1. 11. **2.** 3. **3.** 12. **4.** 117. **5.** 14.
6. 51. **7.** 48. **8.** 36. **9.** 13. **10.** 19.

143.

1. 365. **2.** 245. **3.** 76. **4.** 13.
5. 5000. **6.** 52. **7.** 505; 2½*d.* **8.** 14; 4*s*
9. 7 more bats than bails. **10.** 11.

144.

1. 12. **2.** 23. **3.** 24. **4.** 13. **5.** 87.
6. 140. **7.** 84. **8.** 28. **9.** 14*s.* 0¾*d.* **10.** 5*s.* 5*d.*

145.

1. 3*s.* 4*d.*; 1*s.* 8*d.* **2.** 3*s.*; 9*d.* **3.** A £2. 13*s.* 4*d.*; B £3. 6*s.* 8*d.*; C £4.
4. A £1. 19*s.*; B 19*s.* 6*d.*; C 19*s.* 6*d.*
5. A £1. 5*s.* 9*d.*; B 8*s.* 7*d.*; C £2. 11*s.* 6*d.*
6. A £55. 12*s.* 8*d.*; B £27. 16*s.* 4*d.*; C £13. 18*s.* 2*d.*
7. 16*s.* 8*d.* each boy; 8*s.* 4*d.* each girl.
8. £1. 10*s.* each man; 10*s.* each woman. **9.** 2*s.* 9*d.*; 2*s.* 3*d.*
10. 2*s.* 8*d.*; 1*s.* 10*d.* **11.** 6*s.* 10*d.*; 9*s.* 2*d.* **12.** £1. 9*s.* 3*d.*; £1. 13*s.* 9*d.*
13. A £6; B £6. 17*s.*; C £8. 3*s.* **14.** A 1*s.*; B 2*s.* **15.** C 3*s.*; D 1*s.* 6*d.*
16. A 10*s.*; B £1; C £1. 5*s.* **17.** 20. **18.** 163.
19. 8 shillings; 4 threepenny pieces. **20.** 107 sixpences; 214 shillings.
21. 7 sovereigns; 21 shillings; 28 pennies.
22. 22 pennies; 44 shillings; 132 half-crowns.
23. 42 half-crowns; 126 shillings; 84 florins. **24.** 12.

146.

1. 744. **2.** 41760. **3.** 104. **4.** 13. **5.** 986. **6.** 2400.
7. 120. **8.** 2025. **9.** 30. **10.** 16 min. 40 sec. past 12.
11. 62. **12.** 55. **13.** 9 days 12 hrs. 4 min.
14. 1 hr. 39 min. 15 sec. **15.** 1 day 21 hrs. 55 min.
16. 80 hrs. 2 min. 30 sec. **17.** 2 days 23 hrs. 27 min. $8\frac{4}{7}$ sec.
18. 1 day 50 min. **19.** 2 hrs. 50 min. **20.** 64 days 7 hrs. 20 min.
21. 106 hrs. 37 min. 40 sec. **22.** 473 hrs. 25 min.
23. 11 hrs. 48 min. 20 sec. **24.** 1152.

147.

1. 3360. **2.** 560. **3.** 5376. **4.** 3 cwt. 2 qrs. **5.** 2 tons.
6. 11 cwt. 1 qr. 9 lb. **7.** 8 tons 15 cwt. 1 qr. 14 lb. **8.** 13 lb. 14 oz.
9. 25 lb. 12 oz. **10.** 89 cwt. 2 qrs. 18 lb. **11.** 6 cwt. 1 qr. 8 lb. 9 oz.
12. 39 tons 19 cwt. 1 qr. **13.** 2 qrs. 27 lb. 2 oz. **14.** 1 ton 5 cwt. 1 qr.
15. 10 cwt. 3 qrs. 26 lb. **16.** 80 tons 19 cwt. 4 lb. **17.** 3 tons 6 cwt. 2 qrs.
18. 3 qrs. 26 lb. **19.** 1 ton 9 cwt. 3 qrs. 5 lb. **20.** 12 cwt. 5 lb. 1 oz.

148.

1. 14 cwt. 1 qr. 27 lb. **2.** 9 tons 1 cwt. 2 qrs. 8 lb.
3. 2 tons 4 cwt. 3 qrs. 4 lb. **4.** 11 tons 2 cwt. 7 lb.
5. 2 tons 9 cwt. 9 lb. 8 oz. **6.** 7 lb. 8 oz. **7.** 72 lb.
8. 3 qrs. 9 lb. **9.** 3 tons 7 cwt. 3 qrs. 2 lb. **10.** 13.
11. 256. **12.** 63. **13.** 217. **14.** 8 st. $5\frac{1}{4}$ lb. **15.** 19 st. 10 lb.
16. 9 st. 13 lb. **17.** 10 lb. $1\frac{5}{8}$ oz. **18.** 11 st. 1 lb.
19. 7 oz. each man; $3\frac{1}{2}$ oz. each woman.
20. 10 lb. each man; 5 lb. each woman.

149.

1. 72. **2.** 288. **3.** 45 gall. 2 qts. 1 pt.
4. 32. **5.** 127 gall. 3 qts. 1 pt. **6.** 51 gall. 1 qt. 1 pt.
7. 3 gall. 2 qts. $1\frac{1}{2}$ pt. **8.** 23 gall. 2 qts. $0\frac{1}{4}$ pt. **9.** 89 gall. 1 qt. 1 pt.
10. 39 gall. **11.** 2 gall. 3 qts. **12.** 1 gall. 3 qts. 1 pt.

150.

1. 2 qts. 1 pt. **2.** 12. **3.** 10 gall. 3 qts. 1 pt.
4. 6 gall. 3 qts. 1 pt. **5.** 26 gall. 2 qts. 1 pt. **6.** 198 gall. 2 qts. 1 pt.
7. 10. **8.** 30. **9.** 34 gall. $1\frac{1}{2}$ pt.
10. 4 gall. **11.** 100 gall. 2 qts. 1 pt. **12.** 24.

151.

1. 1638. **2.** 2297. **3.** 2064.
4. 10983 yds. 2 ft. **5.** 5 ft. 4 in. **6.** 31 yds. 2 ft. 2 in.
7. 23 yds. 2 ft. 8 in. **8.** 3 ft. 10 in. **9.** 86 yds. 2 ft.
10. 17 yds. 1 ft. 8 in. **11.** 31 yds. 1 ft. 2 in. **12.** 631 yds. 2 ft. 10 in.
13. 794 yds. 1 ft. 4 in. **14.** 1 yd. 1 ft. 1 in. **15.** 9 yds. 2 ft. 3 in.

152.

1. 14 yds. 2 ft. 11 in. 2. 6 yds. 3. 1080 yds. 4. 675 yds.
5. 15. 6. 72. 7. 62 yds. 1 ft. 8. 4 ft. 8 in.
9. 5 ft. 10. 365 yds. 1 ft. $10\frac{1}{2}$ in. 11. 69 yds. 12. 135 yds.

153.

1. (i) 7920 yds.; (ii) 360 chains. 2. 19 miles 45 chains.
3. 32890 yds. 4. 10 miles. 5. 3 miles 1 fur. 150 yds.
6. 824 yds. 1 ft. 2 in. 7. $5\frac{1}{2}$ yds. 8. 115 chains 11 yds.
9. Viâ B by 90 yds. 10. 236 miles 800 yds. 11. 37 miles 3 fur. 90 yds.
12. 13680. 13. 352. 14. 9812. 15. 12 yds.
16. 66 ft. 17. $12\frac{1}{2}$ miles. 18. 45 miles.

154.

1. 9*d*. 2. £1. 2*s*. 6*d*. 3. 2 cwt. 1 qr.
4. £4. 11*s*. 5. 3720; 1 qr. 1 lb. 1 oz. 6. £90.
7. 27 gall. 8. 4*d*. 9. £1. 10. £15. 7*s*. 6*d*.

155.

1. 9*s*. $4\frac{1}{2}$*d*. 2. £1. 14*s*. 3. 8*s*. 9*d*. 4. £22. 1*s*.
5. £211. 9*s*. 2*d*. 6. £2. 5*s*. 9*d*. 7. 16*s*. 8*d*.
8. £13. 17*s*. $4\frac{1}{2}$*d*. 9. $1\frac{1}{2}$ cwt. 10. 4 tons.

156.

1. 4 sq. in. 2. 16 sq. in. 3. 36 sq. in. 4. 64 sq. in.
5. 100 sq. in. 6. 225 sq. in. 7. 400 sq. in. 8. 484 sq. in.

157.

1. 720. 2. 45. 3. 23255. 4. 223 sq. ft. 34 sq. in.
5. 52 sq. yds. 6 sq. ft. 6. 13 sq. yds. 7 sq. ft. 19 sq. in.
7. 25 sq. yds. 3 sq. ft. 110 sq. in. 8. 9 sq. yds. 6 sq. ft. 124 sq. in.
9. 113 sq. yds. 1 sq. ft. 72 sq. in. 10. 4 sq. yds. 8 sq. ft. 74 sq. in.
11. 500. 12. 44 sq. ft. 18 sq. in.

158.

1. 8 sq. in. 2. 15 sq. in. 3. 28 sq. in. 4. 30 sq. in. 5. 72 sq. in.
6. 91 sq. in. 7. 120 sq. in. 8. 143 sq. in. 9. 132 sq. in.

159.

1. 150 sq. ft. 2. 168 sq. ft. 3. 224 sq. ft. 4. 650 sq. ft.
5. 1862 sq. ft. 6. 2613 sq. ft. 7. 15 ft. 8. 21 ft.
9. 21 ft. 10. 15 ft. 11. 36. 12. 4000.
13. 84; £2. 2*s*. 14. £19. 15. £2. 5*s*.

160.

1. 5 sq. ft.
2. 10 sq. ft. 72 sq. in.
3. 4 sq. ft. 128 sq. in.
4. 497 sq. yds.
5. 31 sq. yds.
6. 2 sq. yds. 2 sq. ft.
7. 80 sq. yds. 5 sq. ft.
8. 67 sq. yds. 5 sq. ft.
9. 3 ft.
10. 5 ft.

161.

1. 640 sq. ft.
2. 630 sq. ft.
3. 1260 sq. ft.
4. 858 sq. ft.
5. 460 sq. ft.
6. 800 sq. ft.
7. 840 sq. ft.
8. 520 sq. ft.
9. £7. 7*s*. 4*d*.
10. £2. 10*s*.
11. £4. 7*s*. 6*d*.
12. £11. 17*s*. 6*d*.

162.

1. 10 ac.
2. 12 ac.
3. 14 ac.
4. 18 ac.
5. 36 ac.
6. 50 ac.
7. 3630 sq. yds.
8. 2420 sq. yds.
9. 242 ac.
10. 95680 ac.
11. 116 sq. miles.
12. 176 yds.
13. 4.

163.

1. 16274 c. in.
2. 2763 c. ft.
3. 109 c. yds.
4. 19 c. ft.
5. 2 c. yds. 26 c. ft. 57 c. in.
6. 94 c. yds. 2 c. ft.
7. 22 c. ft. 1528 c. in.
8. 25 c. yds. 5 c. ft. 360 c. in.
9. 3 c. yds. 25 c. ft. 1600 c. in.
10. 2160.

164.

1. 8 c. in.
2. 64 c. in.
3. 216 c. in.
4. 512 c. in.
5. 8 c. in.
6. 30 c. in.
7. 84 c. in.
8. 120 c. in.
9. 432 c. in.
10. 182 c. in.
11. 360 c. in.
12. 840 c. in.

165.

1. 2400 c. ft.
2. 2736 c. ft.
3. 6480 c. ft.
4. 3510 c. ft.
5. 2197 c. ft.
6. 3375 c. ft.
7. 12 ft.
8. 4 ft.
9. 480 c. yds.
10. 360.
11. 14400.
12. 20 ft.

166.

1. 6·5 cm.
2. 13·6 cm.
3. 10·3 cm.
4. 15·2 cm.
5. 38·0 cm.
6. 70·8 cm.
7. 3 m. 6 dm. 2 cm. 8 mm.
8. 4 m. 0 dm. 7 cm. 2 mm.
9. 3 m. 5 dm. 4 cm. 6 mm.
10. 18 m. 6 dm. 2 cm.
11. 36 m. 5 dm.
12. 4 m. 6 dm. 5 cm. 9 mm.
13. 8·015 Km.
14. 5·705 Km.
15. 3·45 Km.
16. 15·075 Km.
17. 10·507 Km.
18. 16·75 Km.
19. 4765 m.
20. 5076 m.
21. 6708 m.
22. 5830 m.
23. 9800 m.
24. 7080 m.

167.

1. 8·5 cg.	2. 16·8 cg.	3. 95·6 cg.
4. 7 g. 3 dg. 0 cg. 6 mg.	5. 3 g. 7 dg. 1 cg. 4 mg.	6. 7 g. 3 dg. 0 cg. 4 mg.
7. 4·265 Kg.	8. 13·056 Kg.	9. 20·86 Kg.
10. 5875 g.	11. 4730 g.	12. 7400 g.

168.

1. 5·17 Hl.	2. 8·05 Hl.	3. 0·0897 Hl.
4. 4 Hl. 37 l.	5. 84 Hl. 59 l. 7 dl.	6. 3 Hl. 17 l. 2 dl. 5 cl.
7. 8 Hl. 57 l. 5 dl.	8. 1 Hl. 20 l. 1 dl. 5 cl.	9. 4 Hl. 2 l. 5 dl.

169.

1. 313 c.	2. 1404 c.	3. 1010 c.	4. 2550 c.
5. 1 f. 75 c.	6. 10 f. 5 c.	7. 21 f. 50 c.	8. 32 f. 15 c.
9. 25·25 f.	10. 10·1 f.	11. 20·05 f.	12. 7·03 f.
13. 1715 pf.	14. 802 pf.	15. 1980 pf.	16. 2045 pf.
17. 1·15 m.	18. 12·12 m.	19. 6·01 m.	20. 0·15 m.
21. 710 c.	22. 8075 c.	23. 5505 c.	24. 486 c.
25. $9. 67 c.	26. $8. 6 c.	27. $50. 15 c.	28. $100. 65 c.
29. $8.15.	30. $50.1.	31. $65.01.	32. $56.05.

170.

1. ·4.	2. ·7.	3. ·14.	4. ·75.	5. ·03.	6. ·795.
7. ·052.	8. ·008.	9. 9·5.	10. 30·6.	11. 8·53.	12. 2·005.

171.

1. 324·25.	2. ·25.	3. 580·2.	4. 50·02.	5. ·002 Km.
6. 4·5 g.	7. 45 l.	8. ·45 m.	9. 424·5.	10. 6·275.
11. ·025.	12. 5·25.	13. 1010 Km.	14. 0·1 g.	15. 11 l.
16. 1001 m.	17. 37275·1.	18. 5267·5.	19. 52427·8.	20. 47·25.
21. 1 Km.	22. 7·5 g.	23. 15625 l.	24. ·05 m.	

172.

1. 1·6375.	2. ·065.	3. ·0352.	4. 1·35.
5. 12·5025 Km.	6. ·001 g.	7. ·011 l.	8. ·0011 m.
9. ·4775.	10. 3·24005.	11. 3·652.	12. ·000465.
13. ·004 Km.	14. ·0005 g.	15. ·00006 l.	16. ·000005 m.
17. 1·00265.	18. ·365275.	19. ·02556.	20. ·005825.
21. ·0005 Km.	22. ·000025 g.	23. ·0000035 l.	24. ·00004 m.

173.

1. 86·3	2. 6·6.	3. 7·68.	4. 33·011.	5. 61·159.
6. 72·381.	7. 77·434.	8. 226·825.	9. ·461.	10. 87·381.
11. 56·22 m.	12. 46·5 m.	13. 5·331 m.	14. 37·05 m.	15. 199·874 Kg.
16. 56·8835 Hl.	17. 240·58 f.	18. 160·6 f.	19. 104·07 f.	
20. 13·05 f.	21. 35·66 m.	22. 142·64 m.	23. $506.5.	24 $53.05.

174.

1. 2·07.	2. ·071.	3. ·999.	4. ·99.	5. 6·491.
6. 9·325.	7. 10·886.	8. 86·273.	9. 1·097.	10. 2·66 f.
11. 98·97 f.	12. 2·96 f.	13. 2·35 f.	14. $2.88.	15. $1.98.
16. $19.8.	17. 3·47 m.	18. 4·82 m.	19. 1·88 m.	20. 3·52 m.
21. ·19 dm.	22. ·613 m.	23. 4·507 Kg.	24. 17·275 Kg.	25. 4·4 Hl.

175.

1. 1·672 m.	2. 49·2 cm.	3. 278·477 Kg.	4. 58·68 m.; 130·29 f.
5. 3·805 m.	6. 1·5 cm.	7. 4·49 Km.	8. $101.6.

176.

1. ·36.	2. ·35.	3 ·78.	4. 13·02.	5. 24·72.	6. 6·85.	
7. 1·305.	8. ·385.	9 ·084.	10. 42.	11. 45·2.	12. 5·64.	
13. 18·5.	14. 6·4.	15 31·5.	16 6·3.	17. 39.	18. 540.	
19. 14·868.	20. 20·007.	21. 2·052.	22. 648.	23. 22·5.	24. 44·415.	
25. 32·24 m.	26. 4334 55 m.		27. 32·718 m.	28. 467·25 m.		
29. 817·19 Kg.	30. 130·763 Kg.		31. 152·342 Kg.	32. 529·625 Kg.		

177.

1. 60·48.	2. 27·44.	3. 23·04.	4. 1·458.
5. 4·32.	6. 1·68.	7. 211·06.	8. ·104.
9. 874·581.	10. 3201·024.	11. 146·6621.	12. 1293·9375
13. 194·355.	14. 21·106.	15. ·23408.	16. ·0001292.
17. ·0530151.	18. 1·092347.	19. 15·46875.	20. 1·546875.
21. ·1546875.	22. ·01546875.	23. ·380716.	24. 6·384512.
25. ·000504.	26. 17·13768.	27. ·0306.	28. ·000001.

178.

1. 337·05 f.	2. 660 f.	3. 2686·77 f.	4. 207·88 f.
5. 427·55 f.	6. 711·96 f.	7. 424·41 m.	8. 2261·76 m.

179.

1. ·369.	2. ·278.	3. ·196.	4. 2·35.	5. 2·53.
6. 3·45.	7. ·324.	8. ·625.	9. ·372.	10. ·035.
11. ·027.	12. ·058.	13. ·075.	14. ·046.	15. ·095.
16. ·498.	17. 2·17.	18. ·465.	19. 5·14.	20. ·074.
21. ·0856.	22. 3·945.	23. ·3725.	24. ·00368.	25. ·056 m.
26. 8·75 m.	27. ·2875 Kg.	28. 5·305 Kg.	29. 1·125 Hl.	30. 2·065 Hl.

180.

1. 2·51.	2. ·251.	3. ·0251.	4. 4·639.	5. 5·014.	6. ·205.
7. ·0407.	8. ·00123.		9. ·000245.		10. ·000084.
11. ·0000684.	12. ·0000079.		13. ·0013.		14. ·0811.
15. 1·9406.	16. ·9381.		17. ·0193.		18. 5·0379.

181.

1. 12·5.	2. 6·25.	3. 3·125.	4. 1·5625.	5. 2·78125.
6. 1·8.	7. 1·6.	8. 1·5.	9. 1·44.	10. 2·025.
11. 187·25.	12. 98·64.	13. 73·05.	14. 63·025.	15. 62·125.
16. 157·8.	17. 38·12.	18. 1516·48.	19. 335·408.	20. 569·04.
21. ·3125.	22. ·425.	23. ·0125.	24. ·59375.	25. ·078125.
26. ·3̇.	27. ·7̇.	28. ·4̇5̇.	29. ·4̇2̇.	30. ·1̇0̇8̇.
31. ·7̇3̇.	32. ·7̇2̇.	33. ·9̇1̇6̇.	34. ·0̇2̇7̇.	35. ·0̇2̇2̇7̇.

182.

1. 57·18.	2. 12·36.	3. 30·14.	4. 1·65 in.
5. 3·54 in.	6. A 67·75; B 53·4; C 51·24; D 49·33.		

183.

1. 8.	2. 8·7.	3. ·07.	4. ·9.
5. 6·4.	6. ·03.	7. ·65.	8. ·014.
9. ·05.	10. 4·5.	11. 70.	12. 6.
13. 15.	14. ·0513.	15. ·54.	16. 941.
17. 320.	18. 1300.	19. 9·236.	20. 61.
21. ·00638.	22. ·112.	23. ·0112.	24. 3122.
25. ·0000152.	26. ·718.	27. 47·93.	28. 13·756.
29. 253·2.	30. ·000961.	31. ·371.	32. 99·072.
33. 4024·576.	34. ·222.	35. 33·062.	36. 148·280.

184.

1. ·03125.	2. ·06625.	3. 15·63.	4. 1·546875.
5. 15·46875.	6. 7·815.	7. 3134.	8. ·00756.
9. 492·2.	10. 6268.	11. 1·092.	12. ·6268.

185.

1. 2 f. 8 c.	2. 561 Kg.	3. 10 f. 71 c.	4. 732 f.
5. 81 l. 90 cl.	6. 262 f. 50 c.	7. 140 Kg.	8. 7·92 Km.
9. 800 m.	10. £23.	11. £25.	12. 180 m.

186.

1. 37.	2. 55.	3. 78.	4. 70.	5. 51.
6. 3·78.	7. ·37.	8. 9·40.	9. 12·35.	10. 5·10.
11. 357 cm.	12. 15 cm.	13. 92 cm.	14. 390 cm.	15. 37·8 Kg.
16. 15·9 in.	17. 1460 m.	18. 13·9 l.	19. 2·21 lb.	
20. 18500 yds.	21. 928000 miles.	22. ·00376 m.	23. 39·4 in.	

187.

1. 155.	2. 333.	3. 440.	4. 11·4.	5. 7·1.
6. 8·8.	7. 87·8.	8. 42·70.	9. 60·31.	10. ·26.
11. 19·91.	12. 827 cm.	13. 921 cm.	14. 2 cm.	15. 24 cm.

188.

1. 6419·21.	2. 499·27.	3. 35·66.	4. 4·28.
5. ·57.	6. 8558·94.	7. 495·06.	8. 2475·32.
9. 37129·78.	10. 86·64.	11. 9·90.	12. ·74.

189.

1. 23·3.	2. ·007069.	3. 55·794.	4. 1·12.	5. ·864.
6. 4·883.	7. ·01227.	8. ·039.	9. ·013.	10. 20·415.

190.

1. ·5.	2. ·25.	3. ·75.	4. ·125.	5. ·375.
6. ·875.	7. ·1.	8. ·5.	9. ·4.	10. ·7.
11. ·25.	12. ·15.	13. ·75.	14. ·95.	15. ·125.
16. ·875.	17. ·075.	18. ·375.	19. ·775.	20. ·925.
21. ·3375.	22. ·6875.	23. ·3125.	24. ·8625.	25. ·23125.
26. ·54375.	27. ·971875.	28. ·628125.	29. ·359375.	30. ·790625.

191.

1. ·75.	2. ·125.	3. ·675.	4. ·875.
5. ·625.	6. ·77875.	7. ·0634375.	8. ·0588125.
9. ·0015625.	10. ·8375.	11. ·375.	12. ·125.

192.

1. ·25.	2. ·2.	3. ·25.	4. ·75.	5. ·125.
6. ·125.	7. ·075.	8. ·375.	9. ·3875.	10. ·04375.
11. ·34375.	12. ·228125.	13. ·03625.	14. ·430625.	

193.

1. ·9.	2. ·75.	3. ·75.	4. ·875.	5. ·625.
6. ·75.	7. ·6.	8. ·8.	9. ·3.	10. ·625.

194.

1. 7*s.* 3*d.*	2. 5*s.* 9*d.*	3. 3*s.* 1½*d.*	4. 5*s.* 7½*d.*
5. 15*s.* 4½*d.*	6. 15*s.* 9¾*d.*	7. 7*s.* 7½*d.*	8. 5*s.* 6*d.*
9. £5. 6*s.* 9*d.*	10. £48. 8*s.* 1½*d.*	11. £5. 18*s.* 6¾*d.*	12. £4. 10*s.* 6¾*d.*
13. 3 qrs. 3 lb. 8 oz.	14. 7 cwt. 3 qrs. 7 lb.	15. 1 day 15 hrs. 54 min.	
16. 5 cwt. 3 qrs. 14 lb.	17. 12 days 22 hrs. 3 min.	18. 5 yds. 1 ft. 6 in.	
19. 4*s.* 7*d.*	20. 2*s.* 7½*d.*	21. 15*s.* 6*d.*	22. 9 cwt. 3 qrs.
23. 1 cwt. 3 qrs. 18 lb.	24. 228 days 6 hrs. 45 min.	25. 9 lb. 3 oz.	

195.

1. £·125.	2. £·375.	3. £·775.	4. £·925.	5. £·256.
6. £·659.	7. £·457.	8. £·858.	9. £·163.	10. £·567.
11. £·723.	12. £·968.	13. £·527.	14. £·082.	15. £·333.
16. £·634.	17. £·090.	18. £·242.	19. £·695.	20. £·948.

196.

1. 1*s.* 2¼*d.* 2. £3. 14*s.* 9¾*d.* 3. £8. 3*s.* 1¼*d.* 4. £3. 15*s.* 6*d.*
5. £2. 14*s.* 3¾*d.* 6. 1*s.* 4¾*d.* 7. £8. 14*s.* 9*d.* 8. £1. 0*s.* 9¼*d.*
9. £3. 15*s.* 3½*d.* 10. £5. 5*s.* 1¾*d.* 11. £3. 2*s.* 11*d.* 12. 16*s.* 11½*d.*

197.

1. 3907 sq. cm. 2. 50012 sq. cm. 3. 30508 sq. cm.
4. 6 sq. m. 30 sq. dm. 4 sq. cm. 5. 5 sq. m. 9 sq. dm. 21 sq. cm.
6. 5 sq. m. 15 sq. cm. 9 sq. mm. 7. 12·96 sq. cm.
8. 53·29 sq. cm. 9. 73·96 sq. cm. 10. 46·24 sq. cm.
11. 18·62 sq. cm. 12. 8·82 sq. cm. 13. 26·13 sq. cm.
14. 62·78 sq. cm. 15. 178·5 sq. m. 16. 119·7 sq. m.
17. 214·71 sq. m. 18. 210·3101 sq. m. 19. 39·2 m.
20. 15·2 m. 21. 32; 24 f. 22. 2820 f. 6 c.

198.

1. 684900 ares. 2. 50705 ares. 3. 23 Ha. 45 a. 87 ca.
4. 78 sq. Km. 68 Ha. 45 a. 5. 47 Ha. 86 a. 70 ca.
6. 295 Ha. 40 a. 94 ca. 7. 189·4 m.
8. 999 m. 9. 17337 f. 50 c. 10. 152524 f. 72 c.

199.

1. 72571 c.cm. 2. 87025 c.cm.
3. 3152000 c.cm. 4. 37 c.m. 215 c.dm.
5. 1 c.m. 3 c.dm. 401 c.cm. 6. 78 c.m. 64 c.dm. 72 c.cm.
7. 39·304 c.m. 8. 205·379 c.m. 9. 884·736 c.m.
10. 30·039 c.cm. 11. 30·42 c.cm. 12. 706·86 c.cm.
13. 6350·4 c.cm. 14. 1·601613 c.m. 15. 230·656 c.m.

200.

1. 2*s.* 4*d.* 2. 6*s.* 3*d.* 3. 3*s.* 4. 4*s.* 1*d.*
5. 7 in. 6. 6 in. 7. 1 qt. 8. 1 gall. 3 qts. 1 pt.
9. 13 lb. 12 oz. 10. 5 tons 6 cwt. 2 qrs. 11. 1 hr. 1 min. 15 sec.
12. 39 gall. 3 qts. 1 pt. 13. 13 yds. 1 ft. 6 in.

201.

1. $\frac{3}{7}$. 2. $\frac{5}{9}$. 3. $\frac{11}{18}$. 4. $\frac{2}{3}$. 5. $\frac{3}{4}$.
6. $\frac{9}{16}$. 7. $\frac{8}{9}$. 8. $\frac{15}{16}$. 9. $\frac{4}{11}$. 10. $\frac{7}{10}$.
11. $\frac{3}{7}$. 12. $\frac{3}{13}$. 13. $\frac{2}{3}$. 14. $\frac{5}{7}$. 15. $\frac{3}{4}$.
16. $\frac{19}{81}$. 17. $\frac{23}{64}$. 18. $\frac{12}{13}$. 19. $\frac{9}{10}$. 20. $\frac{12}{13}$.
21. $\frac{7}{11}$. 22. $\frac{11}{14}$. 23. $\frac{9}{20}$. 24. $\frac{33}{38}$. 25. $\frac{25}{33}$.

202.

1. $\frac{2}{3}$. 2. $\frac{3}{4}$. 3. $\frac{5}{6}$. 4. $\frac{7}{8}$.
5. $\frac{6}{7}$. 6. $\frac{4}{9}$. 7. $\frac{2}{3}$. 8. $\frac{4}{9}$.

203.

1. $\frac{16}{20}, \frac{15}{20}, \frac{14}{20}$; $\frac{7}{10}, \frac{3}{4}, \frac{4}{5}$.
2. $\frac{48}{168}, \frac{63}{168}, \frac{140}{168}$; $\frac{2}{7}, \frac{3}{8}, \frac{5}{6}$.
3. $\frac{24}{40}, \frac{28}{40}, \frac{25}{40}$; $\frac{3}{5}, \frac{5}{8}, \frac{7}{10}$.
4. $\frac{6}{12}, \frac{8}{12}, \frac{10}{12}, \frac{7}{12}$; $\frac{1}{2}, \frac{7}{12}, \frac{2}{3}, \frac{5}{6}$.
5. $\frac{300}{360}, \frac{225}{360}, \frac{80}{360}, \frac{108}{360}$; $\frac{2}{9}, \frac{3}{10}, \frac{5}{8}, \frac{5}{6}$.
6. $\frac{25}{60}, \frac{40}{60}, \frac{48}{60}, \frac{54}{60}$; $\frac{5}{12}, \frac{2}{3}, \frac{4}{5}, \frac{9}{10}$.
7. $\frac{16}{20}, \frac{6}{20}, \frac{7}{20}$; $\frac{4}{5}, \frac{7}{20}, \frac{3}{10}$.
8. $\frac{30}{36}, \frac{33}{36}, \frac{22}{36}$; $\frac{11}{12}, \frac{5}{6}, \frac{11}{18}$.
9. $\frac{20}{45}, \frac{27}{45}, \frac{24}{45}$; $\frac{3}{5}, \frac{8}{15}, \frac{4}{9}$.
10. $\frac{16}{24}, \frac{20}{24}, \frac{17}{24}, \frac{18}{24}$; $\frac{5}{6}, \frac{3}{4}, \frac{17}{24}, \frac{2}{3}$.
11. $\frac{20}{100}, \frac{75}{100}, \frac{70}{100}, \frac{52}{100}$; $\frac{3}{4}, \frac{7}{10}, \frac{13}{25}, \frac{1}{5}$.
12. $\frac{80}{120}, \frac{105}{120}, \frac{110}{120}, \frac{114}{120}$; $\frac{19}{20}, \frac{11}{12}, \frac{7}{8}, \frac{2}{3}$.

204.

1. $1\frac{5}{6}$. 2. $8\frac{2}{5}$. 3. $7\frac{1}{8}$. 4. 7. 5. $10\frac{2}{7}$.
6. $20\frac{3}{4}$. 7. 9. 8. $13\frac{8}{9}$. 9. $17\frac{1}{7}$. 10. $11\frac{8}{11}$.
11. $17\frac{3}{8}$. 12. $1044\frac{6}{7}$. 13. $121\frac{4}{9}$. 14. $481\frac{5}{11}$. 15. $122\frac{1}{12}$.

205.

1. $\frac{55}{8}$. 2. $\frac{53}{9}$. 3. $\frac{34}{7}$. 4. $\frac{71}{12}$. 5. $\frac{74}{7}$.
6. $\frac{127}{10}$. 7. $\frac{47}{3}$. 8. $\frac{71}{4}$. 9. $\frac{119}{6}$. 10. $\frac{191}{9}$.
11. $\frac{259}{8}$. 12. $\frac{305}{7}$. 13. $\frac{543}{10}$. 14. $\frac{787}{12}$. 15. $\frac{867}{11}$.
16. $\frac{21}{7}$; $\frac{35}{7}$; $\frac{49}{7}$; $\frac{63}{7}$. 17. $\frac{44}{11}$; $\frac{66}{11}$; $\frac{88}{11}$; $\frac{110}{11}$.

206.

1. $\frac{5}{7}$. 2. 1. 3. $\frac{5}{6}$. 4. $\frac{7}{10}$. 5. $\frac{7}{12}$.
6. $\frac{8}{15}$. 7. $\frac{9}{20}$. 8. $\frac{11}{30}$. 9. $\frac{2}{3}$. 10. $\frac{5}{12}$.
11. $\frac{11}{12}$. 12. $\frac{3}{4}$. 13. $2\frac{23}{45}$. 14. $2\frac{4}{5}$. 15. $2\frac{17}{45}$. 16. $2\frac{43}{60}$.

207.

1. $9\frac{5}{9}$. 2. $8\frac{3}{4}$. 3. $18\frac{17}{18}$. 4. $10\frac{63}{88}$.
5. $7\frac{11}{40}$. 6. $17\frac{7}{30}$. 7. $17\frac{55}{72}$. 8. $10\frac{161}{180}$.
9. $21\frac{17}{24}$. 10. $14\frac{1}{4}$. 11. $18\frac{23}{120}$. 12. $12\frac{5}{24}$.

208.

1. $\frac{2}{3}$. 2. $\frac{1}{3}$. 3. $\frac{1}{8}$. 4. $\frac{3}{10}$.
5. $\frac{1}{6}$. 6. $\frac{1}{15}$. 7. $\frac{1}{12}$. 8. $\frac{1}{9}$.
9. $3\frac{1}{6}$. 10. $6\frac{1}{6}$. 11. $3\frac{1}{6}$. 12. $1\frac{7}{10}$.
13. $2\frac{5}{8}$. 14. $7\frac{1}{12}$. 15. $4\frac{1}{36}$. 16. $5\frac{5}{21}$.

209.

1. $11\frac{7}{8}$. 2. $8\frac{2}{3}$. 3. $6\frac{17}{18}$. 4. $4\frac{43}{72}$. 5. $1\frac{22}{35}$. 6. $5\frac{1}{15}$.
7. $9\frac{19}{20}$. 8. $4\frac{23}{40}$. 9. $7\frac{75}{88}$. 10. $9\frac{19}{24}$. 11. $11\frac{41}{60}$. 12. $12\frac{17}{70}$.

210.

1. $\frac{7}{8}$. 2. $1\frac{5}{18}$. 3. $\frac{89}{120}$. 4. $13\frac{35}{36}$. 5. $6\frac{7}{30}$.
6. $6\frac{27}{40}$. 7. $2\frac{2}{5}$. 8. $1\frac{37}{72}$. 9. $5\frac{5}{24}$. 10. $5\frac{59}{120}$.

211.

1. $\frac{4}{11}$; $\frac{6}{11}$. 2. $\frac{25}{28}$; $1\frac{17}{28}$. 3. $4\frac{7}{12}$; $2\frac{1}{12}$.
4. $4\frac{10}{11}$; $6\frac{6}{11}$. 5. $\frac{8}{11}$; $2\frac{2}{3}$. 6. $1\frac{2}{7}$; $2\frac{4}{7}$.
7. 10; $12\frac{1}{2}$. 8. $32\frac{2}{3}$; $51\frac{1}{3}$. 9. $20\frac{2}{5}$; $81\frac{3}{5}$.
10. $29\frac{1}{7}$; $109\frac{2}{7}$. 11. $108\frac{9}{14}$; $158\frac{11}{14}$. 12. $7\frac{6}{11}$; $41\frac{1}{2}$.

212.

1. $\frac{3}{20}$; $\frac{3}{25}$; $\frac{3}{35}$. 2. $\frac{2}{27}$; $\frac{2}{45}$; $\frac{2}{63}$. 3. $\frac{6}{13}$; $\frac{4}{13}$; $\frac{3}{13}$.
4. $\frac{2}{19}$; $\frac{1}{19}$; $\frac{1}{38}$. 5. $\frac{5}{18}$; $\frac{1}{6}$; $\frac{1}{36}$. 6. $\frac{1}{40}$; $\frac{1}{80}$; $\frac{1}{160}$.
7. $1\frac{1}{4}$; $\frac{5}{12}$; $\frac{5}{28}$. 8. $\frac{15}{16}$; $\frac{5}{12}$; $\frac{15}{44}$. 9. $3\frac{1}{3}$; $2\frac{8}{21}$; $1\frac{1}{9}$.
10. $\frac{1}{5}$; $\frac{1}{10}$; $\frac{1}{25}$. 11. $\frac{2}{9}$; $\frac{1}{9}$; $\frac{2}{45}$. 12. $\frac{23}{30}$; $\frac{23}{66}$; $\frac{1}{12}$.

213.

1. $5\frac{3}{16}$. 2. $3\frac{9}{16}$. 3. $5\frac{41}{96}$. 4. $2\frac{17}{72}$. 5. $2\frac{67}{90}$. 6. $7\frac{91}{120}$.

214.

1. $\frac{1}{3}$; $\frac{4}{11}$; $\frac{7}{18}$. 2. $\frac{1}{3}$; $\frac{1}{8}$; $\frac{7}{36}$. 3. $\frac{8}{45}$; $\frac{5}{54}$; $\frac{8}{15}$.
4. $\frac{7}{45}$; $\frac{13}{55}$; $\frac{9}{46}$. 5. $\frac{5}{6}$; $1\frac{1}{4}$; $1\frac{7}{8}$. 6. $1\frac{7}{15}$; $\frac{11}{21}$; $3\frac{1}{18}$.
7. $3\frac{29}{44}$; $1\frac{7}{16}$; 4. 8. $9\frac{3}{8}$; $19\frac{11}{16}$; $28\frac{3}{4}$. 9. $13\frac{3}{4}$; $33\frac{3}{4}$; 21.
10. $19\frac{1}{2}$; $14\frac{5}{8}$; $28\frac{7}{16}$. 11. $\frac{1}{30}$. 12. $\frac{3}{70}$. 13. $\frac{2}{7}$. 14. $4\frac{2}{7}$.
15. $32\frac{5}{8}$. 16. $3\frac{1}{5}$. 17. $33\frac{1}{5}$. 18. $5\frac{4}{7}$.

215.

1. $\frac{3}{4}$; $\frac{27}{32}$; $1\frac{1}{8}$; $\frac{81}{112}$. 2. 10; $\frac{7}{16}$; $1\frac{2}{3}$; $\frac{5}{8}$.
3. $1\frac{5}{27}$; $\frac{3}{5}$; 2; $1\frac{7}{45}$. 4. $1\frac{3}{16}$; $2\frac{1}{3}$; $1\frac{17}{32}$; $2\frac{4}{5}$.
5. $38\frac{2}{3}$; $43\frac{1}{2}$; $50\frac{3}{4}$; $31\frac{5}{12}$. 6. $\frac{3}{44}$; $\frac{9}{56}$; $\frac{5}{56}$; $\frac{9}{160}$.
7. $\frac{2}{5}$; $\frac{5}{57}$; $\frac{2}{19}$; $\frac{1}{3}$. 8. $\frac{19}{22}$; $1\frac{1}{4}$; $\frac{19}{37}$; $1\frac{2}{17}$.
9. $\frac{3}{5}$. 10. $\frac{5}{7}$. 11. $3\frac{3}{4}$. 12. $11\frac{1}{3}$. 13. $\frac{1}{10}$.
14. $2\frac{11}{35}$. 15. 4. 16. $3\frac{1}{2}$. 17. $\frac{5}{8}$.

216.

1. $\frac{5}{6}$. 2. $14\frac{1}{6}$. 3. $\frac{32}{55}$. 4. $4\frac{1}{9}$.
5. $1\frac{82}{93}$. 6. $46\frac{13}{72}$. 7. $17\frac{35}{144}$. 8. $\frac{25}{67}$.

217.

1. $1\frac{1}{4}$. 2. $7\frac{1}{5}$. 3. $1\frac{1}{4}$. 4. $\frac{1}{20}$. 5. $\frac{7}{8}$. 6. 3.

218.

1. $20\frac{5}{12}$. 2. $\frac{1}{10}$. 3. $\frac{11}{28}$. 4. $3\frac{1}{30}$. 5. $7\frac{13}{18}$.
6. $4\frac{2}{15}$. 7. $3\frac{17}{144}$. 8. $5\frac{3}{5}$. 9. 9. 10. $1\frac{7}{10}$.

219.

1. $3\frac{5}{24}$. 2. $25\frac{1}{2}$. 3. 4. 4. $1\frac{5}{46}$. 5. $\frac{7}{8}$.
6. $1\frac{1}{4}$. 7. $1\frac{1}{2}$. 8. 16. 9. $2\frac{2}{9}$. 10. $103\frac{1}{8}$.
11. 6. 12. $1\frac{1}{2}$. 13. $\frac{2}{15}$. 14. 2. 15. $\frac{1}{360}$. 16. 1.

220.

1. $\frac{1}{2}$. 2. $\frac{1}{4}$. 3. $\frac{1}{3}$. 4. $\frac{1}{6}$. 5. $\frac{1}{12}$.
6. $\frac{1}{8}$. 7. $\frac{1}{2}$. 8. $\frac{1}{4}$. 9. $\frac{1}{5}$. 10. $\frac{1}{10}$.
11. $\frac{1}{20}$. 12. $\frac{1}{40}$. 13. $\frac{1}{8}$. 14. $\frac{1}{3}$. 15. $\frac{1}{6}$.
16. $\frac{1}{12}$. 17. $\frac{1}{16}$. 18. $\frac{1}{15}$. 19. $\frac{3}{4}$. 20. $\frac{3}{5}$.
21. $\frac{7}{10}$. 22. $\frac{4}{5}$. 23. $\frac{9}{10}$. 24. $\frac{3}{8}$. 25. $\frac{5}{8}$.
26. $\frac{7}{8}$. 27. $\frac{2}{3}$. 28. $\frac{5}{6}$. 29. $\frac{5}{12}$. 30. $\frac{11}{12}$.

221.

1. $\frac{3}{4}$. 2. $\frac{7}{12}$. 3. $\frac{5}{8}$. 4. $\frac{3}{8}$. 5. $\frac{3}{4}$. 6. $\frac{5}{8}$.
7. $\frac{7}{8}$. 8. $\frac{3}{4}$. 9. $\frac{1}{10}$. 10. $\frac{3}{10}$. 11. $\frac{1}{8}$. 12. $\frac{3}{4}$.

222.

1. $\frac{2}{5}$. 2. $\frac{9}{10}$. 3. $\frac{5}{8}$. 4. $\frac{3}{4}$. 5. $\frac{3}{5}$.
6. $\frac{7}{8}$. 7. $\frac{5}{8}$. 8. $\frac{5}{6}$. 9. $\frac{3}{13}$. 10. $\frac{2}{11}$.

223.

1. £168. 15*s*. 2. £176. 8*s*. 3. £121. 17*s*. 6*d*.
4. £186. 7*s*. 6*d*. 5. £176. 13*s*. 4*d*. 6. £156. 6*s*. 8*d*.
7. £139. 2*s*. 6*d*. 8. £144. 18*s*. 9. £118. 15*s*.
10. £337. 6*s*. 8*d*. 11. £159. 12*s*. 12. £195. 16*s*. 8*d*.

224.

1. £927. 10*s*. 2. £2178. 8*s*. 3. £2258. 2*s*. 4. £2501. 5*s*.
5. £5466. 6*s*. 8*d*. 6. £4973. 3*s*. 4*d*. 7. £5335. 3*s*. 4*d*.
8. £1222. 13*s*. 9. £651. 5*s*. 6*d*. 10. £964. 5*s*. 6*d*.

225.

1. £16. 5*s*. 1½*d*.
2. £13. 10*s*. 5*d*.
3. £1. 12*s*. 9¾*d*.
4. £1. 12*s*. 10*d*.
5. £5. 18*s*. 5*d*.
6. £32. 17*s*.
7. £30. 0*s*. 9*d*.
8. 18*s*. 4*d*.
9. £7. 19*s*. 1*d*.
10. £6. 1*s*. 1½*d*.
11. £45. 7*s*. 3½*d*.
12. £301. 1*s*. 10½*d*.

226.

1. $\frac{1}{12}$.
2. $\frac{27}{80}$.
3. $\frac{5}{36}$.
4. $\frac{4}{21}$.
5. 8*s*.
6. 10*s*. 5*d*.
7. $\frac{13}{40}$.
8. $\frac{1131}{2880}$.
9. $2\frac{14}{29}$ days.
10. $1\frac{34}{71}$ hrs.
11. $17\frac{1}{7}$ min.
12. $1\frac{11}{119}$ hrs.

227.

1. 2*s*. 1*d*.
2. 1*s*. 4*d*.
3. £1. 7*s*.
4. £1. 10*s*.
5. £210.
6. 3*s*. 9*d*.
7. £367. 10*s*.
8. £33. 18*s*. 4*d*.
9. 10*s*.
10. 8*s*. 4*d*.

228.

1. 190 sq. ft. 132 sq. in.; 293 sq. ft. 70 sq. in.
2. 40 ft.
3. 16.
4. 70.
5. 1665 sq. ft., or 185 sq. yds.
6. 127 sq. yds.; £12. 14*s*.
7. 217 c. ft.
8. 4 ft.
9. 30000.
10. 125.

229.

1. $\frac{4}{1}$.
2. $\frac{1}{4}$.
3. $\frac{1}{3}$.
4. $\frac{8}{7}$.
5. $\frac{2}{1}$.
6. $\frac{4}{1}$.
7. $\frac{2}{1}$.
8. $\frac{3}{2}$.
9. $\frac{3}{16}$.
10. $\frac{1}{20}$.
11. $\frac{3}{20}$.
12. $\frac{5}{8}$.
13. $\frac{4}{3}$.
14. $\frac{3}{8}$.
15. $\frac{20}{3}$.
16. $\frac{3}{1}$.
17. $\frac{6}{17}$.
18. $\frac{1}{8}$.
19. $\frac{14}{1}$.
20. $\frac{1}{40}$.

230.

1. 27.
2. 21.
3. 3.
4. 12.
5. 4.
6. 8.
7. 11.
8. 11.
9. 30.
10. 6.
11. $\frac{7}{8}$.
12. 8.

231.

1. 17*s*. 6*d*.
2. 64 miles.
3. 3¾ hrs.
4. 120 miles.
5. 52800 pennies.
6. 6720 pennies.

232.

1. 2½ hrs.
2. 2 hrs. 48 min.
3. 10½ days.
4. 4 days.
5. 9 days.
6. 8 men.

233.

1. 8 days.
2. 5 men.
3. 15 days.
4. 8 hrs.
5. 4 tons.
6. £37. 10*s*.
7. 4 men.
8. £54.
9. £3. 11*s*. 6*d*.
10. 1 hr. 20 min.
11. 7½ min.
12. 3 miles.

234.

1. £3. 15*s.* ; £6. 5*s.*
2. 10 lb. 5 oz. ; 17 lb. 3 oz.
3. 2 ft. 10 in. ; 1 yd. 1 ft. 3 in.
4. £3. 4*s.* ; £4 ; £4. 16*s.*
5. 7·3 in. ; 10·95 in. ; 18·25 in.
6. 50 tons ; $43\frac{3}{4}$ tons ; $6\frac{1}{4}$ tons.
7. 36 lb. ; 60 lb. ; 72 lb.
8. 106 ; 265 ; 477 passengers.
9. *A*, 162 ; *B*, 108 ; *C*, 81.
10. *P*, 78 ; *Q*, 117 ; *R*, 156.

235.

2. London-Bristol, about 120 miles.
3. 4 ft.
4. 2·11 in.
5. 2140 yds.
6. $12\frac{1}{2}$ sq. ft.
7. 26 sq. ft.
8. 0·96 sq. in.

236.

1. $3\frac{1}{4}$ sq. ft.
2. Scale : $\frac{1}{2}$ in. = 1 ft.
3. E, $5\frac{1}{8}$ sq. ft. ; F, $3\frac{7}{8}$ sq. ft. ; H, 5 sq. ft. ; T, $3\frac{1}{4}$ sq. ft.
4. 12 sq. ft.
5. 22 sq. in.
6. 16 sq. in.

238.

1. 50%.
2. 75%.
3. 60%.
4. $37\frac{1}{2}$%.
5. 25%.
6. 80%.
7. 64%.
8. $52\frac{1}{2}$%.
9. $66\frac{2}{3}$%.
10. $16\frac{2}{3}$%.
11. $17\frac{1}{2}$%.
12. $11\frac{1}{4}$%.
13. 78·1%.
14. 35·8%.
15. 71·8%.
16. 64·3%.
17. 61·2%.
18. 74·6%.

239.

1. 25%.
2. $2\frac{1}{2}$%.
3. $16\frac{2}{3}$%.
4. $12\frac{1}{2}$%.
5. $12\frac{1}{2}$%.
6. 25%.
7. $12\frac{1}{2}$%.
8. $12\frac{1}{2}$%.
9. 5%.
10. 75%.

240.

1. 54%.
2. $7\frac{1}{2}$%.
3. 5%.
4. 12%.
5. $12\frac{1}{2}$%.
6. $95\frac{1}{3}$%.
7. 97·5%.
8. 74·8%.
9. 28·6% ; 71·4%.
10. 62·8%.

241.

1. 77·00.
2. 74·62.
3. 68·70.
4. 68·33.
5. 67·50.
6. 64·29.
7. 61·18.
8. 47·06.

241a.

1. 50%.
2. 25%.
3. $33\frac{1}{3}$%.
4. $16\frac{2}{3}$%.
5. $8\frac{1}{3}$%.
6. $12\frac{1}{2}$%.
7. 50%.
8. 25%.
9. 20%.
10. 10%.
11. 5%.
12. $2\frac{1}{2}$%.
13. $12\frac{1}{2}$%.
14. $33\frac{1}{3}$%.
15. $16\frac{2}{3}$%.
16. $8\frac{1}{3}$%
17. $6\frac{1}{4}$%.
18. $6\frac{2}{3}$%.
19. 75%.
20. 60%.
21. 70%.
22. 80%.
23. 90%
24. $37\frac{1}{2}$%.
25. $62\frac{1}{2}$%.
26. $87\frac{1}{2}$%.
27. $66\frac{2}{3}$%.
28. $83\frac{1}{3}$%.
29. $41\frac{2}{3}$%.
30. $91\frac{2}{3}$%.

241b.

1. 75%. 2. $58\frac{1}{3}$%. 3. $62\frac{1}{2}$%. 4. $37\frac{1}{2}$%. 5. 75%.
6. $62\frac{1}{2}$%. 7. $87\frac{1}{2}$%. 8. 75%. 9. 10%. 10. 30%.
11. $12\frac{1}{2}$%. 12. 75%.

241c.

1. 40%. 2. 90%. 3. $62\frac{1}{2}$%. 4. 75%. 5. 60%.
6. $87\frac{1}{2}$%. 7. $62\frac{1}{2}$%. 8. $83\frac{1}{3}$%. 9. $23\frac{1}{13}$%. 10. $18\frac{2}{11}$%.

242.

1. $\frac{1}{2}$. 2. $\frac{1}{4}$. 3. $\frac{1}{5}$. 4. $\frac{3}{20}$. 5. $\frac{1}{10}$.
6. $\frac{1}{3}$. 7. $\frac{1}{8}$. 8. $\frac{3}{40}$. 9. $\frac{3}{80}$. 10. $\frac{1}{40}$.
11. $\frac{1}{80}$. 12. $\frac{2}{5}$. 13. $\frac{4}{5}$. 14. $\frac{3}{10}$. 15. $\frac{2}{3}$.
16. $\frac{3}{8}$. 17. $\frac{5}{8}$. 18. $\frac{7}{8}$. 19. $\frac{1}{16}$. 20. $\frac{3}{16}$.

243.

1. 14. 2. 595. 3. 3. 4. 196. 5. 4301. 6. 1·03% ; 309.

244.

1. £2. 17*s*. 2. £1. 13*s*. 4*d*. 3. £1. 11*s*. 6*d*. 4. £1. 11*s*.
5. £22. 10*s*. 6. 9*s*. 6*d*. 7. 18*s*. 6*d*. 8. £1. 19*s*.
9. 1*s*. 9*d*. 10. 12*s*. 9*d*. 11. £1. 1*s*. 12. £1. 2*s*. 6*d*
13. 10*s*. 14. £1. 9*s*. 4*d*. 15. 10*s*. 9*d*. 16. £1. 3*s*. 8*d*.
17. £1. 3*s*. 18. £2. 7*s*. 3*d*. 19. 17*s*. 6*d*. 20. £1. 10*s*. 5*d*.

245.

1. 1 ton 15 cwt. 3 qrs. 2. 7 lb. 12 oz. 3. 1 hr. 35 min.
4. 2 yds. 2 ft. 9 in. 5. 1 mile 4 fur. 5 ch. 6. 3 qts. 1 pt.
7. 6 sq. ft. 54 sq. in. 8. 12 c. yds. 18 c. ft.

246.

1. 57.
2. 234 ; 273 ; 286.
3. £297.
4. 32 lb. 13 oz., 2 lb. 3 oz.
5. $11\frac{1}{4}$ cwt., $1\frac{1}{2}$ cwt., $2\frac{1}{4}$ cwt.
6. 143543 c. ft., 36340 c. ft.

247.

1. 1*s*. 5*d*. 2. 1*s*. 7*d*. 3. 3*s*. 7*d*. 4. 6*s*. 11*d*.
5. 3*d*. 6. 2*s*. 8*d*. 7. 4*s*. 10*d*. 8. 7*s*. 7*d*.

248.

50% = 10*s*. in the £.
$33\frac{1}{3}$% = 6*s*. 8*d*.....
25% = 5*s*.....
20% = 4*s*.....
15% = 3*s*.....
$12\frac{1}{2}$% = 2*s*. 6*d*.....

10% = 2*s*. in the £.
$7\frac{1}{2}$% = 1*s*. 6*d*. .....
5% = 1*s*. .....
$3\frac{3}{4}$% = 9*d*.....
$2\frac{1}{2}$% = 6*d*.....
$1\frac{1}{4}$% = 3*d*.....

249.

1. £87. 10*s.* **2.** £5. **3.** £137. 15*s.* **4.** £460. 13*s.* 9*d.*
5. £85. 7*s.* 10*d.* **6.** £31. 17*s.* 6*d.* **7.** £1. 6*s.* 3*d.* **8.** 8*s.* 9*d.*; 83⅓%.
9. 8*s.* 9*d.* **10.** £15. 8*s.* 7*d.*

250.

1. 45 min. **2.** 60. **3.** 24. **4.** 63 ft. **5.** 400.
6. 45. **7.** 4945. **8.** £52. **9.** £200. **10.** £1600.

251.

1. 25% gain. **2.** 25% loss. **3.** 5% gain. **4.** 6% loss.
5. 7½% gain. **6.** 15% loss. **7.** 60% gain. **8.** 40% loss.
9. 33⅓% gain. **10.** 12½% loss.

252.

1. 25%. **2.** 10%. **3.** 6⅔%. **4.** £1. 8*s.*; 10%.
5. 68%. **6.** £29. 10*s.* 1*d.*; 11¼%.
7. 8*s.* 10*d.*; 44%. **8.** 8*s.* 4*d.*; $9\frac{1}{11}$%.

253.

1. 3*s.* 9*d.* **2.** 2*s.* 3*d.* **3.** 1*s.* 9*d.* **4.** 4*s.* 9*d.* **5.** 6*s.*
6. 1*s.* 2*d.* **7.** 7*s.* 2*d.* **8.** 3*s.* 11*d.* **9.** 14*s.* 8*d.* **10.** 2*s.* 3*d.*

254.

1. 10*s.* 6*d.*; £2. 2*s.* **2.** £1. 10*s.* **3.** £52; £69; 21%. **4.** 1*s.*
5. 1*s.* 11*d.* **6.** £286. **7.** 4*s.* **8.** 1 f. 62 c.

255.

1. 3*s.* **2.** 3*s.* **3.** 1*s.* 8*d.* **4.** 5*s.* **5.** 1*s.* 4*d.*
6. 1*s.* 4*d.* **7.** 6*s.* 8*d.* **8.** 4*s.* 2*d.* **9.** 4*s.* 7*d.* **10.** 5*s.*

256.

1. £72. **2.** £60. **3.** 576. **4.** 1*s.*
5. £11. 4*s.* **6.** £1. 17*s.* 6*d.* **7.** £15. **8.** 20 centimes.

257.

1. £1. 1*s.* **2.** £2. 6*s.* **3.** 10*s.* 6*d.* **4.** 1*s.* **5.** £13. 6*s.* 8*d.*
6. 5% gain. **7.** 8% gain. **8.** 5% loss. **9.** 10% gain. **10.** 8⅓% gain.

258.

1. £1. 10*s.* **2.** £4. 4*s.* **3.** £5. 12*s.* **4.** £2. 18*s.*
5. £3. 16*s.* **6.** £5. 14*s.* **7.** £7. 8*s.* **8.** £8. 6*s.*
9. £6. 5*s.* **10.** £4. 15*s.* **11.** £8. 3*s.* **12.** £5. 9*s.*
13. £3. 17*s.* **14.** £2. 19*s.* **15.** £7. 7*s.* **16.** £2. 13*s.*

259.

1. £5 ; £105.
2. £10 ; £210.
3. £30 ; £530.
4. £12 ; £162.
5. £29. 8*s.* ; £449. 8*s.*
6. £33. 12*s.* ; £593. 12*s.*
7. £14. 10*s.* ; £304. 10*s.*
8. £31. 5*s.* ; £656. 5*s.*
9. £28. 10*s.* ; £503. 10*s.*
10. £15. 8*s.* ; £400. 8*s.*
11. £38. 3*s.* ; £583. 3*s.*
12. £65. 4*s.* ; £880. 4*s.*

260.

1. £10 ; £110.
2. £30 ; £230.
3. £24 ; £174.
4. £88. 4*s.* ; £508. 4*s.*
5. £67. 4*s.* ; £627. 4*s.*
6. £68. 8*s.* ; £448. 8*s.*
7. £50 ; £675.
8. £195. 12*s.* ; £1010. 12*s.*
9. £76. 6*s.* ; £621. 6*s.*
10. £35. 8*s.* ; £330. 8*s.*

261.

1. £10 ; £210.
2. £5 ; £505.
3. £5 ; £255.
4. £5 ; £255.
5. £22. 16*s.* ; £592. 16*s.*
6. £2. 13*s.* ; £267. 13*s.*
7. £14. 5*s.* ; £489. 5*s.*
8. £12. 18*s.* ; £657. 18*s.*

262.

1. £7. 8*s.* ; £747. 8*s.*
2. £10. 10*s.* ; £535. 10*s.*
3. £11. 14*s.* ; £401. 14*s*
4. £15. 8*s.* ; £400. 8*s.*
5. £26 ; £676.
6. £15. 12*s.* ; £275 12*s.*

263.

1. £47. 7*s.* 6*d.*
2. £50. 3*s.* 6*d.*
3. £36. 8*s.*
4. £46. 17*s.* 6*d.*
5. £29. 5*s.*
6. £121. 14*s.* 3*d.*
7. £27. 11*s.* 9*d.*
8. £2. 15*s.* 9*d.*

264.

1. £5. 10*s.* 11*d.*
2. £129. 12*s.* 9*d.*
3. £10. 19*s.* 4*d.*
4. £897. 7*s.* 2*d.*
5. £234. 17*s.* 1*d.*
6. £78. 17*s.* 4*d.*
7. £34. 8*s.* 3*d.*
8. £718. 8*s.* 9*d.*

265.

1. 65 f. 94 c.
2. 22 f. 36 c.
3. $13. 44 c.
4. $184. 24 c
5. 17 f. 55 c.
6. 228 f. 15 c.
7. $887. 4 c.
8. $782. 58 c.

266.

1. £378. 4*s.*
2. 2*s* 6*d.*
3. £55.
4. £45 in each case.
5. £5. 5*s.*
6. 2 : 3.

267.

1. £45.
2. £78.
3. £84.
4. 1 year.
5. 3 years.
6. 8 months.
7. 5%.
8. 10%.

268.

1. £475. 2. £182. 16*s*. 3*d*. 3. £139. 7*s*. 6*d*. 4. 942 francs.
5. 1 year. 6. 2 years. 7. $3\frac{1}{4}$ years. 8. 7%.
9. 6%. 10. $7\frac{1}{2}$%.

269.

1. 2 years. 2. 3 years. 3. 8 months.
4. 6%. 5. 4%. 6. $6\frac{1}{4}$%.

270.

1. £625. 2. £475. 3. £227. 10*s*.
4. £227. 10*s*. 5. £337. 10*s*. 6. £337. 10*s*.

271.

1. 6*d*. 2. 65%. 3. 120%.
4. 7%. 5. £2000. 6. August 27th.

272.

1. £605 ; £105. 2. £220. 10*s*. ; £20. 10*s*.
3. £270. 8*s*. ; £20. 8*s*. 4. £529. 4*s*. ; £49. 4*s*.
5. £618. 12*s*. 10*d*. ; £57. 10*s*. 4*d*. 6. £1904. 3*s*. 2*d*. ; £143. 13*s*. 2*d*.
7. £4630. 10*s*. ; £630. 10*s*. 8. £1969. 4*s*. 4*d*. ; £218. 11*s*. 10*d*.
9. £994. 18*s*. 3*d*. ; £71. 0*s*. 9*d*. 10. £4969. 3*s*. 9*d*. ; £969. 3*s*. 9*d*.
11. £5705. 16*s*. 7*d*. ; £705. 16*s*. 7*d*. 12. £1214. 6*s*. 10*d*. ; £180. 3*s*. 10*d*.
13. £551. 18*s*. 2*d*. ; £51. 18*s*. 2*d*. 14. £1169. 17*s*. 2*d*. ; £169. 17*s*. 2*d*.

273.

1. 169. 2. 289. 3. 361. 4. 529. 5. 961.
6. 1369. 7. 2209. 8. 2809. 9. 5329. 10. 6241.
11. 7921. 12. 9409. 13. 1936. 14. 5625. 15. 7056.

274.

1. 13. 2. 17. 3. 19. 4. 23. 5. 31. 6. 37.
7. 47. 8. 53. 9. 73. 10. 79. 11. 89. 12. 97.

275.

1. 119. 2. 149. 3. 151. 4. 253.
5. 383. 6. 473. 7. 499. 8. 503.

276.

1. 1·7. 2. 2·3. 3. 8·9. 4. 1·51.
5. 49·9. 6. 0·037. 7. 0·109. 8. 8·04.
9. 0·153. 10. 13·05. 11. 11·01. 12. 0·0031.

277.

1. 7·56. **2.** 0·13. **3.** 0·91. **4.** 1·265. **5.** 0·632.
6. 0·045. **7.** 8·021. **8.** 7·56. **9.** 0·126. **10.** 0·907.
11. 0·0447. **12.** 0·5. **13.** 0·7. **14.** 1·8. **15.** 2·3.

278.

1. 3 ft. 9 in. **2.** 880 yds. **3.** $5\frac{3}{16}$ min. **4.** 16 ft.
5. 26 yds. long, 13 yds. wide. **6.** 10 yds. 2 ft. long, 5 yds. 1 ft. wide.

280.

1. 10 sq. ft. 9 sq. in. **2.** 30 sq. ft. 54 sq. in. **3.** 20 sq. ft. 90 sq. in.
4. 43 sq. ft. 18 sq. in. **5.** 11 ft.

282.

1. 1·875 sq. cm. **2.** 96·6 sq. cm. **3.** 2 sq. ft.
4. 1 sq. ft. 24 sq. in. **5.** 20 sq. ft. 36 sq. in. **6.** 19 sq. ft. 36 sq. in.
7. 33 sq. yds. 3 sq. ft. **8.** 25 sq. yds. **9.** 11 sq. yds. 5 sq. ft.
10. 28 sq. yds. 8 sq. ft. **11.** 2 ft. 9 in. **12.** 18.

283.

1. 17·5 sq. cm. **2.** 6·75 cm. **3.** 2 sq. ft.
4. 32 sq. ft. 117 sq. in. **5.** 3 sq. ft. 108 sq. in. **6.** 93 sq. ft. 138 sq. in.

285.

1. 10 cm. **2.** 13 cm. **3.** 17 in. **4.** 25 in. **5.** 3·5 in.
6. 4·5 in. **7.** 12 cm. **8.** 24 in. **9.** 3·6 cm. **10.** 2·8 in.

286.

1. 7·07 in. **2.** 19·2 in. **3.** 26 yds.
4. 25 ft. **5.** 27 ft. **6.** £15. 8*s.* 4*d.*

288.

1. 125·664 yds. **2.** 9·4248 yds. **3.** 34·5576 yds. **4.** 10·472 yds.
5. 21·9912 yds. **6.** 13·09 yds. **7.** 10 yds. **8.** 40 yds.
9. 14 yds. **10.** 3·5 yds. **11.** 11·25 yds. **12.** 8·75 yds.

289.

1. 11 ft. **2.** 540 ; 5400. **3.** 5 ft. 10 in. **4.** 4 ft. $4\frac{1}{2}$ in.
5. 3. **6.** (i) 44 ft. per sec. ; (ii) 30 miles per hour.

291.

1. 1256·64 sq. yds. **2.** 7·0686 sq. yds. **3.** 95·0334 sq. yds.
4. 8·7267 sq. yds. **5.** 38·4846 sq. yds. **6.** 13·6354 sq. yds.

292.

1. 1 sq. ft. 10 sq. in. **2.** 9 sq. ft. 90 sq. in. **3.** 273 sq. yds. 7 sq. ft.
5. 5·64 cm. **4.** 21 ft. **6.** 21·46 sq. cm

294.

1. $38\frac{1}{2}$ sq. in. **2.** 1 sq. ft. 10 sq. in. **3.** 9 sq. ft. 90 sq. in.
4. 17 sq. ft. 16 sq. in. **5.** 77 sq. yds. **6.** 13 sq. ft. $14\frac{1}{2}$ sq. in.

295.

1. 50·2656 sq. in. **2.** 21·9912 sq. in. **3.** 34·5576 sq. in.
4. 25·1328 sq. in. **5.** 5·4978 sq. in. **6.** 13·54815 sq. in.

296.

1. 6006 sq. yds. **2.** 616 sq. yds. **3.** $269\frac{1}{2}$ sq. ft.
4. 6*s.* $10\frac{1}{2}$*d.* **5.** £2. 15*s.*

298.

1. 1 sq. yd. 8 sq. ft. 72 sq. in **2.** 1 sq. yd. 7 sq. ft. 72 sq. in.
3. 4 sq. yds. 96 sq. in. **4.** 13 sq. yds. 5 sq. ft. 32 sq. in.
5. 12 sq. yds. 68 sq. in. **6.** 8 sq. yds. 2 sq. ft. 80 sq. in.
7. 660. **8.** 1 acre.

299.

1. 466 c. in. **2.** 57·4 c. cm. **3.** 308 c. in.
4. 2701·93 c. cm. **5.** 1540 c. in. **6.** 931·392 c. cm.
7. 693 c. in. **8.** 625·856 c. cm.

300.

1. 7700 c. ft. **2.** 20 ft. **3.** 3850 gallons. **4.** 2·4 mm.
5. 2887·5 grams. **6.** 6109 yds. **7.** 3830 grams.

301.

1. 143 c. cm. **2.** 264 c. in. **3.** 6600 c. cm.
4. 2 c. ft. 702 c. in. **5.** 32 c. ft. 144 c. in. **6.** 1825 grams.

302.

1. 17*s.* 4*d.* **2.** £1. 14*s.* 7*d.* **3.** £1. 14*s.* 6*d.*
4. £1. 11*s.* 1*d.* **5.** 1*s.* 1*d.* **6.** 14*s.* 7*d.*

303.

	Total *Dr.* entries.			Total *Cr.* entries.			Balance.		
	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
1.	10	–	–	8	4	11	1	15	1
2.	1	15	1	7	4	2	5	–	11
	10	10	–						
3.	5	–	11	7	5	3	12	15	8
	15	–	–						
4.	12	15	8	8	13	–	19	17	8
	15	15	–						

306.

1. £67. 1*s.* **2.** (*a*) 6*s.* (*b*) 9*s.* 6*d.* (*c*) £1. 3*s.* (*d*) £1. 6*s.* (*e*) £2. 8*s.* 9*d.*

3. 15*s.* **4.** (*a*) £46. 17*s.* 6*d.* (*b*) £45. 8*s.* (*c*) £32. 3*s.* 4*d.* (*d*) £21. 12*s.* 8*d.*

5. (*a*) £4. 14*s.* 6*d.*, £8. 2*s.*, £5. 1*s.* 3*d.*, £2. 14*s.*, £3. 0*s.* 9*d.*, £1. 7*s.*, £3. 14*s.* 3*d.*, £1. 13*s.* 9*d.*

(*b*) £7. 5*s.* 10*d.*, £12. 10*s.*, £7. 16*s.* 3*d.*, £4. 3*s.* 4*d.*, £4. 13*s.* 9*d.*, £2. 1*s.* 8*d.*, £5. 14*s.* 7*d.*, £2. 12*s.* 1*d.*

Totals, 15*s.*; £30. 7*s.* 6*d.*; £46. 17*s.* 6*d.*

Revision Papers.

1.

1. 88. 2. 13. 3. 65. 4. 80. 5. 32.

2.

1. 89. 2. 15. 3. Tom 24; Harry 19. 4. 72. 5. 11; rem. 3.

3.

1. 88. 15; 13. 3. 57. 4. 72. 5. 23.

4.

1. 82. 2. 16. 3. 36. 4. 14; 6. 5. 12.

5.

1. 81. 2. 15. 3. 15; 16. 4. 12. 5. 87.

6.

1. 91. 2. 72. 3. 72. 4. 6; 33. 5. 18.

7.

1. 178. 2. 49. 3. 18. 4. 131. 5. 57.

8.

1. 222. 2. 289. 3. 189. 4. 840. 5. 245.

9.

1. 366. 2. 188. 3. 868. 4. 36 times. 5. 45.

10.

1. 604. 2. 689. 3. 655. 4. 976; 24. 5. 13 more.

11.

1. 952. 2. 293. 3. 768. 4. 39; rem. 17. 5. 48.

12.

1. 717. 2. 420. 3. 720; 14, rem. 20. 4. 663. 5. 24; 24.

13.

1. 9814. 2. 9504. 3. 305 times. 4. 7824. 5. 1859.

14.

1. 7125. 2. 8172. 3. 206 times. 4. 3000. 5. 25.

15.

1. 6686. 2. 8. 3. 174. 4. 5688. 5. 1948.

16.

1. 7343. 2. 5669. 3. 456. 4. 40. 5. 41.

17.

1. 16129. 2. 4998. 3. 19536. 4. 195. 5. 259.

18.

1. 18934. 2. 1365. 3. 11034. 4. 496. 5. 29952.

19.

1. 87402. 2. 5361. 3. 45021. 4. 23. 5. 17; 19.

20.

1. 83699. 2. 1836. 3. 99991. 4. 80. 5. 315.

21.

1. 870731. 2. 1382. 3. 15223, rem. 49; 15174.
4. 4221 times. 5. 69; 11.

22.

1. 769047. 2. 29942. 3. 147. 4. 249. 5. Blue 914; Red 799.

23.

1. 752724. 2. 293631. 3. 625 times. 4. $135765 \div 5 = 27153$.
5. 65.

24.

1. 216582. 2. 38551. 3. 2358. 4. 470376. 5. 910; 456.

25.

1. $3.5^2.7^2$. 2. 15; 120. 3. 204300; 123375. 4. 25. 5. 16.

26.

1. $2^4.3^4.5$. 2. 420; 3. 3. 20800; 104000.
4. A to B 25 miles, B to C 15 miles, C to D 35 miles. 5. 50; 25.

27.

1. 42. 2. 990; 13. 3. 98532; 23589; 74943.
4. 14759. 5. 47.

28.

1. 75. 2. 36; 360. 3. 83772. 4. 6355. 5. A 19; B 21; C 25.

29.

1. 14. 2. 2; 1260. 3. 128880. 4. 28618. 5. A 115; B 77; C 88.

30.

1. 27; 9. 2. 51975. 3. 3468. 4. 24. 5. 6 years.

31.

1. 94; 15*s.* 6*d.* 2. Latter by 10. 3. 9. 4. 4*s.* 9*d.*; 5*s.* 3*d.*
5. 11*s.* 4*d.*

32.

1. 89; 17*s.* 3*d.* 2. Latter by 1*s.* 2*d.* 3. 51. 4. 7*s.* 6*d.* 5. 1*s.* 7*d.*

33.

1. 85; 18*s.* 11*d.* 2. 13; 1*s.* 9*d.* 3. 95. 4. 3*s.* 9*d.*
5. Marmalade, 5*d.* dearer.

34.

1. 94; 17*s.* 3*d.* 2. 32; 3*s.* 4*d.* 3. 19. 4. 12*s.* 4*d.* 5. 15.

35.

1. 15; rem. 17. 2. £8. 7*s.* 7½*d.* 3. 2*s.* 10¼*d.*; rem. 8*f.*
4. 192. 5. £1. 18*s.* 1½*d.*; £3. 1*s.* 10½*d.*

36.

1. 12; rem. 48. 2. £4. 15*s.* 3¼*d.* 3. 5*s.* 7¼*d.*; rem. 5½*d.*
4. 186. 5. £2. 15*s.* 5*d.*

37.

1. 14; rem. 30. 2. £2. 7*s.* 3. 2*s.* 2¼*d.*; rem. 5¾*d.*
4. 701. 5. £7. 1*s.* 4*d.*; 18*s.* 8*d.*

38.

1. 32; rem. 7. 2. £1. 5*s.* 1½*d.* 3. 3*s.* 10*d.*; rem. 30*d.*
4. 672. 5. £7. 1*s.* 4*d.*

39.

1. 7*d.* 2. £12. 2*s.* 6*d.* 3. 1558.
4. £75. 19*s.* 10½*d.*; 81, rem. 29. 5. 19*s.* 7*d.*

40.

1. £21. 2. £71. 18*s.* 1½*d.*; 107. 3. A 9*s.* 8¼*d.*; B 14*s.* 2¾*d.*
4. £1. 8*s.* 5. £10. 10*s.*

41.

1. £51. 2*s.* 1*d.* 2. 791. 3. 7*s.* 5¾*d.*
4. £16. 12*s.* 6*d.* 5. £4. 4*s.*

42.

1. 61.
2. A £10. 6*s*. 8*d*.; B £16. 4*s*. 4*d*.
3. £2. 18*s*. 6*d*.
4. £51. 12*s*. 6*d*.
5. £18. 18*s*. 9*d*.

43.

1. 21 minutes to 10.
2. 7 hrs. 20 min. 10 sec.
3. £55. 18*s*. 10½*d*.
4. 1224.
5. £5. 5*s*. 4*d*.

44.

1. 6 days 3 hrs. 52 min. 57 sec.
2. 7.
3. £3. 15*s*. 10*d*.
4. £42. 6*s*. 11¾*d*.
5. £82. 10*s*.

45.

1. 56 tons 7 cwt. 2 qrs. 8 lb.
2. 99456.
3. 72.
4. £1. 3*s*. 9*d*.
5. 16.

46.

1. 1 ton 4 cwt. 2 qrs. 12 lb.
2. £53. 17*s*. 8¾*d*.
3. £353. 18*s*. 10½*d*.
4. 8.
5. 1 ton 15 cwt.

47.

1. 34 gall. 1 pt.
2. 150.
3. £100. 12*s*. 8½*d*.; 1*s*. 5½*d*.
4. £520. 12*s*. 6*d*.
5. £2. 6*s*.

48.

1. 2 gall. 3 qts. 1 pt.
2. 38½ gall.
3. 11 times; 3½*d*.
4. £62. 12*s*. 0½*d*.
5. £56. 2*s*. 6*d*.

49.

1. 12 yds. 1 ft. 1 in.
2. 23 yds. 2 ft. 9 in.
3. £70. 9*s*. 9½*d*.
4. £17. 4*s*. 3*d*.
5. 1066.

50.

1. 2 yds. 2 ft. 3 in.
2. 10 yds. 2 ft. 11 in.
3. 19.
4. £4. 15*s*.
5. 19.

51.

1. 2 miles 1444 yds., *or* 2 miles 6 fur. 124 yds.
2. £63. 11*s*. 7¾*d*.
3. 504.
4. A £55. 6*s*. 8*d*.; B £51. 16*s*. 8*d*.; C £50. 6*s*. 8*d*.
5. 5½*d*.

52.

1. 40 chains 14 yds.
2. 325022; 279.
3. £3. 7*s*. 6*d*.
4. 13200.
5. 10½*d*.; £4. 4*s*.

53

1. 1 sq. yd. 3 sq. ft. 92 sq. in. 2. 29 ft. 3. 1 qr. 10 lb. $1\frac{1}{2}$ oz.
4. £2. 2*s*. 1*d*. 5. 4*s*. 6*d*.

54.

1. 360 sq. ft.; 1216 sq. ft. 2. 23255 sq. in. 3. Each result = 654456.
4. 19 gall. 5. 56 lb.

55.

1. 1 ac. 2. 40. 3. 29 ft.
4. 1*s*. 9*d*. 5. Three receive 381 each; the fourth receives 238.

56.

1. 169 sq. miles 214 ac. 2. 330 yds. 3. 1 ft. 10 in.; 1 ft. 2 in.
4. 5*s*. 6*d*. 5. £20.

57.

1. 15 c. yds. 21 c. ft. 2. 18 c. ft. 3. 19941.
4. £1. 6*s*. 4*d*. 5. 12000.

58.

1. 76 c. yds. 8 c. ft. 1650 c. in. 2. 1200 gall. 3. 15 ft.
4. 9500. 5. 32.

59.

1. (*a*) 20; (*b*) 4; (*c*) 60·5 cm.; 8 dm. 5 cm.
2. (*a*) 65 cm., ·45 cm.; (*b*) ·94 dm., ·065 dm. 3. 21·6 cm.
4. (*a*) 755620; (*b*) £12. 0*s*. 5*d*. 5. 1*s*. 6*d*.

60.

1. (*a*) 125 dm., 20 dm., 5 dm.; (*b*) ·152 dm., ·038 dm., ·005 dm.
2. (*a*) 2·01 dm.; (*b*) 4·45 dm. 3. (*a*) ·15; (*b*) ·05; (*c*) ·015; (*d*) ·005.
4. (*a*) 3 qrs. 22 lb.; (*b*) £8. 7*s*. $4\frac{3}{4}d$. 5. £35. 15*s*.

61.

1. 8·438 Kg.; 6·825 Kg. 2. 54 f. 65 c.; 4 f. 95 c. 3. 2·95 in.
4. (*a*) £1285. 18*s*. $10\frac{1}{2}d$.; (*b*) £1003. 15*s*. $7\frac{1}{2}d$. 5. 280.

62.

1. 11 Km. 20 m.; 4 Km. 110 m. 2. \$217. 11 c.; \$4. 95 c.
3. 2·95 Hl. 4. £1. 11*s*. 6*d*. 5. $857\frac{1}{2}$ lb.

63.

1. (*a*) 1510 m.; (*b*) 151 m.; (*c*) 15·1 m.; (*d*) 1·51 m.
2. (*a*) 3·78; (*b*) ·576; (*c*) 58·08. 3. (*a*) 90·055; (*b*) 3·75784.
4. 46; 22 in. 5. 45 tons.

64.

1. (*a*) 30·87 ; (*b*) 19·8 ; (*c*) 21. **2.** (*a*) 2·052 ; (*b*) ·1848 ; (*c*) ·169.
3. 87·397 ; 69·116. **4.** £17. 18*s.* 11¼*d.* ; £60234. 9*s.* 6*d.* **5.** 19, 11, 8.

65.

1. (*a*) 2·587 ; (*b*) ·478 ; (*c*) ·059 ; (*d*) ·095. **2.** (*a*) ·378 ; (*b*) 3·26 ; (*c*) ·056.
3. £388. 10*s.* ; £1. 4*s.* 10¼*d.* **4.** 84431 sec. ; 231 gall. 1 qt. 1 pt. **5.** 33 f. 39 c.

66.

1. (*a*) 8·3 ; (*b*) 5 ; (*c*) ·64 ; (*d*) ·06. **2.** (*a*) ·09 ; (*b*) 9 ; (*c*) 90 ; (*d*) 4010.
3. (*a*) £1205. 6*s.* 6*d.* ; (*b*) £28892. 11*s.* 3¾*d.* **4.** 37 f. 20 c. **5.** 3*d.*

67.

1. £1646. 5*s.* 3*d.* ; rem. 11*d.* **2.** 7·0136 ; 83·1169.
3. 5253 Kg. **4.** 1636 f. 30 c. **5.** 2 f. 80 c.

68.

1. 12*s.* 7*d.* ; rem. 1*s.* 8*d.* **2.** £20. 12*s.* 4*d.* **3.** 1 f. 30 c.
4. 2·9 ; ·089. **5.** 2734 yds. ; 4 Km. 23 m.

69.

1. (*a*) ·625 ; (*b*) ·375 ; (*c*) 1·85. **2.** ·56875 ; ·625.
3. ·075213 ; 22·5. **4.** ·6294. **5.** 620.

70.

1. ·375 ; ·3875. **2.** ·08125. **3.** ·0008 ; 3470.
4. $29. 70 c. **5.** 10164 francs.

71.

1. (*a*) 10½*d.* ; (*b*) 5*s.* 9*d.* ; (*c*) 10*s.* 10½*d.* **2.** £2. 19*s.* 4½*d.* ; £2. 2*s.* 6*d.*
3. 75800 ; ·00758. **4.** 28. **5.** 8 Km. 47 m.

72.

1. (*a*) 1 qr. 14 lb. ; (*b*) 7 cwt. 3 qrs.
2. 2 tons 1 cwt. 2 qrs. 14 lb. ; ·003125. **3.** 3840 ; ·0000384.
4. (*a*) ·6235 ; (*b*) ·1375. **5.** 7 f. 15 c.

73.

1. (*a*) £0·156 ; (*b*) £0·769. **2.** (*a*) 5*s.* 7½*d.* ; (*b*) £2. 19*s.* 5¼*d.*
3. 17*s.* 10*d.* ; 163 lb. **4.** ·00586 ; 5·86. **5.** 363 f. 90 c. ; $787. 8 c.

74.

1. (*a*) (i) 554·19 sq. cm. ; (ii) ·055419 sq. m. ; (*b*) 48 cm.
2. 2588 sq. yds. ; 23292 sq. ft. **3.** ·0125 ; $\frac{1}{40}$.
4. 7000 lb. ; 3 qrs. 3 lb. 8 oz. **5.** 6600 sq. ft.

75.

1. 28·6 sq. m.; 143 francs. 2. 3 ro. 27 per.; 3 ac. 0 ro. 13 per.
3. 38730; ·003873. 4. 4 miles 5 fur. 193 yds. 1 ft.; 1265 yds. 5. 90.

76.

1. (*a*) 2·3322 c. metres; (*b*) 3·25 m. 2. 2646 litres. 3. 2646 Kg.
4. 25 qts. 5. 151·2 sq. in.

77.

1. (*a*) $\frac{5}{12}$, $\frac{1}{2}$, $\frac{3}{4}$; (*b*) 20, 27, 38; (*c*) $4\frac{1}{2}$, $2\frac{3}{4}$, $1\frac{5}{8}$;
(*d*) $\frac{7}{2}$, $\frac{11}{4}$, $\frac{37}{8}$; (*e*) $\frac{1}{2}$, $\frac{3}{4}$, $1\frac{1}{2}$.
2. (*a*) $\frac{7}{8}$ in., $\frac{7}{16}$ in.; (*b*) $1\frac{1}{4}$ in., $\frac{7}{8}$ in.; (*c*) $1\frac{3}{5}$ in., $1\frac{3}{8}$ in.
3. (*a*) $\frac{5}{12}s.$, $\frac{2}{3}s.$, $\frac{3}{4}s.$; (*b*) $\frac{3}{16}$ in., $\frac{3}{4}$ in., $\frac{7}{8}$ in.
4. 840. 5. $3.5.7$; $2^3.3.7$; 3.7^2; 21.

78.

1. $\frac{65}{12}$; $7\frac{7}{12}$; $\frac{5}{6}$. 2. $\frac{21}{24}$, $\frac{20}{24}$; $\frac{7}{8}$; $1\frac{17}{24}$.
3. 60; $\frac{45}{60}$, $\frac{35}{60}$, $\frac{54}{60}$; $\frac{7}{12}$, $\frac{3}{4}$, $\frac{9}{10}$. 4. $2^2.3^1$; $2^1.3^2.5$; 36; $\frac{9}{20}$.
5. £1.

79.

1. (*a*) $2\frac{3}{4}$; (*b*) $7\frac{5}{8}$. 2. (*a*) $\frac{1}{60}$; (*b*) $\frac{5}{8}$; (*c*) $\frac{3}{10}$.
3. (i) $\frac{3}{8}$ in.; (ii) $2\frac{3}{4}$ in.; (iii) $1\frac{5}{8}$ in. 4. 420. 5. 8*d*.

80.

1. (*a*) $11\frac{1}{4}$; (*b*) $4\frac{11}{12}$; (*c*) $2\frac{27}{40}$. 2. (*a*) $3\frac{3}{8}$; (*b*) $1\frac{5}{6}$.
3. $2^2.7^2$; $2.5^2.7$; $2^2.3.7.11$; 14. 4. $\frac{14}{25}$; $\frac{25}{66}$. 5. 240.

81.

1. (*a*) $\frac{7}{16}$ in., $1\frac{1}{4}$ in., $9\frac{1}{2}$ in.; (*b*) $\frac{1}{4}$ in., $\frac{1}{8}$ in., $\frac{1}{4}$ in.
2. $9\frac{17}{66}$; $3\frac{29}{36}$. 3. (*a*) $\frac{1}{8}$; (*b*) $3\frac{1}{5}$. 4. 12; 7560.
5. £1. 2*s*. 8*d*.

82.

1. (*a*) $\frac{1}{16}$ in., $\frac{3}{8}$ in., $\frac{3}{16}$ in.; (*b*) 16; (*c*) 8; (*d*) 2. 2. $16\frac{1}{2}$ in.; 4.
3. (*a*) $1\frac{1}{5}$; (*b*) $3\frac{35}{36}$. 4. (*a*) $\frac{4}{9}$; (*b*) $16\frac{1}{2}$. 5. 25.

83.

1. $2\frac{2}{5}$; $\frac{7}{10}$. 2. $1\frac{5}{9}$; 8. 3. (*a*) $14\frac{2}{7}$; (*b*) 6.
4. 37; $\frac{12}{19}$. 5. 11 st. 10 lb.

84.

1. 4; $\frac{1}{5}$. 2. Sum $1\frac{37}{60}$; difference $\frac{13}{20}$. 3. $1\frac{1}{5}$; $\frac{1}{8}$.
4. $\frac{2}{11}$; $5\frac{2}{5}$. 5. $\frac{3}{8}$; ·074.

85.

1. $1\frac{7}{18}$; $\frac{17}{90}$; $\frac{1}{2}$. 2. $3\frac{3}{28}$. 3. (*a*) $\frac{1}{7}$; (*b*) $\frac{1}{2}$.
4. ·2006875; ·00019. 5. 12*s*. 6*d*.

86.

1. (*a*) $\frac{1}{5}$; (*b*) 4; (*c*) $1\frac{23}{27}$. 2. $2\frac{5}{9}$. 3. $10\frac{2}{3}$.
4. (*a*) $\frac{2}{3}$; (*b*) $\frac{7}{12}$. 5. 23; 17.

87.

1. $\frac{1}{72}$. 2. $\frac{2}{3}$. 3. (*a*) 8; (*b*) $1\frac{5}{12}$.
4. $5\frac{13}{37}$; $\frac{1}{9}$. 5. 16 Kg. 250 g.

88.

1. (*a*) 17*s*. $2\frac{1}{2}$*d*.; (*b*) 6*s*. $0\frac{1}{2}$*d*. 2. £275. 6*s*. 7*d*.; 5*s*. $4\frac{1}{2}$*d*.
3. $\frac{13}{16}$; $\frac{13}{16}$. 4. (*a*) 4540; (*b*) 2·12. 5. 125 lb.

89.

1. $\frac{3}{32}$; ·09375. 2. $\frac{3}{10}$. 3. ·0832. 4. (*a*) $1\frac{1}{104}$; (*b*) $\frac{3}{4}$.
5. 16 min.

90.

1. £2205. 15*s*. 2. 2.3^4; $2^2.3.7^2$; $2^4.3^2.7$; 2.3; $2^4.3^4.7^2$.
3. ·8109375; $\frac{4}{9}$. 4. (*a*) $\frac{1}{10}$; (*b*) 222·9. 5. 18 miles per hour.

91.

1. £2521. 13*s*. 4*d*. 2. ·0256; 1024.
3. £11. 19*s*. $8\frac{1}{4}$*d*. + £1. 9*s*. $3\frac{3}{4}$*d*. = £13. 9*s*. 4. (*a*) 48; (*b*) ·0018.
5. $1\frac{1}{2}$ min.

92.

1. £61. 17*s*. $9\frac{1}{4}$*d*. 2. 7·24581; 5·067. 3. 4; $\frac{3}{14}$.
4. (*a*) 7 mg.; (*b*) ·03125 gall. 5. 23 metres.

93.

1. £1. 19*s*. $10\frac{1}{2}$*d*. 2. $2\frac{31}{48}$. 3. 270 oz.; ·71875.
4. 205 f. 20 c. 5. 240.

94.

1. £2. 2. $2^4.3^5.5.7$; $3.5.7$. 3. ·05897957; ·00919.
4. 8*s*. 3*d*. 5. 45375 gall.

95.

1. $3\frac{3}{7}$ hrs.; A 16*s*., B 12*s*. 2. $\frac{1}{6}$; £260. 3. 100 days.
4. 1650 yds. 5. 3*s*.

96.

1. £2660. 2. 2*s*.; 9*s*. $1\frac{1}{2}$*d*. 3. £4.
4. £318. 16*s*. 6*d*. 5. £5. 7*s*. 4*d*.

97.

1. $8\frac{1}{6}$ sq. ft.; 8 sq. ft. 24 sq. in. 2. 44970 ac. 3. 2·84375 ac.
4. $2^4 . 3^2 . 7^2$; 84; £56. 5. 243 sq. yds.

98.

1. 17 sq. ft. 48 sq. in.; 15 sq. yds. 1 sq. ft. 63 sq. in. 2. 1276 ft.
3. £176. 9*s*. 9*d*.; 9378 f. 75 c. 4. £20. 3*s*. 4*d*. 5. 96 sheep.

99.

1. $12\frac{1}{2}\frac{6}{1}\frac{3}{6}$ c. ft.; 12 c. ft. 1304 c. in. 2. 3264 c. inches.
3. (*a*) $\frac{2}{9}\frac{3}{0}$; (*b*) 600. 4. 3 in. 5. 14000.

100.

1. 26 litres. 2. 69 sq. ft. 96 sq. in. 3. ·47 metre.
4. 2 ft. 6 in. 5. 35 in.

101.

1. 15 cwt. $7\frac{1}{2}$ lb. 2. 2004 sq. cm. 3. (*a*) 5 men; (*b*) $7\frac{1}{2}$ days.
4. ·0536389; ·504. 5. £11. 18*s*. 8*d*.

Examination Papers.

1.

1. (i) 18358; (ii) £4548. 1*s*. 5*d*.
2. (i) 94·605; (ii) 0·309; (iii) (*a*) $\frac{4}{1}\frac{7}{2}\frac{}{0}$; (*b*) $1\frac{5}{1\,2}$.
3.

1 linear yd. = 3 ft.	1 linear ft. = 12 in.
1 square yd. = 9 sq. ft.	1 square ft. = 144 sq. in.
1 cubic yd. = 27 c. ft.	1 cubic ft. = 1728 c. in.

4. (i) 30*s*.; (ii) 6. 5. A £60, B £90, C £30.
6. (i) 3 sq. ft. 108 sq. in.; (ii) 3 sq. ft. 48 sq. in.
7. 71·8%. 8. 7 ft.

2.

1. (i) 771369; (ii) £53807. 16*s*. 9*d*. 2. (i) 140·0841; (ii) 0·97; (iii) $1\frac{2}{2}\frac{3}{4}$.
3. (i) 12; (ii) 0·172; (iii) 0·00001; (iv) 20.
4. (i) 6; (ii) $7\frac{1}{2}$. 5. $\frac{1}{6}$; £260. 6. 48 cm.
7. 353; 1·09. 8. (i) $A = \pi r^2$ square units; (ii) 5·64 in.

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8·2·83

3.

1. (i) 6093041; (ii) 344827. 1*s.* 4*d.*

2. (i) 21·971; (ii) 0·00919; (iii) (*a*) $\frac{6}{7}$, (*b*) $8\frac{8}{9}$.

3.

1 yd. = 36 in.	1 m. = 100 cm.
1 ton = 2240 lb.	1 Kg. = 1000 g.
1 gall. = 8 pints.	1 Hl. = 100 l.
1 mile = 1760 yds.	1 Km. = 1000 m.

4. $4\frac{1}{2}$ in.

5. P £4. 11*s.*, Q £1. 19*s.*

6. (i) 1800 c. ft.; (ii) 66 c. yds. 18 c. ft.

7. (i) 216; (ii) 240.

8. $\pi = 3{\cdot}1$.

4.

1. (i) 27093; (ii) £3456. 14*s.* $8\frac{1}{4}$*d.*

2. (i) 4·30528; (ii) 7·14; (iii) $3\frac{37}{48}$.

3. (i) 16*s.* 6*d.*; (ii) 14*s.* 2*d.*; (iii) 3*s.* $1\frac{1}{2}$*d.*

4. 3·15 in.

5. (i) 0·8177; (ii) £10. 7*s.* 2*d.*

6. 176 sq. ft.; 124 sq. ft.

7. (i) 25%; (ii) £3. 5*s.*

8. 540; 5400.

5.

1. (i) 1666975; (ii) £7. 11*s.* 3*d.*

2. (i) 0·05897957; (ii) 0·089; (iii) $2\frac{31}{48}$.

3. (i) £13; (ii) £2. 2*s.*; (iii) 15*s.*

4. 2·11 in.

5. (i) 0·669; (ii) £5. 13*s.* 11*d.*

6. 151·2 sq. in.

7. (i) 10*s.*; (ii) 5*s.* $7\frac{1}{2}$*d.*

8. (i) 22 ft./sec.; (ii) 15 m.p.h.

6.

1. (i) £21. 14*s.* 8*d.*; (ii) 3542.

2. (i) 703·8657; (ii) 0·0462; (iii) 6.

3. (i) $1386 = 2 \times 3^2 \times 7 \times 11$, $1764 = 2^2 \times 3^2 \times 7^2$; (ii) 126; (iii) 42.

4. 23·5 grams.

5. (i) $8\frac{1}{2}$ in.; (ii) 0·0083.

6. (*a*) $37\frac{1}{2}$ ac., (*b*) 16; 600.

7. 10%.

8. (i) $V = \frac{1}{4}\pi d^2 h$ cubic units; (ii) 96 c. cm.

7.

1. (i) 1744681; (ii) £4. 1*s.* 2*d.*

2. (i) 65·93451; (ii) 0·000854; (iii) $26\frac{1}{8}$.

3. (i) 0·76; 0·58; (ii) $\frac{17}{20}$, $\frac{1}{16}$.

4. 90.

5. 184 yds.

6. 58 tons 11 cwt. 98 lb.

7. £14. 18*s.* 1*d.*

8. 2888 grams.

8.

1. (i) 5206; (ii) £863. 9*s.* 6*d.*

2. (i) 1·262666; (ii) 2·92, (iii) $\frac{23}{90}$.

3. (i) 1 sq. m. $= 10^4$ sq. cm. $= 10^6$ sq. mm.
1 sq. cm. $= 10^2$ sq. mm.

(ii) 1 c. m. $= 10^3$ c. dm. $= 10^6$ c. cm.
1 c. dm. $= 10^3$ c. cm.

4. 720 grams.

5. (i) 0·69375; (ii) £693. 15*s.*

6. 602.

7. 766 f. 83 c.

8. (i) 226·3 c. cm.; (ii) 4023 grams.

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